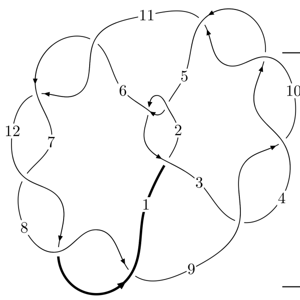
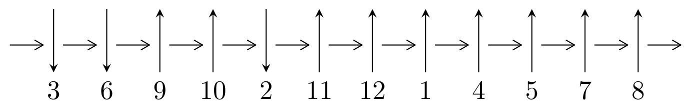
## $12a_{0369}$ $(K12a_{0369})$




A knot diagram[1]

**Linearized knot diagam**




**Solving Sequence**

$$7,11 \xrightarrow[c_{11}]{} 12 \xrightarrow[c_7]{} 8 \xrightarrow[c_{12}]{} 1 \xrightarrow[c_8]{} 9 \xrightarrow[c_6]{} 2,6 \xrightarrow[c_2]{} 3 \xrightarrow[c_5]{} 5 \xrightarrow[c_{10}]{} 10 \xrightarrow[c_4]{} 4 \twoheadrightarrow c_1, c_3, c_9$$

**Ideals for irreducible components[2] of $X_{\mathbf{par}}$**

$$\begin{aligned}
I^u_1 = \langle &-478134295627u^{38} + 149778839704u^{37} + \cdots + 7837471508666b - 1669436053919,\\
&-19258298275u^{38} - 10323980760u^{37} + \cdots + 783747150866a + 8187780738725,\\
&u^{39} - 2u^{38} + \cdots + 10u - 1\rangle\\
I^u_2 = \langle &b + a + u - 1,\ a^2 + 4au - 2a + 3,\ u^2 - u - 1\rangle\\
I^u_3 = \langle &b + u,\ a - 2u - 1,\ u^2 + u - 1\rangle
\end{aligned}$$

* 3 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 45 representations.

[1]The image of knot diagram is generated by the software "**Draw programme**" developed by Andrew Bartholomew(`http://www.layer8.co.uk/maths/draw/index.htm#Running-draw`), where we modified some parts for our purpose(`https://github.com/CATsTAILs/LinksPainter`).

[2]All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

**I.** $\boldsymbol{I^u_1 = \langle -4.78 \times 10^{11}u^{38} + 1.50 \times 10^{11}u^{37} + \cdots + 7.84 \times 10^{11}b - 1.67 \times 10^{12},\ -1.93 \times 10^{10}u^{38} - 1.03 \times 10^{10}u^{37} + \cdots + 7.84 \times 10^{11}a + 8.19 \times 10^{12},\ u^{39} - 2u^{38} + \cdots + 10u - 1\rangle}$

**(i) Arc colorings**

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^3 + 2u \\ u^5 - 3u^3 + u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0.0245721u^{38} + 0.0131726u^{37} + \cdots - 11.4436u - 10.4470 \\ 0.610062u^{38} - 0.191106u^{37} + \cdots - 5.42326u + 2.13007 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -0.819512u^{38} + 1.59433u^{37} + \cdots - 1.16490u - 11.5383 \\ 1.45415u^{38} - 1.77227u^{37} + \cdots - 15.7020u + 3.22140 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 4.72658u^{38} - 5.95208u^{37} + \cdots - 44.4581u + 17.4015 \\ -2.05862u^{38} + 2.32509u^{37} + \cdots + 22.0842u - 3.94672 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -2.46589u^{38} + 3.88914u^{37} + \cdots + 4.27839u - 17.1124 \\ -0.582048u^{38} + 0.868548u^{37} + \cdots + 13.4013u + 0.570488 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.883550u^{38} - 0.877719u^{37} + \cdots - 20.4637u - 8.99829 \\ 1.11038u^{38} - 0.900266u^{37} + \cdots - 10.7236u + 2.76872 \end{pmatrix}$$

**(ii) Obstruction class** $= -1$

**(iii) Cusp Shapes**
$= -\frac{627190064793}{391873575433}u^{38} + \frac{1064333701951}{391873575433}u^{37} + \cdots - \frac{1173686425629}{391873575433}u + \frac{611656068405}{391873575433}$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1$ | $u^{39}+15u^{38}+\cdots+111u+1$ |
| $c_2, c_5$ | $u^{39}+3u^{38}+\cdots+3u+1$ |
| $c_3, c_4, c_9$ $c_{10}$ | $u^{39}-u^{38}+\cdots+4u-4$ |
| $c_6, c_7, c_8$ $c_{11}, c_{12}$ | $u^{39}+2u^{38}+\cdots+10u+1$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1$ | $y^{39} + 25y^{38} + \cdots + 6327y - 1$ |
| $c_2, c_5$ | $y^{39} - 15y^{38} + \cdots + 111y - 1$ |
| $c_3, c_4, c_9$ $c_{10}$ | $y^{39} - 49y^{38} + \cdots + 368y - 16$ |
| $c_6, c_7, c_8$ $c_{11}, c_{12}$ | $y^{39} - 54y^{38} + \cdots + 102y - 1$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_1^u$ | $\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u=-1.020110+0.131184I$<br>$a=-1.009270+0.527233I$<br>$b=\ \ 0.286806-1.184210I$ | $2.30767-2.27298I$ | $10.27248+3.27626I$ |
| $u=-1.020110-0.131184I$<br>$a=-1.009270-0.527233I$<br>$b=\ \ 0.286806+1.184210I$ | $2.30767+2.27298I$ | $10.27248-3.27626I$ |
| $u=-1.04641$<br>$a=\ \ 3.22469$<br>$b=-1.92904$ | $7.09140$ | $13.7060$ |
| $u=\ \ 1.073560+0.109186I$<br>$a=-0.557430-0.305343I$<br>$b=-0.051344-0.282062I$ | $5.55859+1.07065I$ | $15.4669-1.4412I$ |
| $u=\ \ 1.073560-0.109186I$<br>$a=-0.557430+0.305343I$<br>$b=-0.051344+0.282062I$ | $5.55859-1.07065I$ | $15.4669+1.4412I$ |
| $u=\ \ 1.062930+0.284051I$<br>$a=-1.64402-0.62028I$<br>$b=\ \ 0.93533+1.43760I$ | $4.46872+6.29211I$ | $12.7084-7.5348I$ |
| $u=\ \ 1.062930-0.284051I$<br>$a=-1.64402+0.62028I$<br>$b=\ \ 0.93533-1.43760I$ | $4.46872-6.29211I$ | $12.7084+7.5348I$ |
| $u=-1.109570+0.405465I$<br>$a=-2.09516+0.56202I$<br>$b=\ \ 1.43927-1.50328I$ | $12.9852-8.7765I$ | $14.1494+5.9356I$ |
| $u=-1.109570-0.405465I$<br>$a=-2.09516-0.56202I$<br>$b=\ \ 1.43927+1.50328I$ | $12.9852+8.7765I$ | $14.1494-5.9356I$ |
| $u=\ \ 0.801500$<br>$a=\ \ 2.55587$<br>$b=-1.10388$ | $-0.106375$ | $16.7010$ |

| Solutions to $I_1^u$ | $\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 0.476965 + 0.626572I$<br>$a = 1.09357 + 0.93891I$<br>$b = 0.323319 - 1.133120I$ | $9.07010 - 0.79273I$ | $12.42094 - 0.46386I$ |
| $u = 0.476965 - 0.626572I$<br>$a = 1.09357 - 0.93891I$<br>$b = 0.323319 + 1.133120I$ | $9.07010 + 0.79273I$ | $12.42094 + 0.46386I$ |
| $u = 0.783899$<br>$a = 0.595351$<br>$b = -1.19130$ | $5.58284$ | $18.2860$ |
| $u = 0.311989 + 0.683555I$<br>$a = 0.474675 - 0.108825I$<br>$b = -0.86174 - 1.47607I$ | $8.56307 + 5.06640I$ | $11.02566 - 5.09114I$ |
| $u = 0.311989 - 0.683555I$<br>$a = 0.474675 + 0.108825I$<br>$b = -0.86174 + 1.47607I$ | $8.56307 - 5.06640I$ | $11.02566 + 5.09114I$ |
| $u = -1.213320 + 0.300422I$<br>$a = -0.625507 + 1.072000I$<br>$b = -0.039879 - 0.445738I$ | $14.4735 - 2.4041I$ | $15.9891 + 0.I$ |
| $u = -1.213320 - 0.300422I$<br>$a = -0.625507 - 1.072000I$<br>$b = -0.039879 + 0.445738I$ | $14.4735 + 2.4041I$ | $15.9891 + 0.I$ |
| $u = -0.268654 + 0.522744I$<br>$a = 0.312554 + 0.528929I$<br>$b = -0.417052 + 1.216500I$ | $0.30789 - 3.54473I$ | $8.22087 + 8.01132I$ |
| $u = -0.268654 - 0.522744I$<br>$a = 0.312554 - 0.528929I$<br>$b = -0.417052 - 1.216500I$ | $0.30789 + 3.54473I$ | $8.22087 - 8.01132I$ |
| $u = -0.437639 + 0.390994I$<br>$a = 1.41608 - 0.38982I$<br>$b = -0.040008 + 0.660042I$ | $0.876008 + 0.412132I$ | $11.41445 + 0.11588I$ |

| Solutions to $I_1^u$ | $\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u=-0.437639-0.390994I$<br>$a=\ \ 1.41608+0.38982I$<br>$b=-0.040008-0.660042I$ | $0.876008-0.412132I$ | $11.41445-0.11588I$ |
| $u=-0.409866$<br>$a=\ \ 0.871979$<br>$b=-0.319381$ | $0.600340$ | $16.7330$ |
| $u=\ \ 1.59193$<br>$a=-1.99806$<br>$b=\ \ 1.49492$ | $7.67890$ | $0$ |
| $u=-1.60820$<br>$a=-1.60477$<br>$b=\ \ 1.53392$ | $13.8022$ | $0$ |
| $u=\ \ 0.198091+0.297215I$<br>$a=\ \ 0.72132-1.56787I$<br>$b=\ \ 0.125739-0.795889I$ | $-1.45987+0.82313I$ | $-0.72620-2.45968I$ |
| $u=\ \ 0.198091-0.297215I$<br>$a=\ \ 0.72132+1.56787I$<br>$b=\ \ 0.125739+0.795889I$ | $-1.45987-0.82313I$ | $-0.72620+2.45968I$ |
| $u=-1.67666$<br>$a=-2.65337$<br>$b=\ \ 1.89462$ | $8.75767$ | $0$ |
| $u=\ \ 1.73600+0.03045I$<br>$a=\ \ 1.06886+1.00878I$<br>$b=-0.56637-1.46214I$ | $12.24790+2.91315I$ | $0$ |
| $u=\ \ 1.73600-0.03045I$<br>$a=\ \ 1.06886-1.00878I$<br>$b=-0.56637+1.46214I$ | $12.24790-2.91315I$ | $0$ |
| $u=\ \ 1.74422$<br>$a=-3.10810$<br>$b=\ \ 2.23888$ | $17.2035$ | $0$ |

| Solutions to $I_1^u$ | $\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u=-1.74314+0.07213I$<br>$a=\quad 1.88603-1.11976I$<br>$b=-1.29148+1.58672I$ | $14.5204-7.7607I$ | 0 |
| $u=-1.74314-0.07213I$<br>$a=\quad 1.88603+1.11976I$<br>$b=-1.29148-1.58672I$ | $14.5204+7.7607I$ | 0 |
| $u=-1.74624+0.02105I$<br>$a=\quad 0.433151+0.263501I$<br>$b=-0.044228-0.773960I$ | $15.7407-1.5655I$ | 0 |
| $u=-1.74624-0.02105I$<br>$a=\quad 0.433151-0.263501I$<br>$b=-0.044228+0.773960I$ | $15.7407+1.5655I$ | 0 |
| $u=\quad 1.75579+0.11113I$<br>$a=\quad 2.51996+0.96207I$<br>$b=-1.85527-1.45871I$ | $-16.2979+10.9754I$ | 0 |
| $u=\quad 1.75579-0.11113I$<br>$a=\quad 2.51996-0.96207I$<br>$b=-1.85527+1.45871I$ | $-16.2979-10.9754I$ | 0 |
| $u=\quad 1.78103+0.07205I$<br>$a=\quad 0.318355+0.567328I$<br>$b=\quad 0.0141751+0.0635167I$ | $-14.1664+4.0147I$ | 0 |
| $u=\quad 1.78103-0.07205I$<br>$a=\quad 0.318355-0.567328I$<br>$b=\quad 0.0141751-0.0635167I$ | $-14.1664-4.0147I$ | 0 |
| $u=\quad 0.104256$<br>$a=-11.5099$<br>$b=\quad 1.46674$ | 3.32525 | 1.86210 |

**II.** $\boldsymbol{I_2^u = \langle b + a + u - 1,\ a^2 + 4au - 2a + 3,\ u^2 - u - 1\rangle}$

**(i) Arc colorings**

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ -u-1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ -u-1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} a \\ -a-u+1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} a+u \\ -a-2u+1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -a-u \\ a+2u-1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} au-a-u+2 \\ 2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u+1 \\ -a-2u+1 \end{pmatrix}$$

**(ii) Obstruction class** $= 1$

**(iii) Cusp Shapes** $= 12$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_5$ | $(u-1)^4$ |
| $c_2$ | $(u+1)^4$ |
| $c_3, c_4, c_9$<br>$c_{10}$ | $(u^2-2)^2$ |
| $c_6, c_7, c_8$ | $(u^2+u-1)^2$ |
| $c_{11}, c_{12}$ | $(u^2-u-1)^2$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_2, c_5$ | $(y-1)^4$ |
| $c_3, c_4, c_9$<br>$c_{10}$ | $(y-2)^4$ |
| $c_6, c_7, c_8$<br>$c_{11}, c_{12}$ | $(y^2-3y+1)^2$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_2^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -0.618034$<br>$a = 0.821854$<br>$b = 0.796180$ | 4.27683 | 12.0000 |
| $u = -0.618034$<br>$a = 3.65028$<br>$b = -2.03225$ | 4.27683 | 12.0000 |
| $u = 1.61803$<br>$a = -0.821854$<br>$b = 0.203820$ | 12.1725 | 12.0000 |
| $u = 1.61803$<br>$a = -3.65028$<br>$b = 3.03225$ | 12.1725 | 12.0000 |

### III. $I_3^u = \langle b+u,\ a-2u-1,\ u^2+u-1\rangle$

**(i) Arc colorings**

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_{12} = \begin{pmatrix} 1 \\ u-1 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} u \\ -u+1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} u \\ -u \end{pmatrix}$$
$$a_9 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_2 = \begin{pmatrix} 2u+1 \\ -u \end{pmatrix}$$
$$a_6 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_3 = \begin{pmatrix} u+1 \\ 0 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} u+1 \\ 0 \end{pmatrix}$$
$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_4 = \begin{pmatrix} u+1 \\ 0 \end{pmatrix}$$

**(ii) Obstruction class** $= 1$

**(iii) Cusp Shapes** $= 2$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_2$ | $(u-1)^2$ |
| $c_3, c_4, c_9$<br>$c_{10}$ | $u^2$ |
| $c_5$ | $(u+1)^2$ |
| $c_6, c_7, c_8$ | $u^2 - u - 1$ |
| $c_{11}, c_{12}$ | $u^2 + u - 1$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_2, c_5$ | $(y-1)^2$ |
| $c_3, c_4, c_9$ $c_{10}$ | $y^2$ |
| $c_6, c_7, c_8$ $c_{11}, c_{12}$ | $y^2 - 3y + 1$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_3^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 0.618034$<br>$a = 2.23607$<br>$b = -0.618034$ | $-0.657974$ | 2.00000 |
| $u = -1.61803$<br>$a = -2.23607$<br>$b = 1.61803$ | 7.23771 | 2.00000 |

## IV. u-Polynomials

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1$ | $((u-1)^6)(u^{39}+15u^{38}+\cdots+111u+1)$ |
| $c_2$ | $((u-1)^2)(u+1)^4(u^{39}+3u^{38}+\cdots+3u+1)$ |
| $c_3, c_4, c_9$ $c_{10}$ | $u^2(u^2-2)^2(u^{39}-u^{38}+\cdots+4u-4)$ |
| $c_5$ | $((u-1)^4)(u+1)^2(u^{39}+3u^{38}+\cdots+3u+1)$ |
| $c_6, c_7, c_8$ | $(u^2-u-1)(u^2+u-1)^2(u^{39}+2u^{38}+\cdots+10u+1)$ |
| $c_{11}, c_{12}$ | $((u^2-u-1)^2)(u^2+u-1)(u^{39}+2u^{38}+\cdots+10u+1)$ |

## V. Riley Polynomials

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1$ | $((y-1)^6)(y^{39}+25y^{38}+\cdots+6327y-1)$ |
| $c_2, c_5$ | $((y-1)^6)(y^{39}-15y^{38}+\cdots+111y-1)$ |
| $c_3, c_4, c_9$ $c_{10}$ | $y^2(y-2)^4(y^{39}-49y^{38}+\cdots+368y-16)$ |
| $c_6, c_7, c_8$ $c_{11}, c_{12}$ | $((y^2-3y+1)^3)(y^{39}-54y^{38}+\cdots+102y-1)$ |