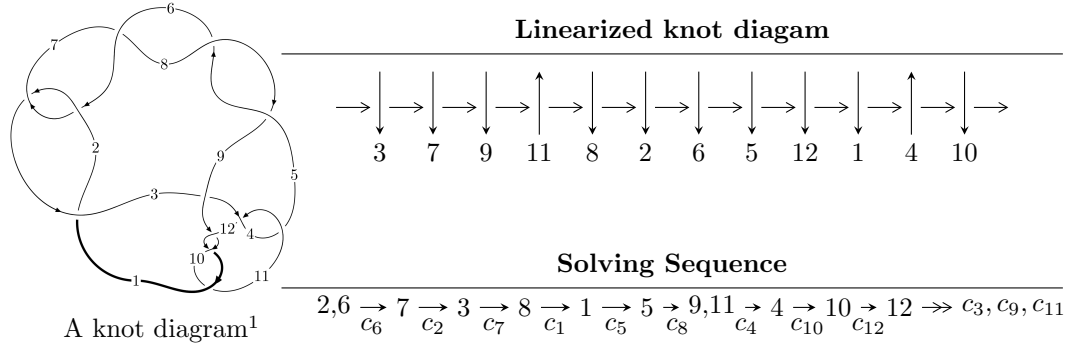


12a<sub>0591</sub> (K12a<sub>0591</sub>)



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle 2u^{58} - 4u^{57} + \dots + b + 2, -u^{58} + 3u^{57} + \dots + a - 3, u^{59} - 2u^{58} + \dots + 6u^2 - 1 \rangle$$

$$I_2^u = \langle -u^2 + b, u^4 + a - u + 1, u^5 + u^4 - u^2 + u + 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 64 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle 2u^{58} - 4u^{57} + \dots + b + 2, -u^{58} + 3u^{57} + \dots + a - 3, u^{59} - 2u^{58} + \dots + 6u^2 - 1 \rangle$$

I.

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^2 + 1 \\ u^2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^3 \\ u^5 - u^3 + u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^4 - u^2 + 1 \\ -u^4 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^6 + u^4 - 2u^2 + 1 \\ u^6 + u^2 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u^{58} - 3u^{57} + \dots + 8u + 3 \\ -2u^{58} + 4u^{57} + \dots - 2u - 2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^{15} - 2u^{13} + 6u^{11} - 8u^9 + 10u^7 - 8u^5 + 4u^3 - 2u \\ -u^{15} + u^{13} - 4u^{11} + 3u^9 - 4u^7 + 2u^5 - 2u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^{58} - 2u^{57} + \dots + 7u + 2 \\ -u^{58} + 2u^{57} + \dots - 2u - 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} u^{58} - u^{57} + \dots + 6u + 2 \\ -u^{56} + 6u^{54} + \dots - 5u^2 - u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $6u^{58} - 9u^{57} + \dots + 11u - 4$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_5, c_7$ $c_8$	$u^{59} + 12u^{58} + \dots + 12u + 1$
$c_2, c_6$	$u^{59} - 2u^{58} + \dots + 6u^2 - 1$
$c_3$	$u^{59} - 2u^{58} + \dots + 770u - 769$
$c_4, c_{11}$	$u^{59} - u^{58} + \dots - 160u - 32$
$c_9, c_{10}, c_{12}$	$u^{59} - 6u^{58} + \dots + 6u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_5, c_7$ $c_8$	$y^{59} + 72y^{58} + \dots + 524y^2 - 1$
$c_2, c_6$	$y^{59} - 12y^{58} + \dots + 12y - 1$
$c_3$	$y^{59} - 12y^{58} + \dots + 8725844y - 591361$
$c_4, c_{11}$	$y^{59} + 33y^{58} + \dots - 512y - 1024$
$c_9, c_{10}, c_{12}$	$y^{59} - 56y^{58} + \dots + 38y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.927503 + 0.362332I$		
$a = 1.043690 + 0.616352I$	$-8.68686 - 0.29443I$	$-15.0428 - 2.5911I$
$b = -0.037342 - 1.128440I$		
$u = -0.927503 - 0.362332I$		
$a = 1.043690 - 0.616352I$	$-8.68686 + 0.29443I$	$-15.0428 + 2.5911I$
$b = -0.037342 + 1.128440I$		
$u = 0.888453 + 0.481598I$		
$a = -0.283319 - 1.162560I$	$-3.19795 - 4.45742I$	$-10.97215 + 6.34811I$
$b = -0.195891 + 1.201930I$		
$u = 0.888453 - 0.481598I$		
$a = -0.283319 + 1.162560I$	$-3.19795 + 4.45742I$	$-10.97215 - 6.34811I$
$b = -0.195891 - 1.201930I$		
$u = 0.809645 + 0.552812I$		
$a = 0.191552 + 0.505222I$	$1.77330 - 2.83842I$	$-2.03651 + 4.98214I$
$b = 0.275906 - 0.423947I$		
$u = 0.809645 - 0.552812I$		
$a = 0.191552 - 0.505222I$	$1.77330 + 2.83842I$	$-2.03651 - 4.98214I$
$b = 0.275906 + 0.423947I$		
$u = -0.851357 + 0.451422I$		
$a = -0.989465 - 0.251556I$	$-1.68118 + 2.03005I$	$-12.04374 - 3.74651I$
$b = -0.627413 + 0.684041I$		
$u = -0.851357 - 0.451422I$		
$a = -0.989465 + 0.251556I$	$-1.68118 - 2.03005I$	$-12.04374 + 3.74651I$
$b = -0.627413 - 0.684041I$		
$u = -0.901916 + 0.511776I$		
$a = 0.912140 + 0.147817I$	$-0.66335 + 6.70936I$	$-8.90418 - 9.33390I$
$b = 0.752079 + 0.229651I$		
$u = -0.901916 - 0.511776I$		
$a = 0.912140 - 0.147817I$	$-0.66335 - 6.70936I$	$-8.90418 + 9.33390I$
$b = 0.752079 - 0.229651I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.949474 + 0.086160I$ $a = -0.813004 - 0.661879I$ $b = 0.916859 - 0.826254I$	$-10.19780 - 5.55174I$	$-17.2725 + 4.5392I$
$u = 0.949474 - 0.086160I$ $a = -0.813004 + 0.661879I$ $b = 0.916859 + 0.826254I$	$-10.19780 + 5.55174I$	$-17.2725 - 4.5392I$
$u = 0.774062 + 0.735471I$ $a = -0.213712 - 0.409713I$ $b = -0.406362 - 0.573958I$	$-1.80101 - 2.68324I$	$-9.89712 + 3.37596I$
$u = 0.774062 - 0.735471I$ $a = -0.213712 + 0.409713I$ $b = -0.406362 + 0.573958I$	$-1.80101 + 2.68324I$	$-9.89712 - 3.37596I$
$u = -0.947787 + 0.525245I$ $a = -0.916835 + 0.152219I$ $b = -0.313188 - 0.887130I$	$-6.72517 + 10.58920I$	$-11.8410 - 8.8623I$
$u = -0.947787 - 0.525245I$ $a = -0.916835 - 0.152219I$ $b = -0.313188 + 0.887130I$	$-6.72517 - 10.58920I$	$-11.8410 + 8.8623I$
$u = 0.666174 + 0.595869I$ $a = -0.674977 + 0.451724I$ $b = 0.190354 + 0.211599I$	$2.23740 - 1.50357I$	$-0.61420 + 3.68395I$
$u = 0.666174 - 0.595869I$ $a = -0.674977 - 0.451724I$ $b = 0.190354 - 0.211599I$	$2.23740 + 1.50357I$	$-0.61420 - 3.68395I$
$u = -0.885590$ $a = -0.363107$ $b = 1.58758$	$-5.76666$	$-16.7450$
$u = 0.880462 + 0.048539I$ $a = 0.431969 + 1.195720I$ $b = -0.367625 + 0.381368I$	$-3.70562 - 2.28031I$	$-15.9939 + 4.5234I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.880462 - 0.048539I$ $a = 0.431969 - 1.195720I$ $b = -0.367625 - 0.381368I$	$-3.70562 + 2.28031I$	$-15.9939 - 4.5234I$
$u = -0.475499 + 0.709477I$ $a = 1.064150 + 0.054307I$ $b = 0.424369 + 0.494166I$	$-5.19700 - 6.02928I$	$-8.35124 + 3.27137I$
$u = -0.475499 - 0.709477I$ $a = 1.064150 - 0.054307I$ $b = 0.424369 - 0.494166I$	$-5.19700 + 6.02928I$	$-8.35124 - 3.27137I$
$u = -0.506438 + 0.625369I$ $a = -0.304837 - 0.259828I$ $b = -0.535812 - 0.823997I$	$0.58647 - 2.42870I$	$-4.78642 + 3.26862I$
$u = -0.506438 - 0.625369I$ $a = -0.304837 + 0.259828I$ $b = -0.535812 + 0.823997I$	$0.58647 + 2.42870I$	$-4.78642 - 3.26862I$
$u = 0.911469 + 0.817194I$ $a = -0.136623 - 0.058652I$ $b = -0.338056 - 0.728570I$	$-1.78315 - 3.05813I$	0
$u = 0.911469 - 0.817194I$ $a = -0.136623 + 0.058652I$ $b = -0.338056 + 0.728570I$	$-1.78315 + 3.05813I$	0
$u = -0.629873 + 0.451932I$ $a = -0.591616 + 0.250732I$ $b = -0.012775 + 1.272100I$	$-0.85526 + 1.48403I$	$-10.53137 - 2.98098I$
$u = -0.629873 - 0.451932I$ $a = -0.591616 - 0.250732I$ $b = -0.012775 - 1.272100I$	$-0.85526 - 1.48403I$	$-10.53137 + 2.98098I$
$u = -0.891610 + 0.896465I$ $a = -2.53033 + 1.42185I$ $b = 3.38449 + 1.43468I$	$5.57557 - 0.69509I$	0

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.891610 - 0.896465I$ $a = -2.53033 - 1.42185I$ $b = 3.38449 - 1.43468I$	$5.57557 + 0.69509I$	0
$u = 0.901021 + 0.888004I$ $a = -2.01271 - 0.18754I$ $b = 2.23237 - 1.26339I$	$6.70724 - 2.05355I$	0
$u = 0.901021 - 0.888004I$ $a = -2.01271 + 0.18754I$ $b = 2.23237 + 1.26339I$	$6.70724 + 2.05355I$	0
$u = 0.891289 + 0.906965I$ $a = 2.27708 + 1.72286I$ $b = -3.68686 + 0.28774I$	$8.48613 + 2.97023I$	0
$u = 0.891289 - 0.906965I$ $a = 2.27708 - 1.72286I$ $b = -3.68686 - 0.28774I$	$8.48613 - 2.97023I$	0
$u = 0.881355 + 0.919934I$ $a = -1.79426 - 2.72825I$ $b = 4.07557 + 0.84147I$	$2.80078 + 7.27092I$	0
$u = 0.881355 - 0.919934I$ $a = -1.79426 + 2.72825I$ $b = 4.07557 - 0.84147I$	$2.80078 - 7.27092I$	0
$u = 0.475104 + 0.546567I$ $a = 1.88094 - 0.67202I$ $b = -0.427394 - 0.066566I$	$-1.93785 + 0.48814I$	$-6.57596 + 0.51918I$
$u = 0.475104 - 0.546567I$ $a = 1.88094 + 0.67202I$ $b = -0.427394 + 0.066566I$	$-1.93785 - 0.48814I$	$-6.57596 - 0.51918I$
$u = 0.944066 + 0.868730I$ $a = 0.67891 + 2.04086I$ $b = -2.18407 - 0.79948I$	$6.56960 - 4.44376I$	0



Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.944066 - 0.868730I$ $a = 0.67891 - 2.04086I$ $b = -2.18407 + 0.79948I$	$6.56960 + 4.44376I$	0
$u = -0.917197 + 0.901529I$ $a = 1.95344 - 0.68617I$ $b = -2.33755 - 1.31220I$	$10.83880 + 1.97378I$	0
$u = -0.917197 - 0.901529I$ $a = 1.95344 + 0.68617I$ $b = -2.33755 + 1.31220I$	$10.83880 - 1.97378I$	0
$u = -0.954974 + 0.867777I$ $a = 1.60650 - 2.23842I$ $b = -3.94977 + 0.62849I$	$5.37300 + 7.21470I$	0
$u = -0.954974 - 0.867777I$ $a = 1.60650 + 2.23842I$ $b = -3.94977 - 0.62849I$	$5.37300 - 7.21470I$	0
$u = -0.943653 + 0.888657I$ $a = -0.91907 + 1.76170I$ $b = 2.60787 - 0.77120I$	$10.75340 + 4.63087I$	0
$u = -0.943653 - 0.888657I$ $a = -0.91907 - 1.76170I$ $b = 2.60787 + 0.77120I$	$10.75340 - 4.63087I$	0
$u = 0.961984 + 0.873239I$ $a = -2.16430 - 2.05346I$ $b = 3.70194 - 0.40138I$	$8.25862 - 9.54022I$	0
$u = 0.961984 - 0.873239I$ $a = -2.16430 + 2.05346I$ $b = 3.70194 + 0.40138I$	$8.25862 + 9.54022I$	0
$u = -0.700512$ $a = -0.0483338$ $b = -0.474525$	-1.03527	-9.24070

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.975652 + 0.873157I$ $a = 2.99487 + 1.42749I$ $b = -4.09520 + 1.68332I$	$2.49582 - 13.87990I$	0
$u = 0.975652 - 0.873157I$ $a = 2.99487 - 1.42749I$ $b = -4.09520 - 1.68332I$	$2.49582 + 13.87990I$	0
$u = -0.941988 + 0.917429I$ $a = -1.46830 - 1.66648I$ $b = -0.16686 + 2.84797I$	$8.35499 + 3.37505I$	0
$u = -0.941988 - 0.917429I$ $a = -1.46830 + 1.66648I$ $b = -0.16686 - 2.84797I$	$8.35499 - 3.37505I$	0
$u = -0.199230 + 0.628329I$ $a = 1.03443 + 1.42099I$ $b = -0.437533 - 0.953757I$	$-6.42147 + 3.73715I$	$-8.91804 - 3.23014I$
$u = -0.199230 - 0.628329I$ $a = 1.03443 - 1.42099I$ $b = -0.437533 + 0.953757I$	$-6.42147 - 3.73715I$	$-8.91804 + 3.23014I$
$u = -0.227842 + 0.411818I$ $a = -1.082040 - 0.120959I$ $b = 0.324797 + 0.691474I$	$-0.462772 + 1.214880I$	$-5.21396 - 5.22189I$
$u = -0.227842 - 0.411818I$ $a = -1.082040 + 0.120959I$ $b = 0.324797 - 0.691474I$	$-0.462772 - 1.214880I$	$-5.21396 + 5.22189I$
$u = 0.399421$ $a = 3.06288$ $b = -0.646854$	$-2.12944$	$-0.328760$

$$\text{II. } I_2^u = \langle -u^2 + b, u^4 + a - u + 1, u^5 + u^4 - u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^2 + 1 \\ u^2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^3 \\ -u^4 - u^3 + u^2 - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^4 - u^2 + 1 \\ -u^4 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^3 \\ u^4 + u^3 - u^2 + 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u^4 + u - 1 \\ u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^4 - u^2 + 1 \\ -u^4 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^4 - u^3 + u - 1 \\ u^4 + u^3 + 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^4 + u - 1 \\ u^2 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-8u^4 - u^3 + 5u^2 + 7u - 18$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_3, c_5$	$u^5 - u^4 + 4u^3 - 3u^2 + 3u - 1$
$c_2$	$u^5 - u^4 + u^2 + u - 1$
$c_4, c_{11}$	$u^5$
$c_6$	$u^5 + u^4 - u^2 + u + 1$
$c_7, c_8$	$u^5 + u^4 + 4u^3 + 3u^2 + 3u + 1$
$c_9, c_{10}$	$(u - 1)^5$
$c_{12}$	$(u + 1)^5$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_3, c_5$ $c_7, c_8$	$y^5 + 7y^4 + 16y^3 + 13y^2 + 3y - 1$
$c_2, c_6$	$y^5 - y^4 + 4y^3 - 3y^2 + 3y - 1$
$c_4, c_{11}$	$y^5$
$c_9, c_{10}, c_{12}$	$(y - 1)^5$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.758138 + 0.584034I$	$0.17487 - 2.21397I$	$-5.34777 + 4.39723I$
$a = 0.487744 + 0.170166I$		
$b = 0.233677 + 0.885557I$		
$u = 0.758138 - 0.584034I$	$0.17487 + 2.21397I$	$-5.34777 - 4.39723I$
$a = 0.487744 - 0.170166I$		
$b = 0.233677 - 0.885557I$		
$u = -0.935538 + 0.903908I$	$9.31336 + 3.33174I$	$-2.87586 - 2.18947I$
$a = 0.92150 + 1.10071I$		
$b = 0.05818 - 1.69128I$		
$u = -0.935538 - 0.903908I$	$9.31336 - 3.33174I$	$-2.87586 + 2.18947I$
$a = 0.92150 - 1.10071I$		
$b = 0.05818 + 1.69128I$		
$u = -0.645200$	$-2.52712$	$-21.5530$
$a = -1.81849$		
$b = 0.416284$		

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_5$	$(u^5 - u^4 + 4u^3 - 3u^2 + 3u - 1)(u^{59} + 12u^{58} + \dots + 12u + 1)$
$c_2$	$(u^5 - u^4 + u^2 + u - 1)(u^{59} - 2u^{58} + \dots + 6u^2 - 1)$
$c_3$	$(u^5 - u^4 + 4u^3 - 3u^2 + 3u - 1)(u^{59} - 2u^{58} + \dots + 770u - 769)$
$c_4, c_{11}$	$u^5(u^{59} - u^{58} + \dots - 160u - 32)$
$c_6$	$(u^5 + u^4 - u^2 + u + 1)(u^{59} - 2u^{58} + \dots + 6u^2 - 1)$
$c_7, c_8$	$(u^5 + u^4 + 4u^3 + 3u^2 + 3u + 1)(u^{59} + 12u^{58} + \dots + 12u + 1)$
$c_9, c_{10}$	$((u - 1)^5)(u^{59} - 6u^{58} + \dots + 6u - 1)$
$c_{12}$	$((u + 1)^5)(u^{59} - 6u^{58} + \dots + 6u - 1)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_5, c_7$ $c_8$	$(y^5 + 7y^4 + 16y^3 + 13y^2 + 3y - 1)(y^{59} + 72y^{58} + \dots + 524y^2 - 1)$
$c_2, c_6$	$(y^5 - y^4 + 4y^3 - 3y^2 + 3y - 1)(y^{59} - 12y^{58} + \dots + 12y - 1)$
$c_3$	$(y^5 + 7y^4 + 16y^3 + 13y^2 + 3y - 1)$ $\cdot (y^{59} - 12y^{58} + \dots + 8725844y - 591361)$
$c_4, c_{11}$	$y^5(y^{59} + 33y^{58} + \dots - 512y - 1024)$
$c_9, c_{10}, c_{12}$	$((y - 1)^5)(y^{59} - 56y^{58} + \dots + 38y - 1)$