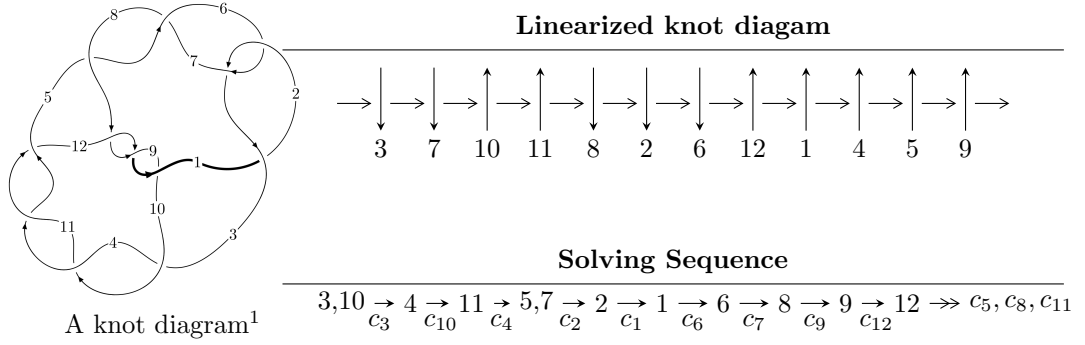


12a<sub>0641</sub> (K12a<sub>0641</sub>)



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle 9.53641 \times 10^{31} u^{44} + 1.44960 \times 10^{32} u^{43} + \dots + 2.63520 \times 10^{32} b + 1.44504 \times 10^{33}, \\ - 9.27193 \times 10^{31} u^{44} - 1.41274 \times 10^{32} u^{43} + \dots + 8.78400 \times 10^{31} a - 1.73256 \times 10^{33}, u^{45} + u^{44} + \dots + 24u \rangle$$

$$I_2^u = \langle 2a^2 - 2au + 5b + 4a + 1, 4a^3 + 4a^2 + 2au + 6a + 7u + 8, u^2 - 2 \rangle$$

$$I_1^v = \langle a, b - v + 1, v^3 - 2v^2 + v - 1 \rangle$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 54 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.

$$I_1^u = \langle 9.54 \times 10^{31} u^{44} + 1.45 \times 10^{32} u^{43} + \dots + 2.64 \times 10^{32} b + 1.45 \times 10^{33}, -9.27 \times 10^{31} u^{44} - 1.41 \times 10^{32} u^{43} + \dots + 8.78 \times 10^{31} a - 1.73 \times 10^{33}, u^{45} + u^{44} + \dots + 24u - 8 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1.05555u^{44} + 1.60831u^{43} + \dots - 15.1462u + 19.7241 \\ -0.361886u^{44} - 0.550090u^{43} + \dots + 5.74395u - 5.48362 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0.930140u^{44} + 1.39744u^{43} + \dots - 12.0451u + 15.7417 \\ -0.248255u^{44} - 0.328783u^{43} + \dots + 4.52710u - 4.10460 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0.681885u^{44} + 1.06866u^{43} + \dots - 7.51802u + 11.6371 \\ -0.248255u^{44} - 0.328783u^{43} + \dots + 4.52710u - 4.10460 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0.373014u^{44} + 0.560645u^{43} + \dots - 4.80141u + 7.41655 \\ -0.155977u^{44} - 0.247269u^{43} + \dots + 2.81501u - 2.80465 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0.927559u^{44} + 1.36065u^{43} + \dots - 12.7981u + 16.1079 \\ -0.251660u^{44} - 0.353713u^{43} + \dots + 2.22074u - 3.09854 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -0.580385u^{44} - 0.859482u^{43} + \dots + 7.87529u - 11.2692 \\ 0.248579u^{44} + 0.372464u^{43} + \dots - 2.39760u + 3.61110 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^3 + 2u \\ u^5 - 3u^3 + u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $0.877910u^{44} + 1.34478u^{43} + \dots - 1.37878u + 23.3266$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_5, c_7$	$u^{45} + 10u^{44} + \dots + 36u + 1$
$c_2, c_6$	$u^{45} - 2u^{44} + \dots - 8u + 1$
$c_3, c_4, c_{10}$ $c_{11}$	$u^{45} + u^{44} + \dots + 24u - 8$
$c_8, c_9, c_{12}$	$u^{45} - 4u^{44} + \dots + 69u + 23$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_5, c_7$	$y^{45} + 54y^{44} + \dots + 420y - 1$
$c_2, c_6$	$y^{45} - 10y^{44} + \dots + 36y - 1$
$c_3, c_4, c_{10}$ $c_{11}$	$y^{45} - 59y^{44} + \dots + 576y - 64$
$c_8, c_9, c_{12}$	$y^{45} - 52y^{44} + \dots + 5635y - 529$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.926963 + 0.277106I$		
$a = -0.09767 - 2.10895I$	$6.86189 - 5.36898I$	$8.49956 + 6.61909I$
$b = 0.912053 + 0.832400I$		
$u = -0.926963 - 0.277106I$		
$a = -0.09767 + 2.10895I$	$6.86189 + 5.36898I$	$8.49956 - 6.61909I$
$b = 0.912053 - 0.832400I$		
$u = 0.941670 + 0.205124I$		
$a = -1.03689 - 1.25346I$	$6.92054 - 0.83550I$	$8.82992 - 1.28747I$
$b = 0.891051 + 0.833627I$		
$u = 0.941670 - 0.205124I$		
$a = -1.03689 + 1.25346I$	$6.92054 + 0.83550I$	$8.82992 + 1.28747I$
$b = 0.891051 - 0.833627I$		
$u = 0.023683 + 0.934020I$		
$a = -0.543202 + 0.459579I$	$11.64400 - 3.29131I$	$8.32249 + 2.32768I$
$b = -0.929182 - 0.891513I$		
$u = 0.023683 - 0.934020I$		
$a = -0.543202 - 0.459579I$	$11.64400 + 3.29131I$	$8.32249 - 2.32768I$
$b = -0.929182 + 0.891513I$		
$u = 0.846105 + 0.378892I$		
$a = -1.22689 - 1.16468I$	$5.23947 + 5.34835I$	$7.95547 - 7.29971I$
$b = -0.954555 + 0.513401I$		
$u = 0.846105 - 0.378892I$		
$a = -1.22689 + 1.16468I$	$5.23947 - 5.34835I$	$7.95547 + 7.29971I$
$b = -0.954555 - 0.513401I$		
$u = -1.039870 + 0.275140I$		
$a = 0.288564 - 0.168386I$	$6.82014 - 0.93085I$	$11.70994 + 0.69834I$
$b = -0.452638 + 0.664033I$		
$u = -1.039870 - 0.275140I$		
$a = 0.288564 + 0.168386I$	$6.82014 + 0.93085I$	$11.70994 - 0.69834I$
$b = -0.452638 - 0.664033I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.966296 + 0.676286I$ $a = 0.61696 + 1.64433I$ $b = 0.973987 - 0.877117I$	$14.4733 + 8.6224I$	0
$u = 0.966296 - 0.676286I$ $a = 0.61696 - 1.64433I$ $b = 0.973987 + 0.877117I$	$14.4733 - 8.6224I$	0
$u = -1.009550 + 0.656162I$ $a = -0.644421 + 0.462034I$ $b = 0.886665 - 0.920023I$	$14.7553 - 1.9983I$	0
$u = -1.009550 - 0.656162I$ $a = -0.644421 - 0.462034I$ $b = 0.886665 + 0.920023I$	$14.7553 + 1.9983I$	0
$u = 0.790311$ $a = 1.71120$ $b = 0.978813$	2.25051	5.01630
$u = -0.585865 + 0.317408I$ $a = -0.78884 + 1.75623I$ $b = -0.805345 - 0.377442I$	$-0.16293 - 3.03748I$	$3.07976 + 9.52409I$
$u = -0.585865 - 0.317408I$ $a = -0.78884 - 1.75623I$ $b = -0.805345 + 0.377442I$	$-0.16293 + 3.03748I$	$3.07976 - 9.52409I$
$u = -1.37682$ $a = -0.746455$ $b = -0.218102$	6.50761	0
$u = 0.131062 + 0.586653I$ $a = 1.24295 + 0.87480I$ $b = 0.739277 + 0.534527I$	$3.06266 - 2.04715I$	$4.85763 + 2.56353I$
$u = 0.131062 - 0.586653I$ $a = 1.24295 - 0.87480I$ $b = 0.739277 - 0.534527I$	$3.06266 + 2.04715I$	$4.85763 - 2.56353I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.46705$ $a = -0.579352$ $b = -0.870479$	4.13430	0
$u = 0.519844 + 0.033701I$ $a = 0.914229 + 0.857221I$ $b = -0.417105 - 0.301945I$	$0.905318 + 0.103320I$	$10.97546 - 0.25661I$
$u = 0.519844 - 0.033701I$ $a = 0.914229 - 0.857221I$ $b = -0.417105 + 0.301945I$	$0.905318 - 0.103320I$	$10.97546 + 0.25661I$
$u = 1.54974 + 0.04851I$ $a = 0.02206 + 1.51683I$ $b = 0.884978 - 0.530874I$	$7.04735 + 4.23377I$	0
$u = 1.54974 - 0.04851I$ $a = 0.02206 - 1.51683I$ $b = 0.884978 + 0.530874I$	$7.04735 - 4.23377I$	0
$u = -1.55369 + 0.03708I$ $a = -0.48020 - 1.33204I$ $b = 0.578245 + 0.609458I$	$8.03802 - 0.09902I$	0
$u = -1.55369 - 0.03708I$ $a = -0.48020 + 1.33204I$ $b = 0.578245 - 0.609458I$	$8.03802 + 0.09902I$	0
$u = 0.001231 + 0.423155I$ $a = -0.622949 - 0.533249I$ $b = -0.883296 + 0.781171I$	$3.96310 + 2.93834I$	$-0.43677 - 3.36885I$
$u = 0.001231 - 0.423155I$ $a = -0.622949 + 0.533249I$ $b = -0.883296 - 0.781171I$	$3.96310 - 2.93834I$	$-0.43677 + 3.36885I$
$u = -0.208519 + 0.357357I$ $a = 1.310670 - 0.182130I$ $b = 0.753961 - 0.173670I$	$-1.242180 + 0.549575I$	$-4.50079 - 0.85707I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.208519 - 0.357357I$ $a = 1.310670 + 0.182130I$ $b = 0.753961 + 0.173670I$	$-1.242180 - 0.549575I$	$-4.50079 + 0.85707I$
$u = 0.378626$ $a = 2.77102$ $b = -0.520904$	0.938924	14.9190
$u = -1.67565$ $a = -0.581072$ $b = -1.13016$	11.0611	0
$u = -1.68688 + 0.09794I$ $a = 0.373353 - 1.216300I$ $b = 1.087160 + 0.497430I$	$14.1681 - 7.1706I$	0
$u = -1.68688 - 0.09794I$ $a = 0.373353 + 1.216300I$ $b = 1.087160 - 0.497430I$	$14.1681 + 7.1706I$	0
$u = 1.70518 + 0.07345I$ $a = 0.48413 - 1.95464I$ $b = -0.959387 + 0.884389I$	$16.2110 + 6.7605I$	0
$u = 1.70518 - 0.07345I$ $a = 0.48413 + 1.95464I$ $b = -0.959387 - 0.884389I$	$16.2110 - 6.7605I$	0
$u = -1.70989 + 0.04785I$ $a = 0.92219 - 1.55641I$ $b = -0.903637 + 0.911957I$	$16.3914 - 0.1365I$	0
$u = -1.70989 - 0.04785I$ $a = 0.92219 + 1.55641I$ $b = -0.903637 - 0.911957I$	$16.3914 + 0.1365I$	0
$u = 1.72676 + 0.05968I$ $a = -0.360921 - 0.923368I$ $b = 0.343715 + 0.875406I$	$16.6790 + 2.2118I$	0



Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.72676 - 0.05968I$ $a = -0.360921 + 0.923368I$ $b = 0.343715 - 0.875406I$	$16.6790 - 2.2118I$	0
$u = -1.72249 + 0.20334I$ $a = -0.05274 + 1.88408I$ $b = -1.020410 - 0.868678I$	$-15.7769 - 12.1806I$	0
$u = -1.72249 - 0.20334I$ $a = -0.05274 - 1.88408I$ $b = -1.020410 + 0.868678I$	$-15.7769 + 12.1806I$	0
$u = 1.74036 + 0.18867I$ $a = 0.891945 + 1.020600I$ $b = -0.845114 - 0.962160I$	$-15.2052 + 5.4595I$	0
$u = 1.74036 - 0.18867I$ $a = 0.891945 - 1.020600I$ $b = -0.845114 + 0.962160I$	$-15.2052 - 5.4595I$	0

$$\text{II. } I_2^u = \langle 2a^2 - 2au + 5b + 4a + 1, 4a^3 + 4a^2 + 2au + 6a + 7u + 8, u^2 - 2 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u \\ -u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} a \\ -\frac{2}{5}a^2 + \frac{2}{5}au - \frac{4}{5}a - \frac{1}{5} \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -\frac{2}{5}a^2u - \frac{1}{5}au + \cdots - \frac{2}{5}a + \frac{1}{5} \\ \frac{2}{5}a^2u + \frac{1}{5}au + \cdots + \frac{2}{5}a - \frac{1}{5} \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -\frac{1}{2}u \\ \frac{2}{5}a^2u + \frac{1}{5}au + \cdots + \frac{2}{5}a - \frac{1}{5} \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -\frac{1}{5}a^2u + \frac{1}{5}au + \cdots + \frac{1}{5}a - \frac{4}{5} \\ \frac{2}{5}a^2u + \frac{3}{5}au + \cdots - \frac{2}{5}a + \frac{3}{5} \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -\frac{1}{2}u \\ \frac{2}{5}a^2u + \frac{1}{5}au + \cdots + \frac{2}{5}a - \frac{1}{5} \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -\frac{1}{2}u \\ \frac{2}{5}a^2u + \frac{1}{5}au + \cdots + \frac{2}{5}a - \frac{1}{5} \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

(ii) Obstruction class = 1

$$\text{(iii) Cusp Shapes} = -\frac{8}{5}a^2 + \frac{8}{5}au - \frac{16}{5}a + \frac{36}{5}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_5$	$(u^3 - u^2 + 2u - 1)^2$
$c_2$	$(u^3 - u^2 + 1)^2$
$c_3, c_4, c_{10}$ $c_{11}$	$(u^2 - 2)^3$
$c_6$	$(u^3 + u^2 - 1)^2$
$c_7$	$(u^3 + u^2 + 2u + 1)^2$
$c_8, c_9$	$(u - 1)^6$
$c_{12}$	$(u + 1)^6$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_5, c_7$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_2, c_6$	$(y^3 - y^2 + 2y - 1)^2$
$c_3, c_4, c_{10}$ $c_{11}$	$(y - 2)^6$
$c_8, c_9, c_{12}$	$(y - 1)^6$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.41421$ $a = -1.50656$ $b = -0.754878$	5.46628	4.98050
$u = 1.41421$ $a = 0.25328 + 1.70473I$ $b = 0.877439 - 0.744862I$	$9.60386 + 2.82812I$	$11.50976 - 2.97945I$
$u = 1.41421$ $a = 0.25328 - 1.70473I$ $b = 0.877439 + 0.744862I$	$9.60386 - 2.82812I$	$11.50976 + 2.97945I$
$u = -1.41421$ $a = -0.683438 + 0.909550I$ $b = 0.877439 - 0.744862I$	$9.60386 + 2.82812I$	$11.50976 - 2.97945I$
$u = -1.41421$ $a = -0.683438 - 0.909550I$ $b = 0.877439 + 0.744862I$	$9.60386 - 2.82812I$	$11.50976 + 2.97945I$
$u = -1.41421$ $a = 0.366877$ $b = -0.754878$	5.46628	4.98050

$$\text{III. } I_1^v = \langle a, b - v + 1, v^3 - 2v^2 + v - 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ v - 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ -v^2 + 2v - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -v^2 + 2v \\ -v^2 + 2v - 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} v - 1 \\ v^2 - v - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} v^2 - 2v \\ v^2 - 2v + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} v^2 - v \\ v^2 - 2v + 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-4v^2 - 2v + 10$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_5$	$u^3 - u^2 + 2u - 1$
$c_2$	$u^3 + u^2 - 1$
$c_3, c_4, c_{10}$ $c_{11}$	$u^3$
$c_6$	$u^3 - u^2 + 1$
$c_7$	$u^3 + u^2 + 2u + 1$
$c_8, c_9$	$(u + 1)^3$
$c_{12}$	$(u - 1)^3$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_5, c_7$	$y^3 + 3y^2 + 2y - 1$
$c_2, c_6$	$y^3 - y^2 + 2y - 1$
$c_3, c_4, c_{10}$ $c_{11}$	$y^3$
$c_8, c_9, c_{12}$	$(y - 1)^3$



(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^v$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$v = 0.122561 + 0.744862I$ $a = 0$ $b = -0.877439 + 0.744862I$	$4.66906 + 2.82812I$	$11.91407 - 2.22005I$
$v = 0.122561 - 0.744862I$ $a = 0$ $b = -0.877439 - 0.744862I$	$4.66906 - 2.82812I$	$11.91407 + 2.22005I$
$v = 1.75488$ $a = 0$ $b = 0.754878$	$0.531480$	$-5.82810$

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_5$	$((u^3 - u^2 + 2u - 1)^3)(u^{45} + 10u^{44} + \dots + 36u + 1)$
$c_2$	$((u^3 - u^2 + 1)^2)(u^3 + u^2 - 1)(u^{45} - 2u^{44} + \dots - 8u + 1)$
$c_3, c_4, c_{10}$ $c_{11}$	$u^3(u^2 - 2)^3(u^{45} + u^{44} + \dots + 24u - 8)$
$c_6$	$(u^3 - u^2 + 1)(u^3 + u^2 - 1)^2(u^{45} - 2u^{44} + \dots - 8u + 1)$
$c_7$	$((u^3 + u^2 + 2u + 1)^3)(u^{45} + 10u^{44} + \dots + 36u + 1)$
$c_8, c_9$	$((u - 1)^6)(u + 1)^3(u^{45} - 4u^{44} + \dots + 69u + 23)$
$c_{12}$	$((u - 1)^3)(u + 1)^6(u^{45} - 4u^{44} + \dots + 69u + 23)$

### V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_5, c_7$	$((y^3 + 3y^2 + 2y - 1)^3)(y^{45} + 54y^{44} + \dots + 420y - 1)$
$c_2, c_6$	$((y^3 - y^2 + 2y - 1)^3)(y^{45} - 10y^{44} + \dots + 36y - 1)$
$c_3, c_4, c_{10}$ $c_{11}$	$y^3(y - 2)^6(y^{45} - 59y^{44} + \dots + 576y - 64)$
$c_8, c_9, c_{12}$	$((y - 1)^9)(y^{45} - 52y^{44} + \dots + 5635y - 529)$