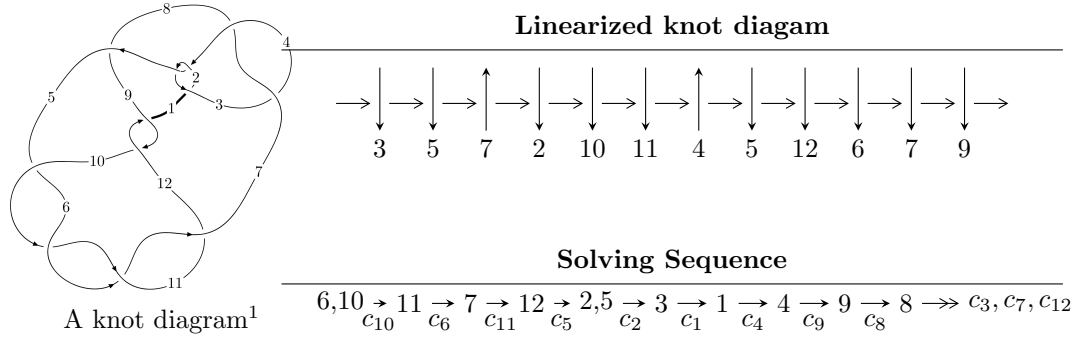


12n<sub>0168</sub> (K12n<sub>0168</sub>)



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -u^{20} + 10u^{18} + \dots + b + 2u, u^{23} - u^{22} + \dots + a - 1, u^{24} - 2u^{23} + \dots - 8u^2 + 1 \rangle$$

$$I_2^u = \langle u^4 - 2u^2 + b + 2u, -u^5 + 3u^3 + a + 1, u^6 + u^5 - 3u^4 - 2u^3 + 2u^2 - u - 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 30 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle -u^{20} + 10u^{18} + \dots + b + 2u, u^{23} - u^{22} + \dots + a - 1, u^{24} - 2u^{23} + \dots - 8u^2 + 1 \rangle \quad \text{I.}$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u^{23} + u^{22} + \dots + 5u + 1 \\ u^{20} - 10u^{18} + \dots - 6u^2 - 2u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -2u^{23} + u^{22} + \dots + 6u + 2 \\ -u^{23} + 12u^{21} + \dots - u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^{10} - 5u^8 + 8u^6 - 5u^4 + 3u^2 - 1 \\ u^{12} - 6u^{10} + 12u^8 - 8u^6 + u^4 - 2u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^{19} + 10u^{17} + \dots + 5u + 1 \\ u^{23} - u^{22} + \dots - 2u - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^6 + 3u^4 - 2u^2 + 1 \\ -u^8 + 4u^6 - 4u^4 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^{10} + 5u^8 - 8u^6 + 5u^4 - 3u^2 + 1 \\ -u^{10} + 4u^8 - 3u^6 - 2u^4 - u^2 \end{pmatrix}$$

(ii) Obstruction class = -1

$$\begin{aligned} \text{(iii) Cusp Shapes} &= -4u^{23} + 7u^{22} + 45u^{21} - 76u^{20} - 213u^{19} + 331u^{18} + 563u^{17} - \\ &727u^{16} - 961u^{15} + 840u^{14} + 1203u^{13} - 550u^{12} - 1159u^{11} + 373u^{10} + 788u^9 - 284u^8 - \\ &414u^7 + 79u^6 + 239u^5 - 78u^4 - 63u^3 + 41u^2 + 22u - 6 \end{aligned}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{24} + u^{23} + \dots + 33u + 1$
$c_2, c_4$	$u^{24} - 7u^{23} + \dots - 9u + 1$
$c_3, c_7$	$u^{24} - u^{23} + \dots + 64u + 64$
$c_5, c_6, c_{10}$ $c_{11}$	$u^{24} - 2u^{23} + \dots - 8u^2 + 1$
$c_8$	$u^{24} + 2u^{23} + \dots + 7956u + 4721$
$c_9, c_{12}$	$u^{24} - 2u^{23} + \dots + 2u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{24} + 51y^{23} + \dots - 789y + 1$
$c_2, c_4$	$y^{24} - y^{23} + \dots - 33y + 1$
$c_3, c_7$	$y^{24} - 39y^{23} + \dots - 65536y + 4096$
$c_5, c_6, c_{10}$ $c_{11}$	$y^{24} - 26y^{23} + \dots - 16y + 1$
$c_8$	$y^{24} + 94y^{23} + \dots - 805269180y + 22287841$
$c_9, c_{12}$	$y^{24} + 34y^{23} + \dots - 16y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.546136 + 0.704998I$ $a = 0.218919 + 0.545111I$ $b = -1.58637 + 0.30753I$	$13.8157 + 6.6372I$	$-5.51662 - 4.82221I$
$u = -0.546136 - 0.704998I$ $a = 0.218919 - 0.545111I$ $b = -1.58637 - 0.30753I$	$13.8157 - 6.6372I$	$-5.51662 + 4.82221I$
$u = -0.490937 + 0.721217I$ $a = -0.42824 + 1.49833I$ $b = -0.020820 - 0.317218I$	$13.98230 - 1.87133I$	$-5.11127 - 0.61495I$
$u = -0.490937 - 0.721217I$ $a = -0.42824 - 1.49833I$ $b = -0.020820 + 0.317218I$	$13.98230 + 1.87133I$	$-5.11127 + 0.61495I$
$u = 0.581256 + 0.532393I$ $a = 0.124238 + 0.261688I$ $b = 1.180950 + 0.505513I$	$2.81700 - 3.48253I$	$-5.09473 + 5.87430I$
$u = 0.581256 - 0.532393I$ $a = 0.124238 - 0.261688I$ $b = 1.180950 - 0.505513I$	$2.81700 + 3.48253I$	$-5.09473 - 5.87430I$
$u = 0.371044 + 0.593072I$ $a = 0.614735 + 1.078340I$ $b = -0.138383 + 0.111177I$	$3.46440 - 0.36969I$	$-3.22680 + 1.74990I$
$u = 0.371044 - 0.593072I$ $a = 0.614735 - 1.078340I$ $b = -0.138383 - 0.111177I$	$3.46440 + 0.36969I$	$-3.22680 - 1.74990I$
$u = -0.560055$ $a = -0.358547$ $b = -0.698358$	$-0.920303$	$-10.4000$
$u = -1.44707 + 0.16024I$ $a = -0.1025080 + 0.0199673I$ $b = -0.075688 - 0.613179I$	$-2.38092 + 3.00213I$	$-7.53165 - 2.42924I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.44707 - 0.16024I$ $a = -0.1025080 - 0.0199673I$ $b = -0.075688 + 0.613179I$	$-2.38092 - 3.00213I$	$-7.53165 + 2.42924I$
$u = 1.46661 + 0.06456I$ $a = -1.18298 + 1.35435I$ $b = -1.81615 + 1.04191I$	$-6.76285 - 2.21001I$	$-11.76840 + 2.54579I$
$u = 1.46661 - 0.06456I$ $a = -1.18298 - 1.35435I$ $b = -1.81615 - 1.04191I$	$-6.76285 + 2.21001I$	$-11.76840 - 2.54579I$
$u = -1.47571$ $a = 3.35878$ $b = 4.21798$	$-8.10337$	$-9.82550$
$u = 1.50201 + 0.24602I$ $a = 0.543477 - 0.750165I$ $b = 1.09231 - 1.65332I$	$7.50585 - 1.64558I$	$-8.27015 + 0.73963I$
$u = 1.50201 - 0.24602I$ $a = 0.543477 + 0.750165I$ $b = 1.09231 + 1.65332I$	$7.50585 + 1.64558I$	$-8.27015 - 0.73963I$
$u = -0.344587 + 0.302443I$ $a = -1.119750 + 0.097114I$ $b = 0.366639 + 0.663744I$	$-0.814955 + 1.024630I$	$-8.91038 - 6.27818I$
$u = -0.344587 - 0.302443I$ $a = -1.119750 - 0.097114I$ $b = 0.366639 - 0.663744I$	$-0.814955 - 1.024630I$	$-8.91038 + 6.27818I$
$u = 1.53592 + 0.23426I$ $a = 2.64852 - 0.49160I$ $b = 3.50928 + 0.08824I$	$6.98523 - 10.07590I$	$-8.82304 + 4.75008I$
$u = 1.53592 - 0.23426I$ $a = 2.64852 + 0.49160I$ $b = 3.50928 - 0.08824I$	$6.98523 + 10.07590I$	$-8.82304 - 4.75008I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.55778$ $a = 2.20275$ $b = 2.47794$	-8.18010	-9.41090
$u = -1.55100 + 0.14681I$ $a = -2.45672 - 0.03517I$ $b = -2.88742 + 0.30740I$	$-4.30018 + 5.91154I$	$-8.53274 - 5.50143I$
$u = -1.55100 - 0.14681I$ $a = -2.45672 + 0.03517I$ $b = -2.88742 - 0.30740I$	$-4.30018 - 5.91154I$	$-8.53274 + 5.50143I$
$u = 0.323756$ $a = 2.07762$ $b = -1.24627$	-2.07138	3.20840

**II.**

$$I_2^u = \langle u^4 - 2u^2 + b + 2u, -u^5 + 3u^3 + a + 1, u^6 + u^5 - 3u^4 - 2u^3 + 2u^2 - u - 1 \rangle$$

**(i) Arc colorings**

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^5 - 3u^3 - 1 \\ -u^4 + 2u^2 - 2u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^5 - 3u^3 + u - 1 \\ -u^4 + 2u^2 - u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u \\ -u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^5 - 3u^3 + u - 1 \\ -u^4 + 2u^2 - u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^5 - 2u^3 - u \\ u^5 - 3u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

**(ii) Obstruction class = 1**

$$\text{(iii) Cusp Shapes} = 3u^5 + u^4 - 14u^3 - u^2 + 14u - 18$$



(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$(u - 1)^6$
$c_3, c_7$	$u^6$
$c_4$	$(u + 1)^6$
$c_5, c_6$	$u^6 - u^5 - 3u^4 + 2u^3 + 2u^2 + u - 1$
$c_8, c_{12}$	$u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1$
$c_9$	$u^6 + u^5 + 3u^4 + 2u^3 + 2u^2 + u - 1$
$c_{10}, c_{11}$	$u^6 + u^5 - 3u^4 - 2u^3 + 2u^2 - u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y - 1)^6$
$c_3, c_7$	$y^6$
$c_5, c_6, c_{10}$ $c_{11}$	$y^6 - 7y^5 + 17y^4 - 16y^3 + 6y^2 - 5y + 1$
$c_8, c_9, c_{12}$	$y^6 + 5y^5 + 9y^4 + 4y^3 - 6y^2 - 5y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.493180 + 0.575288I$ $a = 0.011399 - 0.918055I$ $b = -0.847526 + 0.083869I$	$1.31531 - 1.97241I$	$-6.43930 + 3.48596I$
$u = 0.493180 - 0.575288I$ $a = 0.011399 + 0.918055I$ $b = -0.847526 - 0.083869I$	$1.31531 + 1.97241I$	$-6.43930 - 3.48596I$
$u = -0.483672$ $a = -0.687021$ $b = 1.38049$	$-2.38379$	$-23.4460$
$u = -1.52087 + 0.16310I$ $a = 1.98288 + 0.88048I$ $b = 2.63293 + 0.95019I$	$-5.34051 + 4.59213I$	$-10.66600 - 2.48468I$
$u = -1.52087 - 0.16310I$ $a = 1.98288 - 0.88048I$ $b = 2.63293 - 0.95019I$	$-5.34051 - 4.59213I$	$-10.66600 + 2.48468I$
$u = 1.53904$ $a = -3.30155$ $b = -3.95130$	$-9.30502$	$-18.3430$

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u - 1)^6)(u^{24} + u^{23} + \dots + 33u + 1)$
$c_2$	$((u - 1)^6)(u^{24} - 7u^{23} + \dots - 9u + 1)$
$c_3, c_7$	$u^6(u^{24} - u^{23} + \dots + 64u + 64)$
$c_4$	$((u + 1)^6)(u^{24} - 7u^{23} + \dots - 9u + 1)$
$c_5, c_6$	$(u^6 - u^5 - 3u^4 + 2u^3 + 2u^2 + u - 1)(u^{24} - 2u^{23} + \dots - 8u^2 + 1)$
$c_8$	$(u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1)(u^{24} + 2u^{23} + \dots + 7956u + 4721)$
$c_9$	$(u^6 + u^5 + 3u^4 + 2u^3 + 2u^2 + u - 1)(u^{24} - 2u^{23} + \dots + 2u + 1)$
$c_{10}, c_{11}$	$(u^6 + u^5 - 3u^4 - 2u^3 + 2u^2 - u - 1)(u^{24} - 2u^{23} + \dots - 8u^2 + 1)$
$c_{12}$	$(u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1)(u^{24} - 2u^{23} + \dots + 2u + 1)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y - 1)^6)(y^{24} + 51y^{23} + \dots - 789y + 1)$
$c_2, c_4$	$((y - 1)^6)(y^{24} - y^{23} + \dots - 33y + 1)$
$c_3, c_7$	$y^6(y^{24} - 39y^{23} + \dots - 65536y + 4096)$
$c_5, c_6, c_{10}$ $c_{11}$	$(y^6 - 7y^5 + \dots - 5y + 1)(y^{24} - 26y^{23} + \dots - 16y + 1)$
$c_8$	$(y^6 + 5y^5 + 9y^4 + 4y^3 - 6y^2 - 5y + 1)$ $\cdot (y^{24} + 94y^{23} + \dots - 805269180y + 22287841)$
$c_9, c_{12}$	$(y^6 + 5y^5 + \dots - 5y + 1)(y^{24} + 34y^{23} + \dots - 16y + 1)$