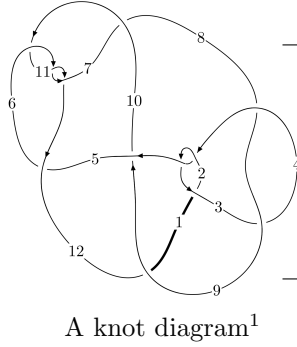
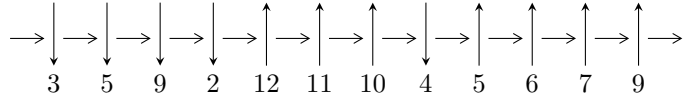


12n<sub>0241</sub> (K12n<sub>0241</sub>)



**Linearized knot diagram**



**Solving Sequence**

$$3,9 \xrightarrow{c_3} 4,5 \xrightarrow{c_9} 10 \xrightarrow{c_2} 2 \xrightarrow{c_1} 1 \xrightarrow{c_8} 8 \xrightarrow{c_7} 7 \xrightarrow{c_{12}} 12 \xrightarrow{c_5} 6 \xrightarrow{c_{11}} 11 \rightsquigarrow c_4, c_6, c_{10}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -1.03137 \times 10^{66} u^{36} + 6.98280 \times 10^{65} u^{35} + \dots + 2.95081 \times 10^{68} b - 1.06268 \times 10^{68}, \\ 3.77823 \times 10^{67} u^{36} - 2.87660 \times 10^{67} u^{35} + \dots + 2.36065 \times 10^{69} a + 2.08922 \times 10^{69}, \\ u^{37} - u^{36} + \dots + 128u + 256 \rangle$$

$$I_1^v = \langle a, b - 1, v^8 - v^7 - v^6 + 2v^5 + v^4 - 2v^3 + 2v - 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 45 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle -1.03 \times 10^{66} u^{36} + 6.98 \times 10^{65} u^{35} + \dots + 2.95 \times 10^{68} b - 1.06 \times 10^{68}, 3.78 \times 10^{67} u^{36} - 2.88 \times 10^{67} u^{35} + \dots + 2.36 \times 10^{69} a + 2.09 \times 10^{69}, u^{37} - u^{36} + \dots + 128u + 256 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -0.0160050u^{36} + 0.0121856u^{35} + \dots - 8.06518u - 0.885020 \\ 0.00349520u^{36} - 0.00236640u^{35} + \dots + 1.79889u + 0.360133 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -0.0133199u^{36} + 0.0418348u^{35} + \dots + 0.507839u + 12.6265 \\ 0.00381941u^{36} - 0.0132824u^{35} + \dots + 0.836374u - 4.09729 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.0160050u^{36} + 0.0121856u^{35} + \dots - 8.06518u - 0.885020 \\ 0.00596782u^{36} - 0.00520306u^{35} + \dots + 2.78728u + 0.617635 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -0.0100372u^{36} + 0.00698258u^{35} + \dots - 5.27790u - 0.267385 \\ 0.00596782u^{36} - 0.00520306u^{35} + \dots + 2.78728u + 0.617635 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -0.0154489u^{36} + 0.0231599u^{35} + \dots - 4.79943u + 5.30651 \\ -0.000302840u^{36} + 0.00840424u^{35} + \dots + 2.35144u + 2.54004 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -0.0100372u^{36} + 0.00698258u^{35} + \dots - 5.27790u - 0.267385 \\ 0.0110751u^{36} - 0.00839995u^{35} + \dots + 5.74781u + 1.39963 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -0.0440610u^{36} + 0.0321408u^{35} + \dots - 23.2326u - 5.35412 \\ 0.0221974u^{36} - 0.0147892u^{35} + \dots + 12.6696u + 3.48847 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -0.0148066u^{36} + 0.00829427u^{35} + \dots - 11.4914u - 1.08615 \\ -0.00186082u^{36} + 0.00671544u^{35} + \dots + 3.81289u + 3.67874 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $-0.0751690u^{36} + 0.0952069u^{35} + \dots - 23.9143u + 7.79267$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{37} + 5u^{36} + \dots - 5u + 1$
$c_2, c_4$	$u^{37} - 9u^{36} + \dots - 7u + 1$
$c_3, c_8$	$u^{37} - u^{36} + \dots + 128u + 256$
$c_5, c_7$	$u^{37} + 6u^{36} + \dots + 35u + 5$
$c_6, c_{10}, c_{11}$	$u^{37} - 2u^{36} + \dots + 3u + 1$
$c_9$	$u^{37} + 2u^{36} + \dots + 3u + 1$
$c_{12}$	$u^{37} - 8u^{36} + \dots + 2082719u - 154033$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{37} + 63y^{36} + \dots - y - 1$
$c_2, c_4$	$y^{37} - 5y^{36} + \dots - 5y - 1$
$c_3, c_8$	$y^{37} + 51y^{36} + \dots - 475136y - 65536$
$c_5, c_7$	$y^{37} + 16y^{36} + \dots + 735y - 25$
$c_6, c_{10}, c_{11}$	$y^{37} - 32y^{36} + \dots + 27y - 1$
$c_9$	$y^{37} - 48y^{36} + \dots + 27y - 1$
$c_{12}$	$y^{37} - 84y^{36} + \dots + 1848165923715y - 23726165089$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.878368 + 0.497461I$		
$a = 0.626443 + 0.343621I$	$-0.66892 + 2.27408I$	$1.99793 - 4.76069I$
$b = 0.227102 - 0.673098I$		
$u = -0.878368 - 0.497461I$		
$a = 0.626443 - 0.343621I$	$-0.66892 - 2.27408I$	$1.99793 + 4.76069I$
$b = 0.227102 + 0.673098I$		
$u = -0.756357 + 0.569885I$		
$a = 0.466384 - 0.081501I$	$0.61705 - 4.41139I$	$2.87485 + 3.50022I$
$b = 1.080620 + 0.363590I$		
$u = -0.756357 - 0.569885I$		
$a = 0.466384 + 0.081501I$	$0.61705 + 4.41139I$	$2.87485 - 3.50022I$
$b = 1.080620 - 0.363590I$		
$u = -0.491761 + 0.952107I$		
$a = 0.695040 + 0.682945I$	$6.82750 - 1.30632I$	$10.37535 + 1.68426I$
$b = -0.267990 - 0.719272I$		
$u = -0.491761 - 0.952107I$		
$a = 0.695040 - 0.682945I$	$6.82750 + 1.30632I$	$10.37535 - 1.68426I$
$b = -0.267990 + 0.719272I$		
$u = 0.886724 + 0.087993I$		
$a = 0.586081 - 0.184026I$	$2.29551 + 0.52381I$	$5.03075 + 0.94889I$
$b = 0.553122 + 0.487673I$		
$u = 0.886724 - 0.087993I$		
$a = 0.586081 + 0.184026I$	$2.29551 - 0.52381I$	$5.03075 - 0.94889I$
$b = 0.553122 - 0.487673I$		
$u = -0.366615 + 0.752594I$		
$a = 0.451654 - 0.034286I$	$0.11703 + 2.14687I$	$4.21868 - 4.14096I$
$b = 1.201400 + 0.167112I$		
$u = -0.366615 - 0.752594I$		
$a = 0.451654 + 0.034286I$	$0.11703 - 2.14687I$	$4.21868 + 4.14096I$
$b = 1.201400 - 0.167112I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.040136 + 0.830344I$ $a = 1.19957 - 1.19424I$ $b = -0.581327 + 0.416814I$	$1.73939 + 7.30260I$	$6.40314 - 7.32057I$
$u = 0.040136 - 0.830344I$ $a = 1.19957 + 1.19424I$ $b = -0.581327 - 0.416814I$	$1.73939 - 7.30260I$	$6.40314 + 7.32057I$
$u = 0.563224 + 0.605095I$ $a = 0.466156 + 0.056547I$ $b = 1.114100 - 0.256450I$	$-3.61155 + 1.05730I$	$-2.65114 + 0.19593I$
$u = 0.563224 - 0.605095I$ $a = 0.466156 - 0.056547I$ $b = 1.114100 + 0.256450I$	$-3.61155 - 1.05730I$	$-2.65114 - 0.19593I$
$u = 0.981996 + 0.654437I$ $a = 0.572407 - 0.396675I$ $b = 0.180218 + 0.817886I$	$4.10036 - 5.71187I$	$7.23097 + 5.43034I$
$u = 0.981996 - 0.654437I$ $a = 0.572407 + 0.396675I$ $b = 0.180218 - 0.817886I$	$4.10036 + 5.71187I$	$7.23097 - 5.43034I$
$u = -0.066568 + 0.771395I$ $a = 1.33592 + 1.03307I$ $b = -0.531570 - 0.362239I$	$-2.71712 - 3.28265I$	$1.87410 + 4.96573I$
$u = -0.066568 - 0.771395I$ $a = 1.33592 - 1.03307I$ $b = -0.531570 + 0.362239I$	$-2.71712 + 3.28265I$	$1.87410 - 4.96573I$
$u = 0.419103 + 0.595983I$ $a = 0.910599 - 0.447643I$ $b = -0.115558 + 0.434784I$	$1.109940 + 0.478757I$	$7.85612 - 2.59030I$
$u = 0.419103 - 0.595983I$ $a = 0.910599 + 0.447643I$ $b = -0.115558 - 0.434784I$	$1.109940 - 0.478757I$	$7.85612 + 2.59030I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.077849 + 0.690240I$ $a = 1.50612 - 0.77029I$ $b = -0.473705 + 0.269170I$	$0.539059 - 0.667337I$	$5.56096 - 0.61157I$
$u = 0.077849 - 0.690240I$ $a = 1.50612 + 0.77029I$ $b = -0.473705 - 0.269170I$	$0.539059 + 0.667337I$	$5.56096 + 0.61157I$
$u = -0.407194$ $a = 0.525318$ $b = 0.903607$	$-1.31151$	$-10.2650$
$u = 0.39124 + 1.82665I$ $a = -0.068005 + 0.975912I$ $b = -1.07106 - 1.01973I$	$9.29659 - 4.13983I$	$0$
$u = 0.39124 - 1.82665I$ $a = -0.068005 - 0.975912I$ $b = -1.07106 + 1.01973I$	$9.29659 + 4.13983I$	$0$
$u = 0.24251 + 1.87487I$ $a = 0.000094 + 0.931629I$ $b = -0.99989 - 1.07339I$	$9.54816 - 3.51968I$	$0$
$u = 0.24251 - 1.87487I$ $a = 0.000094 - 0.931629I$ $b = -0.99989 + 1.07339I$	$9.54816 + 3.51968I$	$0$
$u = -0.48778 + 1.83564I$ $a = -0.120525 - 0.976988I$ $b = -1.12438 + 1.00821I$	$6.99297 + 8.38148I$	$0$
$u = -0.48778 - 1.83564I$ $a = -0.120525 + 0.976988I$ $b = -1.12438 - 1.00821I$	$6.99297 - 8.38148I$	$0$
$u = -0.13124 + 1.93107I$ $a = 0.037828 - 0.886275I$ $b = -0.95193 + 1.12627I$	$7.58072 - 0.60625I$	$0$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.13124 - 1.93107I$		
$a = 0.037828 + 0.886275I$	$7.58072 + 0.60625I$	0
$b = -0.95193 - 1.12627I$		
$u = 0.53822 + 1.86696I$		
$a = -0.147624 + 0.960917I$	$12.0430 - 12.4368I$	0
$b = -1.15619 - 1.01668I$		
$u = 0.53822 - 1.86696I$		
$a = -0.147624 - 0.960917I$	$12.0430 + 12.4368I$	0
$b = -1.15619 + 1.01668I$		
$u = 0.09684 + 2.00647I$		
$a = 0.036196 + 0.850721I$	$12.76300 + 4.48472I$	0
$b = -0.95008 - 1.17335I$		
$u = 0.09684 - 2.00647I$		
$a = 0.036196 - 0.850721I$	$12.76300 - 4.48472I$	0
$b = -0.95008 + 1.17335I$		
$u = -0.35556 + 2.00636I$		
$a = -0.066983 - 0.886854I$	$16.7972 + 4.0807I$	0
$b = -1.08468 + 1.12119I$		
$u = -0.35556 - 2.00636I$		
$a = -0.066983 + 0.886854I$	$16.7972 - 4.0807I$	0
$b = -1.08468 - 1.12119I$		



$$\text{II. } I_1^v = \langle a, b - 1, v^8 - v^7 - v^6 + 2v^5 + v^4 - 2v^3 + 2v - 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} v \\ -v \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -v^3 + v \\ v^3 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} v^2 \\ -1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} v^4 \\ -v^2 + 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -v^7 + v^6 + 2v^5 - v^4 - 2v^3 + 2v^2 + 2v - 1 \\ v^7 - 2v^5 + 2v^3 - 2v \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $6v^7 - v^6 - 11v^5 + 7v^4 + 12v^3 - 6v^2 - 6v + 10$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$(u - 1)^8$
$c_3, c_8$	$u^8$
$c_4$	$(u + 1)^8$
$c_5, c_7$	$u^8 + 3u^7 + 7u^6 + 10u^5 + 11u^4 + 10u^3 + 6u^2 + 4u + 1$
$c_6$	$u^8 - u^7 - 3u^6 + 2u^5 + 3u^4 - 2u - 1$
$c_9, c_{12}$	$u^8 - u^7 - u^6 + 2u^5 + u^4 - 2u^3 + 2u - 1$
$c_{10}, c_{11}$	$u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y - 1)^8$
$c_3, c_8$	$y^8$
$c_5, c_7$	$y^8 + 5y^7 + 11y^6 + 6y^5 - 17y^4 - 34y^3 - 22y^2 - 4y + 1$
$c_6, c_{10}, c_{11}$	$y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1$
$c_9, c_{12}$	$y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^v$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$v = 0.570868 + 0.730671I$ $a = 0$ $b = 1.00000$	$-0.604279 - 1.131230I$	$-1.074136 + 0.216470I$
$v = 0.570868 - 0.730671I$ $a = 0$ $b = 1.00000$	$-0.604279 + 1.131230I$	$-1.074136 - 0.216470I$
$v = -0.855237 + 0.665892I$ $a = 0$ $b = 1.00000$	$-3.80435 - 2.57849I$	$-3.22623 + 3.25417I$
$v = -0.855237 - 0.665892I$ $a = 0$ $b = 1.00000$	$-3.80435 + 2.57849I$	$-3.22623 - 3.25417I$
$v = -1.09818$ $a = 0$ $b = 1.00000$	4.85780	7.89920
$v = 1.031810 + 0.655470I$ $a = 0$ $b = 1.00000$	$0.73474 + 6.44354I$	$2.34782 - 4.54733I$
$v = 1.031810 - 0.655470I$ $a = 0$ $b = 1.00000$	$0.73474 - 6.44354I$	$2.34782 + 4.54733I$
$v = 0.603304$ $a = 0$ $b = 1.00000$	-0.799899	7.00590

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u - 1)^8)(u^{37} + 5u^{36} + \dots - 5u + 1)$
$c_2$	$((u - 1)^8)(u^{37} - 9u^{36} + \dots - 7u + 1)$
$c_3, c_8$	$u^8(u^{37} - u^{36} + \dots + 128u + 256)$
$c_4$	$((u + 1)^8)(u^{37} - 9u^{36} + \dots - 7u + 1)$
$c_5, c_7$	$(u^8 + 3u^7 + 7u^6 + 10u^5 + 11u^4 + 10u^3 + 6u^2 + 4u + 1)$ $\cdot (u^{37} + 6u^{36} + \dots + 35u + 5)$
$c_6$	$(u^8 - u^7 - 3u^6 + 2u^5 + 3u^4 - 2u - 1)(u^{37} - 2u^{36} + \dots + 3u + 1)$
$c_9$	$(u^8 - u^7 + \dots + 2u - 1)(u^{37} + 2u^{36} + \dots + 3u + 1)$
$c_{10}, c_{11}$	$(u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1)(u^{37} - 2u^{36} + \dots + 3u + 1)$
$c_{12}$	$(u^8 - u^7 - u^6 + 2u^5 + u^4 - 2u^3 + 2u - 1)$ $\cdot (u^{37} - 8u^{36} + \dots + 2082719u - 154033)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y - 1)^8)(y^{37} + 63y^{36} + \dots - y - 1)$
$c_2, c_4$	$((y - 1)^8)(y^{37} - 5y^{36} + \dots - 5y - 1)$
$c_3, c_8$	$y^8(y^{37} + 51y^{36} + \dots - 475136y - 65536)$
$c_5, c_7$	$(y^8 + 5y^7 + 11y^6 + 6y^5 - 17y^4 - 34y^3 - 22y^2 - 4y + 1)$ $\cdot (y^{37} + 16y^{36} + \dots + 735y - 25)$
$c_6, c_{10}, c_{11}$	$(y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1)$ $\cdot (y^{37} - 32y^{36} + \dots + 27y - 1)$
$c_9$	$(y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1)$ $\cdot (y^{37} - 48y^{36} + \dots + 27y - 1)$
$c_{12}$	$(y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1)$ $\cdot (y^{37} - 84y^{36} + \dots + 1848165923715y - 23726165089)$