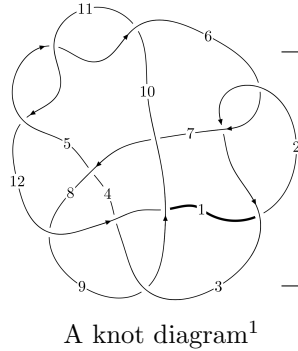
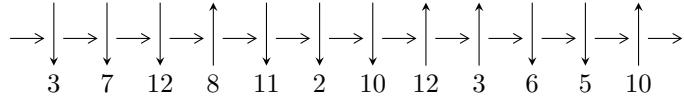


12n<sub>0561</sub> (K12n<sub>0561</sub>)



**Linearized knot diagram**



**Solving Sequence**

$$2,6 \xrightarrow{c_6} 7 \xrightarrow{c_2} 3,10 \xrightarrow{c_7} 8 \xrightarrow{c_{10}} 11 \xrightarrow{c_1} 1 \xrightarrow{c_5} 5 \xrightarrow{c_4} 4 \xrightarrow{c_9} 9 \xrightarrow{c_{12}} 12 \Rightarrow c_3, c_8, c_{11}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle 19514643174787u^{34} - 4572969237385u^{33} + \dots + 35208391096192b - 115134997157437, \\ - 44181034772241u^{34} - 93208096612611u^{33} + \dots + 176041955480960a - 34686663928811, \\ u^{35} + u^{34} + \dots - 4u - 5 \rangle$$

$$I_2^u = \langle -u^2a - u^2 + b + a, u^3a - 2u^2a + u^3 + a^2 - au + 2a - 1, u^4 - u^2 + 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 43 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.

$$I_1^u = \langle 1.95 \times 10^{13} u^{34} - 4.57 \times 10^{12} u^{33} + \dots + 3.52 \times 10^{13} b - 1.15 \times 10^{14}, -4.42 \times 10^{13} u^{34} - 9.32 \times 10^{13} u^{33} + \dots + 1.76 \times 10^{14} a - 3.47 \times 10^{13}, u^{35} + u^{34} + \dots - 4u - 5 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0.250969u^{34} + 0.529465u^{33} + \dots + 2.80218u + 0.197036 \\ -0.554261u^{34} + 0.129883u^{33} + \dots + 0.868164u + 3.27010 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0.791772u^{34} + 0.423142u^{33} + \dots + 3.34243u - 2.71039 \\ -0.523385u^{34} + 0.0869282u^{33} + \dots + 0.937584u + 2.29735 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0.805230u^{34} + 0.399582u^{33} + \dots + 1.93402u - 3.07306 \\ -0.554261u^{34} + 0.129883u^{33} + \dots + 0.868164u + 3.27010 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^3 \\ u^5 - u^3 + u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -0.472936u^{34} + 0.0799123u^{33} + \dots + 0.438637u + 2.87741 \\ -0.556115u^{34} - 0.173472u^{33} + \dots - 2.34087u + 2.38011 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -0.816166u^{34} + 0.145199u^{33} + \dots + 1.79451u + 3.42292 \\ -1.06658u^{34} - 0.102045u^{33} + \dots - 3.07330u + 6.87995 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0.601350u^{34} + 0.306853u^{33} + \dots + 2.54938u - 1.70567 \\ -0.455033u^{34} + 0.135445u^{33} + \dots + 0.580899u + 2.30784 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -0.528222u^{34} + 0.0504600u^{33} + \dots - 1.61488u + 4.49023 \\ 0.662513u^{34} + 0.0170577u^{33} + \dots + 1.34187u - 2.86220 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -\frac{3308824036235}{17604195548096} u^{34} + \frac{2338411391075}{17604195548096} u^{33} + \dots + \frac{68902332437973}{8802097774048} u - \frac{143545741438237}{17604195548096}$$

(iv) u-Polynomials at the component

| Crossings             | u-Polynomials at each crossing            |
|-----------------------|---|
| $c_1$                 | $u^{35} + 21u^{34} + \dots - 74u + 25$    |
| $c_2, c_6$            | $u^{35} - u^{34} + \dots - 4u + 5$        |
| $c_3$                 | $u^{35} - 5u^{34} + \dots + 296u + 28$    |
| $c_4, c_9$            | $u^{35} - u^{34} + \dots - 16u + 4$       |
| $c_5, c_{10}, c_{11}$ | $u^{35} + u^{34} + \dots - 6u + 1$        |
| $c_7$                 | $u^{35} - 3u^{34} + \dots + 6720u - 1472$ |
| $c_8$                 | $u^{35} - 3u^{34} + \dots + 6u + 5$       |
| $c_{12}$              | $u^{35} + 3u^{34} + \dots + 12u + 1$      |

(v) Riley Polynomials at the component

| Crossings             | Riley Polynomials at each crossing                |
|-----------------------|---|
| $c_1$                 | $y^{35} - 9y^{34} + \dots + 39626y - 625$         |
| $c_2, c_6$            | $y^{35} - 21y^{34} + \dots - 74y - 25$            |
| $c_3$                 | $y^{35} - 57y^{34} + \dots - 32784y - 784$        |
| $c_4, c_9$            | $y^{35} + 43y^{34} + \dots + 184y - 16$           |
| $c_5, c_{10}, c_{11}$ | $y^{35} + 39y^{34} + \dots + 34y - 1$             |
| $c_7$                 | $y^{35} - 31y^{34} + \dots + 22678016y - 2166784$ |
| $c_8$                 | $y^{35} + 39y^{34} + \dots + 126y - 25$           |
| $c_{12}$              | $y^{35} + 51y^{34} + \dots + 154y - 1$            |

(vi) Complex Volumes and Cusp Shapes

| Solutions to $I_1^u$   | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape             |
|--|---------------------------------------|------------------------|
| $u = -0.203911 + 0.973360I$<br>$a = -0.519106 + 0.963683I$<br>$b = -0.26045 - 1.53546I$  | $-1.43478 - 6.20512I$                 | $-1.86669 + 2.78198I$  |
| $u = -0.203911 - 0.973360I$<br>$a = -0.519106 - 0.963683I$<br>$b = -0.26045 + 1.53546I$  | $-1.43478 + 6.20512I$                 | $-1.86669 - 2.78198I$  |
| $u = 0.066320 + 0.984954I$<br>$a = -0.448541 - 0.317717I$<br>$b = -0.750046 + 0.533829I$ | $-8.19601 + 2.49744I$                 | $-5.06408 - 2.60195I$  |
| $u = 0.066320 - 0.984954I$<br>$a = -0.448541 + 0.317717I$<br>$b = -0.750046 - 0.533829I$ | $-8.19601 - 2.49744I$                 | $-5.06408 + 2.60195I$  |
| $u = 0.801249 + 0.559810I$<br>$a = 1.36028 - 1.03030I$<br>$b = 0.01451 + 1.59501I$       | $8.83899 - 2.24678I$                  | $4.64462 + 3.81835I$   |
| $u = 0.801249 - 0.559810I$<br>$a = 1.36028 + 1.03030I$<br>$b = 0.01451 - 1.59501I$       | $8.83899 + 2.24678I$                  | $4.64462 - 3.81835I$   |
| $u = 1.014440 + 0.137733I$<br>$a = -1.44237 + 0.41096I$<br>$b = -0.391237 + 0.679863I$   | $-2.29834 - 0.76813I$                 | $-7.21756 - 0.78239I$  |
| $u = 1.014440 - 0.137733I$<br>$a = -1.44237 - 0.41096I$<br>$b = -0.391237 - 0.679863I$   | $-2.29834 + 0.76813I$                 | $-7.21756 + 0.78239I$  |
| $u = 0.910408 + 0.541398I$<br>$a = -0.378721 + 0.991836I$<br>$b = -0.212665 - 0.039123I$ | $-1.75949 - 2.05479I$                 | $-11.09061 + 3.13209I$ |
| $u = 0.910408 - 0.541398I$<br>$a = -0.378721 - 0.991836I$<br>$b = -0.212665 + 0.039123I$ | $-1.75949 + 2.05479I$                 | $-11.09061 - 3.13209I$ |

| Solutions to $I_1^u$  | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape            |
|---|---------------------------------------|-----------------------|
| $u = -0.922478 + 0.127300I$<br>$a = -1.96495 + 0.18361I$<br>$b = -0.05491 + 1.64507I$   | $5.90539 + 0.64255I$                  | $-5.75392 + 0.64646I$ |
| $u = -0.922478 - 0.127300I$<br>$a = -1.96495 - 0.18361I$<br>$b = -0.05491 - 1.64507I$   | $5.90539 - 0.64255I$                  | $-5.75392 - 0.64646I$ |
| $u = -0.812370 + 0.424392I$<br>$a = 0.773162 + 0.362489I$<br>$b = 0.109550 - 0.685718I$ | $1.05382 + 1.86722I$                  | $3.18128 - 4.59075I$  |
| $u = -0.812370 - 0.424392I$<br>$a = 0.773162 - 0.362489I$<br>$b = 0.109550 + 0.685718I$ | $1.05382 - 1.86722I$                  | $3.18128 + 4.59075I$  |
| $u = 0.912261$<br>$a = 1.05610$<br>$b = 0.463900$                                       | $-1.34236$                            | $-8.12570$            |
| $u = -0.842169 + 0.777146I$<br>$a = -0.89892 - 1.28817I$<br>$b = -0.050273 + 1.397230I$ | $3.07114 + 2.89052I$                  | $-2.21100 - 2.95071I$ |
| $u = -0.842169 - 0.777146I$<br>$a = -0.89892 + 1.28817I$<br>$b = -0.050273 - 1.397230I$ | $3.07114 - 2.89052I$                  | $-2.21100 + 2.95071I$ |
| $u = -1.137850 + 0.335834I$<br>$a = -1.53286 + 0.05803I$<br>$b = -0.587577 + 0.359881I$ | $-3.27817 + 4.26600I$                 | $-8.05552 - 6.74990I$ |
| $u = -1.137850 - 0.335834I$<br>$a = -1.53286 - 0.05803I$<br>$b = -0.587577 - 0.359881I$ | $-3.27817 - 4.26600I$                 | $-8.05552 + 6.74990I$ |
| $u = -1.152990 + 0.388555I$<br>$a = 0.878906 - 1.034180I$<br>$b = 0.002872 - 1.337860I$ | $2.24498 + 1.33286I$                  | $-3.24945 - 0.69819I$ |

| Solutions to $I_1^u$        | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape            |
|-----------------------------|---------------------------------------|-----------------------|
| $u = -1.152990 - 0.388555I$ |                                       |                       |
| $a = 0.878906 + 1.034180I$  | $2.24498 - 1.33286I$                  | $-3.24945 + 0.69819I$ |
| $b = 0.002872 + 1.337860I$  |                                       |                       |
| $u = 0.098226 + 0.753815I$  |                                       |                       |
| $a = 0.468379 + 0.420561I$  | $5.88674 + 2.52249I$                  | $1.23506 - 2.87985I$  |
| $b = 0.09057 - 1.46639I$    |                                       |                       |
| $u = 0.098226 - 0.753815I$  |                                       |                       |
| $a = 0.468379 - 0.420561I$  | $5.88674 - 2.52249I$                  | $1.23506 + 2.87985I$  |
| $b = 0.09057 + 1.46639I$    |                                       |                       |
| $u = 1.194500 + 0.470305I$  |                                       |                       |
| $a = -1.80401 - 0.60694I$   | $2.66330 - 7.05772I$                  | $-2.51905 + 5.88692I$ |
| $b = -0.19201 - 1.46324I$   |                                       |                       |
| $u = 1.194500 - 0.470305I$  |                                       |                       |
| $a = -1.80401 + 0.60694I$   | $2.66330 + 7.05772I$                  | $-2.51905 - 5.88692I$ |
| $b = -0.19201 + 1.46324I$   |                                       |                       |
| $u = 1.330550 + 0.340353I$  |                                       |                       |
| $a = 0.175592 - 0.683458I$  | $-6.45071 + 1.73894I$                 | $-5.86337 - 0.49500I$ |
| $b = 0.33732 - 1.47223I$    |                                       |                       |
| $u = 1.330550 - 0.340353I$  |                                       |                       |
| $a = 0.175592 + 0.683458I$  | $-6.45071 - 1.73894I$                 | $-5.86337 + 0.49500I$ |
| $b = 0.33732 + 1.47223I$    |                                       |                       |
| $u = -1.252450 + 0.585255I$ |                                       |                       |
| $a = 1.89880 + 0.08113I$    | $-4.64525 + 11.83980I$                | $-4.29756 - 5.89282I$ |
| $b = 0.27080 - 1.59119I$    |                                       |                       |
| $u = -1.252450 - 0.585255I$ |                                       |                       |
| $a = 1.89880 - 0.08113I$    | $-4.64525 - 11.83980I$                | $-4.29756 + 5.89282I$ |
| $b = 0.27080 + 1.59119I$    |                                       |                       |
| $u = -1.326130 + 0.448294I$ |                                       |                       |
| $a = 0.903307 + 0.583518I$  | $-12.58030 + 2.55008I$                | $-8.43526 - 0.60252I$ |
| $b = 0.842224 + 0.446428I$  |                                       |                       |

| Solutions to $I_1^u$        | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape            |
|-----------------------------|---------------------------------------|-----------------------|
| $u = -1.326130 - 0.448294I$ |                                       |                       |
| $a = 0.903307 - 0.583518I$  | $-12.58030 - 2.55008I$                | $-8.43526 + 0.60252I$ |
| $b = 0.842224 - 0.446428I$  |                                       |                       |
| $u = 1.297550 + 0.525631I$  |                                       |                       |
| $a = 1.45281 - 0.36988I$    | $-11.9977 - 7.9047I$                  | $-7.36295 + 5.50788I$ |
| $b = 0.780540 + 0.640043I$  |                                       |                       |
| $u = 1.297550 - 0.525631I$  |                                       |                       |
| $a = 1.45281 + 0.36988I$    | $-11.9977 + 7.9047I$                  | $-7.36295 - 5.50788I$ |
| $b = 0.780540 - 0.640043I$  |                                       |                       |
| $u = -0.019019 + 0.438455I$ |                                       |                       |
| $a = 0.850181 + 0.183537I$  | $-0.204022 - 1.109310I$               | $-3.01108 + 6.25704I$ |
| $b = 0.318829 + 0.391878I$  |                                       |                       |
| $u = -0.019019 - 0.438455I$ |                                       |                       |
| $a = 0.850181 - 0.183537I$  | $-0.204022 + 1.109310I$               | $-3.01108 - 6.25704I$ |
| $b = 0.318829 - 0.391878I$  |                                       |                       |



$$\text{II. } \Gamma_2^u = \langle -u^2a - u^2 + b + a, u^3a - 2u^2a + u^3 + a^2 - au + 2a - 1, u^4 - u^2 + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} a \\ u^2a + u^2 - a \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^3a - u^2a + u^3 - au + 2a \\ -u^3a + u^2a + u^2 - a - u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u^2a - u^2 + 2a \\ u^2a + u^2 - a \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^3 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^3a + u^2a + u^3 - au - u^2 - 2u + 1 \\ -u^3 + au + u + 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -2u^3a + 2u^2a + u^3 - 2au - u^2 - 2u + 2 \\ -2u^3 + 2au + u^2 - a + 2u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^2 + a + 1 \\ u^2a + 2u^2 - a \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^3a + u^3 + au + u^2 - a - u - 1 \\ -u^2a - u^3 - u^2 + a \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $4u^2 - 4$

(iv) u-Polynomials at the component

| Crossings             | u-Polynomials at each crossing                                 |
|-----------------------|--|
| $c_1$                 | $(u^2 - u + 1)^4$  |
| $c_2, c_6, c_8$       | $(u^4 - u^2 + 1)^2$  |
| $c_3$                 | $u^8 + 4u^7 + 16u^6 + 34u^5 + 57u^4 + 62u^3 + 46u^2 + 20u + 4$ |
| $c_4, c_9$            | $(u^2 + 1)^4$  |
| $c_5, c_{10}, c_{11}$ | $(u^4 + 3u^2 + 1)^2$   |
| $c_7$                 | $u^8 - 8u^7 + 25u^6 - 38u^5 + 33u^4 - 28u^3 + 28u^2 - 16u + 4$ |
| $c_{12}$              | $(u^2 + u - 1)^4$  |

(v) Riley Polynomials at the component

| Crossings             | Riley Polynomials at each crossing                                  |
|-----------------------|---|
| $c_1$                 | $(y^2 + y + 1)^4$   |
| $c_2, c_6, c_8$       | $(y^2 - y + 1)^4$   |
| $c_3$                 | $y^8 + 16y^7 + 98y^6 + 264y^5 + 353y^4 + 168y^3 + 92y^2 - 32y + 16$ |
| $c_4, c_9$            | $(y + 1)^8$   |
| $c_5, c_{10}, c_{11}$ | $(y^2 + 3y + 1)^4$  |
| $c_7$                 | $y^8 - 14y^7 + 83y^6 - 186y^5 + 113y^4 + 48y^3 + 152y^2 - 32y + 16$ |
| $c_{12}$              | $(y^2 - 3y + 1)^4$  |

(vi) Complex Volumes and Cusp Shapes

| Solutions to $I_2^u$  | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape            |
|---|---------------------------------------|-----------------------|
| $u = 0.866025 + 0.500000I$<br>$a = 0.901259 + 0.057008I$<br>$b = 1.61803I$    | $7.23771 - 2.02988I$                  | $-2.00000 + 3.46410I$ |
| $u = 0.866025 + 0.500000I$<br>$a = -1.03523 + 1.17504I$<br>$b = -0.618034I$   | $-0.65797 - 2.02988I$                 | $-2.00000 + 3.46410I$ |
| $u = 0.866025 - 0.500000I$<br>$a = 0.901259 - 0.057008I$<br>$b = -1.61803I$   | $7.23771 + 2.02988I$                  | $-2.00000 - 3.46410I$ |
| $u = 0.866025 - 0.500000I$<br>$a = -1.03523 - 1.17504I$<br>$b = 0.618034I$    | $-0.65797 + 2.02988I$                 | $-2.00000 - 3.46410I$ |
| $u = -0.866025 + 0.500000I$<br>$a = 0.035233 - 0.557008I$<br>$b = -0.618034I$ | $-0.65797 + 2.02988I$                 | $-2.00000 - 3.46410I$ |
| $u = -0.866025 + 0.500000I$<br>$a = -1.90126 - 1.67504I$<br>$b = 1.61803I$    | $7.23771 + 2.02988I$                  | $-2.00000 - 3.46410I$ |
| $u = -0.866025 - 0.500000I$<br>$a = 0.035233 + 0.557008I$<br>$b = 0.618034I$  | $-0.65797 - 2.02988I$                 | $-2.00000 + 3.46410I$ |
| $u = -0.866025 - 0.500000I$<br>$a = -1.90126 + 1.67504I$<br>$b = -1.61803I$   | $7.23771 - 2.02988I$                  | $-2.00000 + 3.46410I$ |

### III. u-Polynomials

| Crossings             | u-Polynomials at each crossing  |
|-----------------------|---|
| $c_1$                 | $((u^2 - u + 1)^4)(u^{35} + 21u^{34} + \dots - 74u + 25)$   |
| $c_2, c_6$            | $((u^4 - u^2 + 1)^2)(u^{35} - u^{34} + \dots - 4u + 5)$   |
| $c_3$                 | $(u^8 + 4u^7 + 16u^6 + 34u^5 + 57u^4 + 62u^3 + 46u^2 + 20u + 4)$<br>$\cdot (u^{35} - 5u^{34} + \dots + 296u + 28)$    |
| $c_4, c_9$            | $((u^2 + 1)^4)(u^{35} - u^{34} + \dots - 16u + 4)$  |
| $c_5, c_{10}, c_{11}$ | $((u^4 + 3u^2 + 1)^2)(u^{35} + u^{34} + \dots - 6u + 1)$  |
| $c_7$                 | $(u^8 - 8u^7 + 25u^6 - 38u^5 + 33u^4 - 28u^3 + 28u^2 - 16u + 4)$<br>$\cdot (u^{35} - 3u^{34} + \dots + 6720u - 1472)$ |
| $c_8$                 | $((u^4 - u^2 + 1)^2)(u^{35} - 3u^{34} + \dots + 6u + 5)$  |
| $c_{12}$              | $((u^2 + u - 1)^4)(u^{35} + 3u^{34} + \dots + 12u + 1)$   |

#### IV. Riley Polynomials

| Crossings             | Riley Polynomials at each crossing   |
|-----------------------|--|
| $c_1$                 | $((y^2 + y + 1)^4)(y^{35} - 9y^{34} + \dots + 39626y - 625)$   |
| $c_2, c_6$            | $((y^2 - y + 1)^4)(y^{35} - 21y^{34} + \dots - 74y - 25)$  |
| $c_3$                 | $(y^8 + 16y^7 + 98y^6 + 264y^5 + 353y^4 + 168y^3 + 92y^2 - 32y + 16)$<br>$\cdot (y^{35} - 57y^{34} + \dots - 32784y - 784)$        |
| $c_4, c_9$            | $((y + 1)^8)(y^{35} + 43y^{34} + \dots + 184y - 16)$   |
| $c_5, c_{10}, c_{11}$ | $((y^2 + 3y + 1)^4)(y^{35} + 39y^{34} + \dots + 34y - 1)$  |
| $c_7$                 | $(y^8 - 14y^7 + 83y^6 - 186y^5 + 113y^4 + 48y^3 + 152y^2 - 32y + 16)$<br>$\cdot (y^{35} - 31y^{34} + \dots + 22678016y - 2166784)$ |
| $c_8$                 | $((y^2 - y + 1)^4)(y^{35} + 39y^{34} + \dots + 126y - 25)$   |
| $c_{12}$              | $((y^2 - 3y + 1)^4)(y^{35} + 51y^{34} + \dots + 154y - 1)$   |