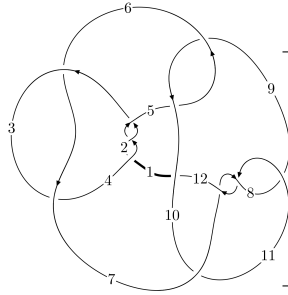
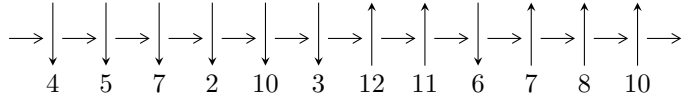


12n<sub>0678</sub> (K12n<sub>0678</sub>)



A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**

$$7, 12 \xrightarrow{c_7} 4, 8 \xrightarrow{c_3} 3 \xrightarrow{c_6} 6 \xrightarrow{c_{11}} 11 \xrightarrow{c_{10}} 10 \xrightarrow{c_{12}} 1 \xrightarrow{c_5} 5 \xrightarrow{c_2} 2 \xrightarrow{c_9} 9 \twoheadrightarrow c_1, c_4, c_8$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -171832171743933u^{44} + 565195223234702u^{43} + \dots + 319912709247818b - 45090425568559, \\ 221527977237717u^{44} - 489493690784658u^{43} + \dots + 319912709247818a + 1085130159951208, \\ u^{45} - 3u^{44} + \dots - 8u + 1 \rangle$$

$$I_2^u = \langle -au + b - u, u^2a + a^2 + au - u^2 + 4a, u^3 + u^2 + 2u + 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 51 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

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$$I_1^u = \langle -1.72 \times 10^{14} u^{44} + 5.65 \times 10^{14} u^{43} + \dots + 3.20 \times 10^{14} b - 4.51 \times 10^{13}, 2.22 \times 10^{14} u^{44} - 4.89 \times 10^{14} u^{43} + \dots + 3.20 \times 10^{14} a + 1.09 \times 10^{15}, u^{45} - 3u^{44} + \dots - 8u + 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_7 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_4 &= \begin{pmatrix} -0.692464u^{44} + 1.53009u^{43} + \dots - 21.6689u - 3.39196 \\ 0.537122u^{44} - 1.76672u^{43} + \dots + 4.08754u + 0.140946 \end{pmatrix} \\ a_8 &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} -0.155342u^{44} - 0.236632u^{43} + \dots - 17.5814u - 3.25101 \\ 0.537122u^{44} - 1.76672u^{43} + \dots + 4.08754u + 0.140946 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1.72119u^{44} - 3.14555u^{43} + \dots + 18.4989u + 1.81534 \\ -2.01801u^{44} + 5.92336u^{43} + \dots - 15.5848u + 1.72119 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} -u \\ u^3 + u \end{pmatrix} \\ a_{10} &= \begin{pmatrix} -u^3 - 2u \\ u^3 + u \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^7 + 4u^5 + 4u^3 \\ -u^7 - 3u^5 - 2u^3 + u \end{pmatrix} \\ a_5 &= \begin{pmatrix} -0.470191u^{44} + 1.87005u^{43} + \dots + 7.18359u + 3.39261 \\ -0.462878u^{44} + 1.23328u^{43} + \dots - 3.91246u + 0.140946 \end{pmatrix} \\ a_2 &= \begin{pmatrix} -1.21496u^{44} + 3.02326u^{43} + \dots - 22.9486u - 0.0362322 \\ 0.462878u^{44} - 1.23328u^{43} + \dots + 3.91246u - 0.140946 \end{pmatrix} \\ a_9 &= \begin{pmatrix} u^2 + 1 \\ -u^4 - 2u^2 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -\frac{124420232195109}{159956354623909} u^{44} + \frac{585183168556345}{319912709247818} u^{43} + \dots - \frac{6164660264148967}{319912709247818} u - \frac{3111644555281919}{319912709247818}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_4$	$u^{45} - 4u^{44} + \dots + 3u - 1$
$c_3, c_6$	$u^{45} + 4u^{44} + \dots - u + 1$
$c_5, c_9$	$u^{45} + 3u^{44} + \dots + 160u + 64$
$c_7, c_8, c_{11}$	$u^{45} + 3u^{44} + \dots - 8u - 1$
$c_{10}$	$u^{45} - 3u^{44} + \dots - 542u - 97$
$c_{12}$	$u^{45} + 19u^{44} + \dots + 653792u - 13633$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$y^{45} - 34y^{44} + \dots + 5y - 1$
$c_3, c_6$	$y^{45} - 6y^{44} + \dots + 5y - 1$
$c_5, c_9$	$y^{45} + 35y^{44} + \dots + 58368y - 4096$
$c_7, c_8, c_{11}$	$y^{45} + 37y^{44} + \dots + 120y - 1$
$c_{10}$	$y^{45} - 39y^{44} + \dots + 1081792y - 9409$
$c_{12}$	$y^{45} - 59y^{44} + \dots + 530299074808y - 185858689$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.583612 + 0.749370I$		
$a = -0.145378 - 1.180080I$	$-0.22746 - 3.52607I$	$-0.53223 + 9.05892I$
$b = 0.314212 + 0.693704I$		
$u = -0.583612 - 0.749370I$		
$a = -0.145378 + 1.180080I$	$-0.22746 + 3.52607I$	$-0.53223 - 9.05892I$
$b = 0.314212 - 0.693704I$		
$u = 0.884188 + 0.167276I$		
$a = -1.44217 + 3.00622I$	$4.46171 + 9.77082I$	$-1.55814 - 6.22355I$
$b = 1.01437 - 1.10209I$		
$u = 0.884188 - 0.167276I$		
$a = -1.44217 - 3.00622I$	$4.46171 - 9.77082I$	$-1.55814 + 6.22355I$
$b = 1.01437 + 1.10209I$		
$u = -0.862556 + 0.189586I$		
$a = -0.10994 + 1.47934I$	$1.49082 - 1.15898I$	$5.48142 + 4.79165I$
$b = -0.099189 - 0.621407I$		
$u = -0.862556 - 0.189586I$		
$a = -0.10994 - 1.47934I$	$1.49082 + 1.15898I$	$5.48142 - 4.79165I$
$b = -0.099189 + 0.621407I$		
$u = 0.879134 + 0.073302I$		
$a = 1.69262 - 2.92394I$	$8.52509 + 4.00354I$	$1.85103 - 2.75584I$
$b = -1.05605 + 1.10417I$		
$u = 0.879134 - 0.073302I$		
$a = 1.69262 + 2.92394I$	$8.52509 - 4.00354I$	$1.85103 + 2.75584I$
$b = -1.05605 - 1.10417I$		
$u = 0.834305 + 0.026519I$		
$a = -2.03456 - 2.80224I$	$4.18745 + 1.80014I$	$0.034492 - 1.194140I$
$b = 1.11558 + 1.09067I$		
$u = 0.834305 - 0.026519I$		
$a = -2.03456 + 2.80224I$	$4.18745 - 1.80014I$	$0.034492 + 1.194140I$
$b = 1.11558 - 1.09067I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.481295 + 1.085350I$ $a = -1.52416 - 1.39190I$ $b = 0.840283 + 1.091050I$	$1.64682 - 4.91710I$	$-3.75732 + 2.86803I$
$u = 0.481295 - 1.085350I$ $a = -1.52416 + 1.39190I$ $b = 0.840283 - 1.091050I$	$1.64682 + 4.91710I$	$-3.75732 - 2.86803I$
$u = -0.027576 + 1.213260I$ $a = 0.097830 + 0.955725I$ $b = 0.987113 - 0.071189I$	$-4.06865 - 0.00938I$	$-7.42868 + 0.I$
$u = -0.027576 - 1.213260I$ $a = 0.097830 - 0.955725I$ $b = 0.987113 + 0.071189I$	$-4.06865 + 0.00938I$	$-7.42868 + 0.I$
$u = -0.310904 + 1.181390I$ $a = -0.21426 - 1.54867I$ $b = 0.294215 + 0.670186I$	$-1.38332 - 2.86199I$	$0. + 3.33644I$
$u = -0.310904 - 1.181390I$ $a = -0.21426 + 1.54867I$ $b = 0.294215 - 0.670186I$	$-1.38332 + 2.86199I$	$0. - 3.33644I$
$u = 0.159859 + 1.260150I$ $a = 1.07101 - 1.51572I$ $b = -1.63864 + 0.16074I$	$-11.89460 + 2.24035I$	$-8.35575 + 0.I$
$u = 0.159859 - 1.260150I$ $a = 1.07101 + 1.51572I$ $b = -1.63864 - 0.16074I$	$-11.89460 - 2.24035I$	$-8.35575 + 0.I$
$u = 0.433209 + 1.205320I$ $a = 1.56995 + 1.36216I$ $b = -0.89067 - 1.16692I$	$5.04167 + 0.68814I$	$0$
$u = 0.433209 - 1.205320I$ $a = 1.56995 - 1.36216I$ $b = -0.89067 + 1.16692I$	$5.04167 - 0.68814I$	$0$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.091883 + 1.292420I$ $a = 0.68882 + 2.04860I$ $b = 0.100880 - 0.832088I$	$-5.73773 - 2.01190I$	0
$u = -0.091883 - 1.292420I$ $a = 0.68882 - 2.04860I$ $b = 0.100880 + 0.832088I$	$-5.73773 + 2.01190I$	0
$u = 0.378074 + 1.248800I$ $a = -0.30888 + 2.43252I$ $b = 1.23412 - 0.93095I$	$0.40714 + 2.55603I$	0
$u = 0.378074 - 1.248800I$ $a = -0.30888 - 2.43252I$ $b = 1.23412 + 0.93095I$	$0.40714 - 2.55603I$	0
$u = -0.233067 + 1.286920I$ $a = -1.54727 - 1.27295I$ $b = -0.472121 + 0.195549I$	$-4.28958 - 3.06247I$	$-27.0240 + 0.I$
$u = -0.233067 - 1.286920I$ $a = -1.54727 + 1.27295I$ $b = -0.472121 - 0.195549I$	$-4.28958 + 3.06247I$	$-27.0240 + 0.I$
$u = 0.376537 + 1.291220I$ $a = -1.67227 - 1.26273I$ $b = 0.99238 + 1.22341I$	$0.08117 + 6.15166I$	0
$u = 0.376537 - 1.291220I$ $a = -1.67227 + 1.26273I$ $b = 0.99238 - 1.22341I$	$0.08117 - 6.15166I$	0
$u = -0.594574 + 0.259179I$ $a = -0.442778 + 1.247600I$ $b = -0.223270 - 0.529291I$	$1.36466 - 0.79548I$	$4.15835 + 2.62510I$
$u = -0.594574 - 0.259179I$ $a = -0.442778 - 1.247600I$ $b = -0.223270 + 0.529291I$	$1.36466 + 0.79548I$	$4.15835 - 2.62510I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.173829 + 1.352770I$ $a = -0.626070 + 0.431699I$ $b = -0.604107 - 0.273968I$	$-3.74207 - 3.36112I$	0
$u = -0.173829 - 1.352770I$ $a = -0.626070 - 0.431699I$ $b = -0.604107 + 0.273968I$	$-3.74207 + 3.36112I$	0
$u = -0.615344$ $a = -2.94504$ $b = -0.368354$	$-0.259777$	$-41.6450$
$u = 0.399717 + 1.327110I$ $a = 0.22643 - 2.45843I$ $b = -1.17155 + 1.01534I$	$4.14207 + 8.58803I$	0
$u = 0.399717 - 1.327110I$ $a = 0.22643 + 2.45843I$ $b = -1.17155 - 1.01534I$	$4.14207 - 8.58803I$	0
$u = -0.40967 + 1.36131I$ $a = 0.064099 + 1.291930I$ $b = -0.432795 - 0.645865I$	$-3.33493 - 5.79576I$	0
$u = -0.40967 - 1.36131I$ $a = 0.064099 - 1.291930I$ $b = -0.432795 + 0.645865I$	$-3.33493 + 5.79576I$	0
$u = 0.38534 + 1.38238I$ $a = -0.15294 + 2.46124I$ $b = 1.12554 - 1.06328I$	$-0.4279 + 14.3330I$	0
$u = 0.38534 - 1.38238I$ $a = -0.15294 - 2.46124I$ $b = 1.12554 + 1.06328I$	$-0.4279 - 14.3330I$	0
$u = -0.11768 + 1.50035I$ $a = -0.220688 - 0.588458I$ $b = 0.779647 + 0.534509I$	$-7.68575 - 5.74695I$	0



Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.11768 - 1.50035I$ $a = -0.220688 + 0.588458I$ $b = 0.779647 - 0.534509I$	$-7.68575 + 5.74695I$	0
$u = 0.466619$ $a = 4.22060$ $b = -1.54831$	$-8.02557$	$-22.5480$
$u = -0.280147 + 0.187854I$ $a = 2.57065 + 1.29057I$ $b = 0.441345 - 0.382694I$	$-1.25055 - 0.68721I$	$-6.23111 - 2.03128I$
$u = -0.280147 - 0.187854I$ $a = 2.57065 - 1.29057I$ $b = 0.441345 + 0.382694I$	$-1.25055 + 0.68721I$	$-6.23111 + 2.03128I$
$u = 0.0963991$ $a = -5.35560$ $b = 0.614065$	$-0.870395$	$-12.0080$

$$\text{II. } I_2^u = \langle -au + b - u, u^2a + a^2 + au - u^2 + 4a, u^3 + u^2 + 2u + 1 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a \\ au + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} au + a + u \\ au + u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^2 - a + u + 1 \\ -au - u - 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^2 + 1 \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^2 - a + u + 1 \\ -au - u - 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} au + u^2 - a + 2u \\ -au - u - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ -u^2 - u - 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $3u^2a + 9u^2 - 3a + u - 1$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_3$	$(u^2 + u - 1)^3$
$c_4, c_6$	$(u^2 - u - 1)^3$
$c_5, c_9$	$u^6$
$c_7, c_8$	$(u^3 + u^2 + 2u + 1)^2$
$c_{10}, c_{12}$	$(u^3 + u^2 - 1)^2$
$c_{11}$	$(u^3 - u^2 + 2u - 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_6$	$(y^2 - 3y + 1)^3$
$c_5, c_9$	$y^6$
$c_7, c_8, c_{11}$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_{10}, c_{12}$	$(y^3 - y^2 + 2y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.215080 + 1.307140I$ $a = -0.924253 + 0.460350I$ $b = -0.618034$	$-4.01109 - 2.82812I$	$-8.01769 - 5.87116I$
$u = -0.215080 + 1.307140I$ $a = -1.19831 - 1.20521I$ $b = 1.61803$	$-11.90680 - 2.82812I$	$-8.63833 + 7.89410I$
$u = -0.215080 - 1.307140I$ $a = -0.924253 - 0.460350I$ $b = -0.618034$	$-4.01109 + 2.82812I$	$-8.01769 + 5.87116I$
$u = -0.215080 - 1.307140I$ $a = -1.19831 + 1.20521I$ $b = 1.61803$	$-11.90680 + 2.82812I$	$-8.63833 - 7.89410I$
$u = -0.569840$ $a = 0.0845740$ $b = -0.618034$	0.126494	1.18130
$u = -0.569840$ $a = -3.83945$ $b = 1.61803$	-7.76919	9.13080

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$((u^2 + u - 1)^3)(u^{45} - 4u^{44} + \dots + 3u - 1)$
$c_3$	$((u^2 + u - 1)^3)(u^{45} + 4u^{44} + \dots - u + 1)$
$c_4$	$((u^2 - u - 1)^3)(u^{45} - 4u^{44} + \dots + 3u - 1)$
$c_5, c_9$	$u^6(u^{45} + 3u^{44} + \dots + 160u + 64)$
$c_6$	$((u^2 - u - 1)^3)(u^{45} + 4u^{44} + \dots - u + 1)$
$c_7, c_8$	$((u^3 + u^2 + 2u + 1)^2)(u^{45} + 3u^{44} + \dots - 8u - 1)$
$c_{10}$	$((u^3 + u^2 - 1)^2)(u^{45} - 3u^{44} + \dots - 542u - 97)$
$c_{11}$	$((u^3 - u^2 + 2u - 1)^2)(u^{45} + 3u^{44} + \dots - 8u - 1)$
$c_{12}$	$((u^3 + u^2 - 1)^2)(u^{45} + 19u^{44} + \dots + 653792u - 13633)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$((y^2 - 3y + 1)^3)(y^{45} - 34y^{44} + \dots + 5y - 1)$
$c_3, c_6$	$((y^2 - 3y + 1)^3)(y^{45} - 6y^{44} + \dots + 5y - 1)$
$c_5, c_9$	$y^6(y^{45} + 35y^{44} + \dots + 58368y - 4096)$
$c_7, c_8, c_{11}$	$((y^3 + 3y^2 + 2y - 1)^2)(y^{45} + 37y^{44} + \dots + 120y - 1)$
$c_{10}$	$((y^3 - y^2 + 2y - 1)^2)(y^{45} - 39y^{44} + \dots + 1081792y - 9409)$
$c_{12}$	$(y^3 - y^2 + 2y - 1)^2$ $\cdot (y^{45} - 59y^{44} + \dots + 530299074808y - 185858689)$