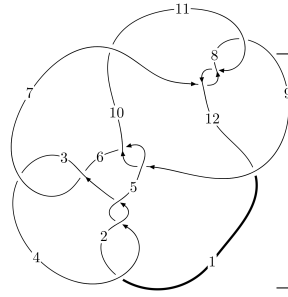
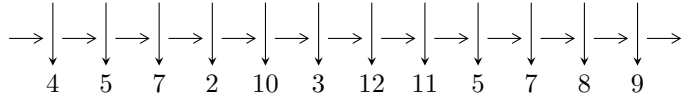


12n<sub>0680</sub> (K12n<sub>0680</sub>)



A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**

$$7,12 \xrightarrow{c_7} 4,8 \xrightarrow{c_3} 3 \xrightarrow{c_6} 6 \xrightarrow{c_{11}} 11 \xrightarrow{c_8} 9 \xrightarrow{c_{12}} 1 \xrightarrow{c_{10}} 10 \xrightarrow{c_5} 5 \xrightarrow{c_2} 2 \rightsquigarrow c_1, c_4, c_9$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle 3449u^{20} + 3669u^{19} + \dots + 71498b - 42457, -23077u^{20} - 82531u^{19} + \dots + 71498a + 43556, u^{21} + 4u^{20} + \dots - 2u + 1 \rangle$$

$$I_2^u = \langle b, u^5 + 2u^4 + 4u^3 + 4u^2 + a + 3u + 2, u^6 + u^5 + 3u^4 + 2u^3 + 2u^2 + u - 1 \rangle$$

$$I_3^u = \langle -au + b - u, -u^2a + a^2 + au - 3u^2 + 2u - 4, u^3 - u^2 + 2u - 1 \rangle$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 33 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\mathbf{I. } I_1^u = \langle 3449u^{20} + 3669u^{19} + \dots + 71498b - 42457, -23077u^{20} - 82531u^{19} + \dots + 71498a + 43556, u^{21} + 4u^{20} + \dots - 2u + 1 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.322764u^{20} + 1.15431u^{19} + \dots + 4.24348u - 0.609192 \\ -0.0482391u^{20} - 0.0513161u^{19} + \dots - 0.0347422u + 0.593821 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.274525u^{20} + 1.10300u^{19} + \dots + 4.20873u - 0.0153711 \\ -0.0482391u^{20} - 0.0513161u^{19} + \dots - 0.0347422u + 0.593821 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -0.224552u^{20} - 1.13479u^{19} + \dots - 2.45920u + 0.935718 \\ 0.166117u^{20} + 0.596254u^{19} + \dots + 0.165739u - 0.324387 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^5 - 2u^3 - u \\ -u^7 - 3u^5 - 2u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^3 + 2u \\ u^3 + u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -0.406179u^{20} - 1.57648u^{19} + \dots - 2.97737u + 0.847101 \\ -0.0482391u^{20} - 0.0513161u^{19} + \dots - 0.0347422u - 0.406179 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0.0868835u^{20} + 0.255951u^{19} + \dots + 1.16347u + 0.290428 \\ 0.0482391u^{20} + 0.0513161u^{19} + \dots + 0.0347422u + 0.406179 \end{pmatrix}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = -\frac{58626}{35749}u^{20} - \frac{459843}{71498}u^{19} + \dots + \frac{105825}{71498}u - \frac{777975}{71498}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_4$	$u^{21} - 10u^{20} + \dots - 4u - 1$
$c_3, c_6$	$u^{21} + 4u^{20} + \dots - 128u + 64$
$c_5, c_9$	$u^{21} - 2u^{20} + \dots + 224u + 64$
$c_7, c_8, c_{11}$	$u^{21} - 4u^{20} + \dots - 2u - 1$
$c_{10}, c_{12}$	$u^{21} + 4u^{20} + \dots - 304u - 97$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$y^{21} - 4y^{20} + \dots + 152y - 1$
$c_3, c_6$	$y^{21} + 30y^{20} + \dots + 90112y - 4096$
$c_5, c_9$	$y^{21} + 28y^{20} + \dots + 82944y - 4096$
$c_7, c_8, c_{11}$	$y^{21} + 22y^{20} + \dots - 10y - 1$
$c_{10}, c_{12}$	$y^{21} + 14y^{20} + \dots - 131266y - 9409$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.904622 + 0.417722I$ $a = 1.45274 - 0.01148I$ $b = 0.92182 - 2.09929I$	$6.05963 + 7.34750I$	$-14.4555 - 4.2838I$
$u = -0.904622 - 0.417722I$ $a = 1.45274 + 0.01148I$ $b = 0.92182 + 2.09929I$	$6.05963 - 7.34750I$	$-14.4555 + 4.2838I$
$u = -0.766571 + 0.752408I$ $a = -1.037380 + 0.144450I$ $b = 0.29405 + 2.35369I$	$7.09375 - 1.76941I$	$-12.87851 - 0.26190I$
$u = -0.766571 - 0.752408I$ $a = -1.037380 - 0.144450I$ $b = 0.29405 - 2.35369I$	$7.09375 + 1.76941I$	$-12.87851 + 0.26190I$
$u = 0.182461 + 1.208850I$ $a = -0.276557 - 0.201585I$ $b = -0.119527 + 0.423528I$	$2.74625 - 2.07596I$	$-5.86030 + 3.17371I$
$u = 0.182461 - 1.208850I$ $a = -0.276557 + 0.201585I$ $b = -0.119527 - 0.423528I$	$2.74625 + 2.07596I$	$-5.86030 - 3.17371I$
$u = -0.734798$ $a = -1.01042$ $b = -1.32151$	$-10.5256$	$-25.3380$
$u = 0.200947 + 1.339400I$ $a = -1.31937 + 2.00455I$ $b = -0.552581 - 0.279603I$	$1.78822 - 2.54403I$	$-24.8123 + 5.5170I$
$u = 0.200947 - 1.339400I$ $a = -1.31937 - 2.00455I$ $b = -0.552581 + 0.279603I$	$1.78822 + 2.54403I$	$-24.8123 - 5.5170I$
$u = -0.341638 + 1.336290I$ $a = 0.828590 - 0.644391I$ $b = -1.256020 - 0.461833I$	$-6.25683 + 3.88389I$	$-16.6815 - 2.9719I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.341638 - 1.336290I$ $a = 0.828590 + 0.644391I$ $b = -1.256020 + 0.461833I$	$-6.25683 - 3.88389I$	$-16.6815 + 2.9719I$
$u = 0.512237$ $a = -5.60059$ $b = -0.289328$	$-2.52473$	$-76.9450$
$u = -0.35905 + 1.50519I$ $a = 0.78634 + 2.07126I$ $b = 1.39494 - 1.98909I$	$12.2217 + 11.9532I$	$-11.90704 - 5.00504I$
$u = -0.35905 - 1.50519I$ $a = 0.78634 - 2.07126I$ $b = 1.39494 + 1.98909I$	$12.2217 - 11.9532I$	$-11.90704 + 5.00504I$
$u = 0.06735 + 1.55292I$ $a = -1.37220 + 0.85690I$ $b = 1.86679 - 0.98776I$	$5.79361 - 0.73866I$	$-10.20116 + 0.28003I$
$u = 0.06735 - 1.55292I$ $a = -1.37220 - 0.85690I$ $b = 1.86679 + 0.98776I$	$5.79361 + 0.73866I$	$-10.20116 - 0.28003I$
$u = -0.21280 + 1.64374I$ $a = -0.63755 - 2.25205I$ $b = -0.62112 + 3.12830I$	$15.1889 + 1.8962I$	$-10.30157 - 0.70895I$
$u = -0.21280 - 1.64374I$ $a = -0.63755 + 2.25205I$ $b = -0.62112 - 3.12830I$	$15.1889 - 1.8962I$	$-10.30157 + 0.70895I$
$u = 0.334401$ $a = -0.860564$ $b = 0.297521$	$-0.669543$	$-14.6190$
$u = 0.077997 + 0.278544I$ $a = -0.18883 + 1.76255I$ $b = 0.728300 - 0.059767I$	$-0.764279 + 0.134030I$	$-11.95123 + 0.33972I$

	Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u =$	$0.077997 - 0.278544I$		
$a =$	$-0.18883 - 1.76255I$	$-0.764279 - 0.134030I$	$-11.95123 - 0.33972I$
$b =$	$0.728300 + 0.059767I$		

**II.**

$$I_2^u = \langle b, u^5 + 2u^4 + 4u^3 + 4u^2 + a + 3u + 2, u^6 + u^5 + 3u^4 + 2u^3 + 2u^2 + u - 1 \rangle$$

**(i) Arc colorings**

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^5 - 2u^4 - 4u^3 - 4u^2 - 3u - 2 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^5 - 2u^4 - 4u^3 - 4u^2 - 3u - 2 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^5 - 2u^3 - u \\ -u^5 - u^4 - 2u^3 - u^2 - u + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^3 + 2u \\ u^3 + u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^5 + 2u^3 + u \\ u^5 + u^4 + 2u^3 + u^2 + u - 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -2u^5 - 2u^4 - 6u^3 - 4u^2 - 4u - 2 \\ -u^5 - u^4 - 2u^3 - u^2 - u + 1 \end{pmatrix}$$

**(ii) Obstruction class = 1**

**(iii) Cusp Shapes =  $u^5 - u^4 + 5u^3 + 4u^2 + 7u - 7$**



(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$(u - 1)^6$
$c_3, c_6$	$u^6$
$c_4$	$(u + 1)^6$
$c_5$	$u^6 - u^5 - 3u^4 + 2u^3 + 2u^2 + u - 1$
$c_7, c_8$	$u^6 + u^5 + 3u^4 + 2u^3 + 2u^2 + u - 1$
$c_9, c_{10}, c_{12}$	$u^6 + u^5 - 3u^4 - 2u^3 + 2u^2 - u - 1$
$c_{11}$	$u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y - 1)^6$
$c_3, c_6$	$y^6$
$c_5, c_9, c_{10}$ $c_{12}$	$y^6 - 7y^5 + 17y^4 - 16y^3 + 6y^2 - 5y + 1$
$c_7, c_8, c_{11}$	$y^6 + 5y^5 + 9y^4 + 4y^3 - 6y^2 - 5y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.873214$ $a = -0.422181$ $b = 0$	-9.30502	-14.4810
$u = 0.138835 + 1.234450I$ $a = 0.26610 + 1.72116I$ $b = 0$	$1.31531 - 1.97241I$	$-15.7816 + 4.5012I$
$u = 0.138835 - 1.234450I$ $a = 0.26610 - 1.72116I$ $b = 0$	$1.31531 + 1.97241I$	$-15.7816 - 4.5012I$
$u = -0.408802 + 1.276380I$ $a = -0.417699 - 0.090629I$ $b = 0$	$-5.34051 + 4.59213I$	$-11.43321 - 5.39767I$
$u = -0.408802 - 1.276380I$ $a = -0.417699 + 0.090629I$ $b = 0$	$-5.34051 - 4.59213I$	$-11.43321 + 5.39767I$
$u = 0.413150$ $a = -4.27462$ $b = 0$	-2.38379	-3.08970

$$\text{III. } I_3^u = \langle -au + b - u, -u^2a + a^2 + au - 3u^2 + 2u - 4, u^3 - u^2 + 2u - 1 \rangle$$

(i) Arc colorings

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a \\ au + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} au + a + u \\ au + u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^2 - a + u - 3 \\ -au - u - 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u \\ u^2 - u + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ u^2 - u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^2 + 1 \\ u^2 - u + 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^2 - a + u - 3 \\ -au - u - 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} au - u^2 - a + 2u - 4 \\ -au - u - 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-3u^2a - 5u^2 + 3a + 7u - 15$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_3$	$(u^2 + u - 1)^3$
$c_4, c_6$	$(u^2 - u - 1)^3$
$c_5, c_9$	$u^6$
$c_7, c_8$	$(u^3 - u^2 + 2u - 1)^2$
$c_{10}, c_{12}$	$(u^3 - u^2 + 1)^2$
$c_{11}$	$(u^3 + u^2 + 2u + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_6$	$(y^2 - 3y + 1)^3$
$c_5, c_9$	$y^6$
$c_7, c_8, c_{11}$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_{10}, c_{12}$	$(y^3 - y^2 + 2y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.215080 + 1.307140I$ $a = -1.075750 + 0.460350I$ $b = -0.618034$	$2.03717 - 2.82812I$	$-12.9982 + 11.8301I$
$u = 0.215080 + 1.307140I$ $a = -0.80169 - 1.20521I$ $b = 1.61803$	$-5.85852 - 2.82812I$	$-13.61882 - 1.93520I$
$u = 0.215080 - 1.307140I$ $a = -1.075750 - 0.460350I$ $b = -0.618034$	$2.03717 + 2.82812I$	$-12.9982 - 11.8301I$
$u = 0.215080 - 1.307140I$ $a = -0.80169 + 1.20521I$ $b = 1.61803$	$-5.85852 + 2.82812I$	$-13.61882 + 1.93520I$
$u = 0.569840$ $a = 1.83945$ $b = 1.61803$	$-9.99610$	$-8.90830$
$u = 0.569840$ $a = -2.08457$ $b = -0.618034$	$-2.10041$	$-16.8580$

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$((u-1)^6)(u^2+u-1)^3(u^{21}-10u^{20}+\dots-4u-1)$
$c_3$	$u^6(u^2+u-1)^3(u^{21}+4u^{20}+\dots-128u+64)$
$c_4$	$((u+1)^6)(u^2-u-1)^3(u^{21}-10u^{20}+\dots-4u-1)$
$c_5$	$u^6(u^6-u^5+\dots+u-1)(u^{21}-2u^{20}+\dots+224u+64)$
$c_6$	$u^6(u^2-u-1)^3(u^{21}+4u^{20}+\dots-128u+64)$
$c_7, c_8$	$(u^3-u^2+2u-1)^2(u^6+u^5+3u^4+2u^3+2u^2+u-1)$ $\cdot (u^{21}-4u^{20}+\dots-2u-1)$
$c_9$	$u^6(u^6+u^5+\dots-u-1)(u^{21}-2u^{20}+\dots+224u+64)$
$c_{10}, c_{12}$	$(u^3-u^2+1)^2(u^6+u^5-3u^4-2u^3+2u^2-u-1)$ $\cdot (u^{21}+4u^{20}+\dots-304u-97)$
$c_{11}$	$(u^3+u^2+2u+1)^2(u^6-u^5+3u^4-2u^3+2u^2-u-1)$ $\cdot (u^{21}-4u^{20}+\dots-2u-1)$



## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$((y-1)^6)(y^2-3y+1)^3(y^{21}-4y^{20}+\dots+152y-1)$
$c_3, c_6$	$y^6(y^2-3y+1)^3(y^{21}+30y^{20}+\dots+90112y-4096)$
$c_5, c_9$	$y^6(y^6-7y^5+17y^4-16y^3+6y^2-5y+1)$ $\cdot (y^{21}+28y^{20}+\dots+82944y-4096)$
$c_7, c_8, c_{11}$	$(y^3+3y^2+2y-1)^2(y^6+5y^5+9y^4+4y^3-6y^2-5y+1)$ $\cdot (y^{21}+22y^{20}+\dots-10y-1)$
$c_{10}, c_{12}$	$(y^3-y^2+2y-1)^2(y^6-7y^5+17y^4-16y^3+6y^2-5y+1)$ $\cdot (y^{21}+14y^{20}+\dots-131266y-9409)$