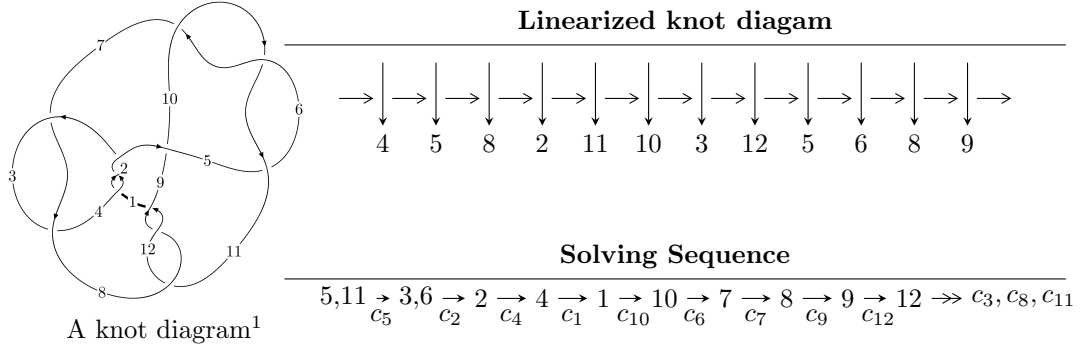


12n₀₆₈₉ (K12n₀₆₈₉)



Ideals for irreducible components² of X_{par}

$$I_1^u = \langle -249863221u^{14} - 685643435u^{13} + \dots + 7149815356b + 6972584040, \\ -9675904336u^{14} - 21495350106u^{13} + \dots + 21449446068a + 126465591430, \\ u^{15} + 2u^{14} + \dots - 4u + 4 \rangle$$

$$I_2^u = \langle b + 1, 2u^5 + 4u^4 + 7u^3 + 8u^2 + 3a + 6u + 5, u^6 + u^5 + 3u^4 + 2u^3 + 2u^2 + u - 1 \rangle$$

$$I_3^u = \langle -au + 3b + 2a - 3, 2a^2 - au - 2a - 2u - 1, u^2 + 2 \rangle$$

$$I_1^v = \langle a, b - v - 2, v^2 + 3v + 1 \rangle$$

* 4 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 27 representations.

¹The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

²All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.

$$I_1^u = \langle -2.50 \times 10^8 u^{14} - 6.86 \times 10^8 u^{13} + \dots + 7.15 \times 10^9 b + 6.97 \times 10^9, -9.68 \times 10^9 u^{14} - 2.15 \times 10^{10} u^{13} + \dots + 2.14 \times 10^{10} a + 1.26 \times 10^{11}, u^{15} + 2u^{14} + \dots - 4u + 4 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.451103u^{14} + 1.00214u^{13} + \dots - 17.5109u - 5.89598 \\ 0.0349468u^{14} + 0.0958967u^{13} + \dots - 1.33742u - 0.975212 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0.486050u^{14} + 1.09804u^{13} + \dots - 18.8483u - 6.87120 \\ 0.0349468u^{14} + 0.0958967u^{13} + \dots - 1.33742u - 0.975212 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.294788u^{14} + 0.657010u^{13} + \dots - 14.7148u - 4.23924 \\ -0.0459078u^{14} - 0.0956177u^{13} + \dots - 0.878005u - 0.288978 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0.130153u^{14} + 0.257727u^{13} + \dots - 6.22583u - 2.21939 \\ -0.0158172u^{14} - 0.0711109u^{13} + \dots + 0.500235u - 0.0616192 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0.201267u^{14} + 0.468199u^{13} + \dots - 7.66259u - 2.24374 \\ 0.0648205u^{14} + 0.172140u^{13} + \dots - 1.47894u - 0.348628 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^3 + 2u \\ u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0.0813622u^{14} + 0.162342u^{13} + \dots - 5.15784u - 1.63915 \\ -0.0550842u^{14} - 0.133716u^{13} + \dots + 1.02581u + 0.255966 \end{pmatrix}$$

(ii) Obstruction class = -1

$$(iii) \text{ Cusp Shapes} = \frac{20309339548}{16087084551} u^{14} + \frac{5027559920}{1787453839} u^{13} + \dots - \frac{319023539806}{5362361517} u - \frac{592867371580}{16087084551}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_2, c_4	$u^{15} - 10u^{14} + \dots - 57u - 9$
c_3, c_7	$u^{15} - 2u^{14} + \dots - 192u + 576$
c_5, c_6, c_{10}	$u^{15} + 2u^{14} + \dots - 4u + 4$
c_8, c_{11}, c_{12}	$u^{15} + 4u^{14} + \dots - 37u - 19$
c_9	$u^{15} - 2u^{14} + \dots - 6348u + 2116$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_2, c_4	$y^{15} + 52y^{13} + \dots + 3177y - 81$
c_3, c_7	$y^{15} + 66y^{14} + \dots + 4349952y - 331776$
c_5, c_6, c_{10}	$y^{15} + 24y^{14} + \dots + 336y - 16$
c_8, c_{11}, c_{12}	$y^{15} - 2y^{14} + \dots + 4561y - 361$
c_9	$y^{15} + 144y^{14} + \dots + 53534800y - 4477456$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.016193 + 1.236613I$ $a = 0.649765 - 0.309826I$ $b = -0.019483 + 0.380184I$	$3.11226 + 1.37153I$	$-7.52048 - 4.70500I$
$u = -0.016193 - 1.236613I$ $a = 0.649765 + 0.309826I$ $b = -0.019483 - 0.380184I$	$3.11226 - 1.37153I$	$-7.52048 + 4.70500I$
$u = -0.382234 + 0.648228I$ $a = 0.093614 + 0.314483I$ $b = -0.910726 - 0.608079I$	$-0.561570 - 0.602510I$	$-12.89865 + 0.16991I$
$u = -0.382234 - 0.648228I$ $a = 0.093614 - 0.314483I$ $b = -0.910726 + 0.608079I$	$-0.561570 + 0.602510I$	$-12.89865 - 0.16991I$
$u = 0.481765$ $a = 0.876017$ $b = 1.52879$	-10.9150	-27.4910
$u = 0.43189 + 1.48192I$ $a = -0.381495 - 0.081254I$ $b = 1.32115 - 0.59294I$	$-5.53981 - 3.37298I$	$-14.3863 + 0.4326I$
$u = 0.43189 - 1.48192I$ $a = -0.381495 + 0.081254I$ $b = 1.32115 + 0.59294I$	$-5.53981 + 3.37298I$	$-14.3863 - 0.4326I$
$u = -1.01904 + 1.21960I$ $a = 0.793015 + 0.629146I$ $b = 1.05096 - 1.67638I$	$4.94872 + 5.24403I$	$-12.95978 - 2.95789I$
$u = -1.01904 - 1.21960I$ $a = 0.793015 - 0.629146I$ $b = 1.05096 + 1.67638I$	$4.94872 - 5.24403I$	$-12.95978 + 2.95789I$
$u = -0.358485$ $a = 0.777590$ $b = -0.141539$	-0.594411	-16.4650

Solutions to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.269953$ $a = -9.24911$ $b = -0.853104$	-2.86090	-50.4000
$u = -0.39688 + 1.74630I$ $a = 0.064779 + 1.248157I$ $b = 1.80094 - 0.90555I$	$14.2231 + 11.2321I$	$-13.44172 - 4.13184I$
$u = -0.39688 - 1.74630I$ $a = 0.064779 - 1.248157I$ $b = 1.80094 + 0.90555I$	$14.2231 - 11.2321I$	$-13.44172 + 4.13184I$
$u = 0.18584 + 2.25780I$ $a = -0.088591 - 0.982699I$ $b = 1.49008 + 2.48794I$	$18.1439 - 0.8477I$	$-12.17052 + 0.18757I$
$u = 0.18584 - 2.25780I$ $a = -0.088591 + 0.982699I$ $b = 1.49008 - 2.48794I$	$18.1439 + 0.8477I$	$-12.17052 - 0.18757I$

II.

$$I_2^u = \langle b+1, 2u^5+4u^4+7u^3+8u^2+3a+6u+5, u^6+u^5+3u^4+2u^3+2u^2+u-1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -\frac{2}{3}u^5 - \frac{4}{3}u^4 + \dots - 2u - \frac{5}{3} \\ -1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -\frac{2}{3}u^5 - \frac{4}{3}u^4 + \dots - 2u - \frac{8}{3} \\ -1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -\frac{2}{3}u^5 - \frac{4}{3}u^4 + \dots - 2u - \frac{5}{3} \\ -1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^3 + 2u \\ u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^5 - 2u^3 - u \\ -u^5 - u^4 - 2u^3 - u^2 - u + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = $\frac{7}{9}u^5 - \frac{31}{9}u^4 - \frac{10}{9}u^3 - \frac{41}{9}u^2 - 2u - \frac{110}{9}$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_2	$(u - 1)^6$
c_3, c_7	u^6
c_4	$(u + 1)^6$
c_5, c_6	$u^6 + u^5 + 3u^4 + 2u^3 + 2u^2 + u - 1$
c_8	$u^6 - u^5 - 3u^4 + 2u^3 + 2u^2 + u - 1$
c_9, c_{11}, c_{12}	$u^6 + u^5 - 3u^4 - 2u^3 + 2u^2 - u - 1$
c_{10}	$u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_2, c_4	$(y - 1)^6$
c_3, c_7	y^6
c_5, c_6, c_{10}	$y^6 + 5y^5 + 9y^4 + 4y^3 - 6y^2 - 5y + 1$
c_8, c_9, c_{11} c_{12}	$y^6 - 7y^5 + 17y^4 - 16y^3 + 6y^2 - 5y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_2^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.873214$ $a = -0.836730$ $b = -1.00000$	-9.30502	-15.6070
$u = 0.138835 + 1.234445I$ $a = -0.366605 + 0.544193I$ $b = -1.00000$	$1.31531 - 1.97241I$	$-11.11410 + 3.48248I$
$u = 0.138835 - 1.234445I$ $a = -0.366605 - 0.544193I$ $b = -1.00000$	$1.31531 + 1.97241I$	$-11.11410 - 3.48248I$
$u = -0.408802 + 1.276377I$ $a = 0.031424 - 0.540243I$ $b = -1.00000$	$-5.34051 + 4.59213I$	$-13.8624 - 6.6392I$
$u = -0.408802 - 1.276377I$ $a = 0.031424 + 0.540243I$ $b = -1.00000$	$-5.34051 - 4.59213I$	$-13.8624 + 6.6392I$
$u = 0.413150$ $a = -3.15957$ $b = -1.00000$	-2.38379	-13.9950

$$\text{III. } I_3^u = \langle -au + 3b + 2a - 3, 2a^2 - au - 2a - 2u - 1, u^2 + 2 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} a \\ \frac{1}{3}au - \frac{2}{3}a + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} \frac{1}{3}au + \frac{1}{3}a + 1 \\ \frac{1}{3}au - \frac{1}{3}a + 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -\frac{1}{3}au + \frac{2}{3}a + \frac{1}{2}u \\ -\frac{1}{3}au + \frac{2}{3}a - 2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} \frac{1}{2}u \\ -\frac{1}{3}au + \frac{2}{3}a - 2 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ -u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} \frac{1}{2}u \\ -\frac{1}{3}au + \frac{2}{3}a - 2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} \frac{1}{2}u \\ -\frac{1}{3}au + \frac{2}{3}a + u - 2 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -16

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_2, c_7	$(u^2 + u - 1)^2$
c_3, c_4	$(u^2 - u - 1)^2$
c_5, c_6, c_9 c_{10}	$(u^2 + 2)^2$
c_8	$(u + 1)^4$
c_{11}, c_{12}	$(u - 1)^4$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_2, c_3 c_4, c_7	$(y^2 - 3y + 1)^2$
c_5, c_6, c_9 c_{10}	$(y + 2)^4$
c_8, c_{11}, c_{12}	$(y - 1)^4$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.414210I$ $a = -0.618034 - 0.437016I$ $b = 1.61803$	-5.59278	-16.0000
$u = 1.414210I$ $a = 1.61803 + 1.14412I$ $b = -0.618034$	2.30291	-16.0000
$u = -1.414210I$ $a = -0.618034 + 0.437016I$ $b = 1.61803$	-5.59278	-16.0000
$u = -1.414210I$ $a = 1.61803 - 1.14412I$ $b = -0.618034$	2.30291	-16.0000

$$\text{IV. } I_1^v = \langle a, b - v - 2, v^2 + 3v + 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ v + 2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} v + 2 \\ v + 2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -v - 2 \\ -v - 3 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ -v - 3 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ v + 3 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} v - 1 \\ -v - 3 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -6

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_2, c_3	$u^2 + u - 1$
c_4, c_7	$u^2 - u - 1$
c_5, c_6, c_9 c_{10}	u^2
c_8	$(u - 1)^2$
c_{11}, c_{12}	$(u + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_2, c_3 c_4, c_7	$y^2 - 3y + 1$
c_5, c_6, c_9 c_{10}	y^2
c_8, c_{11}, c_{12}	$(y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_1^v	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$v = -0.381966$ $a = 0$ $b = 1.61803$	-10.5276	-6.00000
$v = -2.61803$ $a = 0$ $b = -0.618034$	-2.63189	-6.00000

V. u-Polynomials

Crossings	u-Polynomials at each crossing
c_1, c_2	$((u-1)^6)(u^2+u-1)^3(u^{15}-10u^{14}+\dots-57u-9)$
c_3	$u^6(u^2-u-1)^2(u^2+u-1)(u^{15}-2u^{14}+\dots-192u+576)$
c_4	$((u+1)^6)(u^2-u-1)^3(u^{15}-10u^{14}+\dots-57u-9)$
c_5, c_6	$u^2(u^2+2)^2(u^6+u^5+\dots+u-1)(u^{15}+2u^{14}+\dots-4u+4)$
c_7	$u^6(u^2-u-1)(u^2+u-1)^2(u^{15}-2u^{14}+\dots-192u+576)$
c_8	$(u-1)^2(u+1)^4(u^6-u^5-3u^4+2u^3+2u^2+u-1)$ $\cdot (u^{15}+4u^{14}+\dots-37u-19)$
c_9	$u^2(u^2+2)^2(u^6+u^5-3u^4-2u^3+2u^2-u-1)$ $\cdot (u^{15}-2u^{14}+\dots-6348u+2116)$
c_{10}	$u^2(u^2+2)^2(u^6-u^5+\dots-u-1)(u^{15}+2u^{14}+\dots-4u+4)$
c_{11}, c_{12}	$(u-1)^4(u+1)^2(u^6+u^5-3u^4-2u^3+2u^2-u-1)$ $\cdot (u^{15}+4u^{14}+\dots-37u-19)$

VI. Riley Polynomials

Crossings	Riley Polynomials at each crossing
c_1, c_2, c_4	$((y - 1)^6)(y^2 - 3y + 1)^3(y^{15} + 52y^{13} + \dots + 3177y - 81)$
c_3, c_7	$y^6(y^2 - 3y + 1)^3(y^{15} + 66y^{14} + \dots + 4349952y - 331776)$
c_5, c_6, c_{10}	$y^2(y + 2)^4(y^6 + 5y^5 + 9y^4 + 4y^3 - 6y^2 - 5y + 1)$ $\cdot (y^{15} + 24y^{14} + \dots + 336y - 16)$
c_8, c_{11}, c_{12}	$(y - 1)^6(y^6 - 7y^5 + 17y^4 - 16y^3 + 6y^2 - 5y + 1)$ $\cdot (y^{15} - 2y^{14} + \dots + 4561y - 361)$
c_9	$y^2(y + 2)^4(y^6 - 7y^5 + 17y^4 - 16y^3 + 6y^2 - 5y + 1)$ $\cdot (y^{15} + 144y^{14} + \dots + 53534800y - 4477456)$