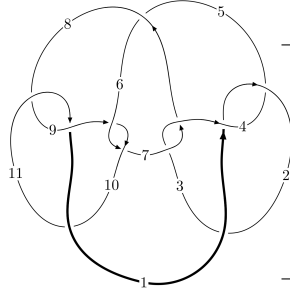
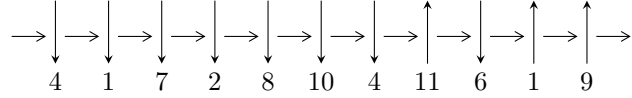


11n<sub>23</sub> (K11n<sub>23</sub>)



A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**

$$9,11 \xrightarrow{c_{11}} 1,4 \xrightarrow{c_1} 2 \xrightarrow{c_4} 5 \xrightarrow{c_8} 8 \xrightarrow{c_7} 7 \xrightarrow{c_3} 3 \xrightarrow{c_{10}} 10 \xrightarrow{c_6} 6 \longrightarrow c_2, c_5, c_9$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -68646u^{21} - 209213u^{20} + \dots + 105421b - 71719, \\ -179395u^{21} - 675073u^{20} + \dots + 210842a - 1064489, u^{22} + 4u^{21} + \dots + 8u + 1 \rangle$$

$$I_2^u = \langle -u^4 - u^3 + b + u, u^4 + u^3 + a - u, u^6 + u^5 - u^4 - 2u^3 + u + 1 \rangle$$

$$I_3^u = \langle b + 1, a^2 - a - 1, u - 1 \rangle$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 30 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.

$$I_1^u = \langle -6.86 \times 10^4 u^{21} - 2.09 \times 10^5 u^{20} + \dots + 1.05 \times 10^5 b - 7.17 \times 10^4, -1.79 \times 10^5 u^{21} - 6.75 \times 10^5 u^{20} + \dots + 2.11 \times 10^5 a - 1.06 \times 10^6, u^{22} + 4u^{21} + \dots + 8u + 1 \rangle$$

(i) Arc colorings

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.850850u^{21} + 3.20180u^{20} + \dots + 4.49725u + 5.04875 \\ 0.651161u^{21} + 1.98455u^{20} + \dots + 2.21543u + 0.680310 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0.315042u^{21} + 1.37186u^{20} + \dots + 0.488432u + 3.18691 \\ 0.182947u^{21} + 0.441795u^{20} + \dots - 0.201113u + 0.0840297 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0.146826u^{21} + 1.01918u^{20} + \dots + 1.95979u + 3.15823 \\ 0.351163u^{21} + 0.794481u^{20} + \dots - 1.67248u + 0.112710 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -0.112710u^{21} - 0.0996765u^{20} + \dots - 0.646560u - 2.57416 \\ 0.431873u^{21} + 0.691897u^{20} + \dots + 1.98362u - 0.146826 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.688814u^{21} + 2.58804u^{20} + \dots + 1.89815u + 3.21457 \\ -0.166694u^{21} - 0.538261u^{20} + \dots - 2.05866u - 0.194885 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^2 + 1 \\ u^4 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -0.317366u^{21} - 1.15089u^{20} + \dots - 1.03139u - 2.97993 \\ -0.180623u^{21} - 0.662766u^{20} + \dots + 0.744069u - 0.291009 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -0.317366u^{21} - 1.15089u^{20} + \dots - 1.03139u - 2.97993 \\ -0.180623u^{21} - 0.662766u^{20} + \dots + 0.744069u - 0.291009 \end{pmatrix}$$

(ii) Obstruction class = -1

$$(iii) \text{ Cusp Shapes} = \frac{98871}{105421}u^{21} + \frac{399034}{105421}u^{20} + \dots + \frac{1542002}{105421}u - \frac{1060356}{105421}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_4$	$u^{22} - 8u^{21} + \dots - 10u + 1$
$c_2$	$u^{22} + 36u^{21} + \dots + 6u + 1$
$c_3, c_7$	$u^{22} + 2u^{21} + \dots + 128u^2 + 64$
$c_5$	$u^{22} - 3u^{21} + \dots - u + 1$
$c_6, c_9$	$u^{22} + 2u^{21} + \dots + 28u + 4$
$c_8, c_{11}$	$u^{22} + 4u^{21} + \dots + 8u + 1$
$c_{10}$	$u^{22} - 8u^{21} + \dots - 64u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$y^{22} - 36y^{21} + \dots - 6y + 1$
$c_2$	$y^{22} - 92y^{21} + \dots + 3898y + 1$
$c_3, c_7$	$y^{22} - 42y^{21} + \dots + 16384y + 4096$
$c_5$	$y^{22} - 49y^{21} + \dots - 17y + 1$
$c_6, c_9$	$y^{22} - 18y^{21} + \dots - 264y + 16$
$c_8, c_{11}$	$y^{22} - 8y^{21} + \dots - 64y + 1$
$c_{10}$	$y^{22} + 16y^{21} + \dots - 3112y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.949302 + 0.242875I$ $a = 0.659054 + 0.118437I$ $b = -0.606013 - 0.510709I$	$1.72824 + 0.76607I$	$3.12936 - 1.22783I$
$u = 0.949302 - 0.242875I$ $a = 0.659054 - 0.118437I$ $b = -0.606013 + 0.510709I$	$1.72824 - 0.76607I$	$3.12936 + 1.22783I$
$u = 1.06873$ $a = 2.88944$ $b = -2.44319$	$0.373053$	$-36.4230$
$u = -0.611771 + 0.692060I$ $a = 0.790435 - 0.254867I$ $b = 0.122623 + 0.098224I$	$-2.04648 + 0.07308I$	$-6.61841 + 0.32192I$
$u = -0.611771 - 0.692060I$ $a = 0.790435 + 0.254867I$ $b = 0.122623 - 0.098224I$	$-2.04648 - 0.07308I$	$-6.61841 - 0.32192I$
$u = -0.831560$ $a = -0.842263$ $b = -0.991565$	$-7.60774$	$-21.1720$
$u = -1.040460 + 0.605021I$ $a = 0.213779 - 0.135252I$ $b = -0.431084 + 0.709735I$	$-0.69505 - 5.13446I$	$-2.61215 + 4.09914I$
$u = -1.040460 - 0.605021I$ $a = 0.213779 + 0.135252I$ $b = -0.431084 - 0.709735I$	$-0.69505 + 5.13446I$	$-2.61215 - 4.09914I$
$u = -0.837414 + 0.879532I$ $a = -0.820247 + 0.387550I$ $b = 0.79848 - 1.81489I$	$-6.52288 - 1.21996I$	$-10.21847 + 1.61822I$
$u = -0.837414 - 0.879532I$ $a = -0.820247 - 0.387550I$ $b = 0.79848 + 1.81489I$	$-6.52288 + 1.21996I$	$-10.21847 - 1.61822I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.908141 + 0.818565I$ $a = 1.68712 + 1.20696I$ $b = -0.40292 - 2.59906I$	$-12.75240 + 3.06700I$	$-8.27093 - 2.19380I$
$u = 0.908141 - 0.818565I$ $a = 1.68712 - 1.20696I$ $b = -0.40292 + 2.59906I$	$-12.75240 - 3.06700I$	$-8.27093 + 2.19380I$
$u = -0.522764 + 1.140680I$ $a = 1.47562 - 0.18862I$ $b = -0.27191 + 1.80152I$	$-18.8826 + 3.9450I$	$-10.61812 - 1.00166I$
$u = -0.522764 - 1.140680I$ $a = 1.47562 + 0.18862I$ $b = -0.27191 - 1.80152I$	$-18.8826 - 3.9450I$	$-10.61812 + 1.00166I$
$u = -0.982971 + 0.827014I$ $a = -1.20538 + 1.02888I$ $b = -0.113345 - 1.400720I$	$-6.06672 - 5.11915I$	$-9.50515 + 3.92885I$
$u = -0.982971 - 0.827014I$ $a = -1.20538 - 1.02888I$ $b = -0.113345 + 1.400720I$	$-6.06672 + 5.11915I$	$-9.50515 - 3.92885I$
$u = 0.569732 + 0.260828I$ $a = -2.62584 - 0.99559I$ $b = 1.40111 + 0.75463I$	$-0.990371 + 0.924237I$	$-8.66470 - 0.43219I$
$u = 0.569732 - 0.260828I$ $a = -2.62584 + 0.99559I$ $b = 1.40111 - 0.75463I$	$-0.990371 - 0.924237I$	$-8.66470 + 0.43219I$
$u = -1.22965 + 0.77380I$ $a = 0.90635 - 1.49914I$ $b = 0.11518 + 2.75329I$	$-16.6374 - 10.8083I$	$-8.69774 + 4.99684I$
$u = -1.22965 - 0.77380I$ $a = 0.90635 + 1.49914I$ $b = 0.11518 - 2.75329I$	$-16.6374 + 10.8083I$	$-8.69774 - 4.99684I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.49238$ $a = -0.327992$ $b = -1.18058$	-10.9429	-8.28670
$u = -0.133848$ $a = 4.11903$ $b = 0.391099$	-0.845350	-11.9660

$$\text{II. } I_2^u = \langle -u^4 - u^3 + b + u, u^4 + u^3 + a - u, u^6 + u^5 - u^4 - 2u^3 + u + 1 \rangle$$

(i) Arc colorings

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^4 - u^3 + u \\ u^4 + u^3 - u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u^4 - u^3 + u + 1 \\ u^4 + u^3 - u^2 - u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^4 - u^3 + u \\ u^4 + u^3 - u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^2 + 1 \\ u^4 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^4 + u^2 - 1 \\ u^4 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^4 + u^2 - 1 \\ u^4 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-3u^4 + 2u^3 + 3u^2 + 2u - 11$



(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$(u - 1)^6$
$c_2, c_4$	$(u + 1)^6$
$c_3, c_7$	$u^6$
$c_5$	$u^6 - 3u^5 + 5u^4 - 4u^3 + 2u^2 - u + 1$
$c_6, c_{11}$	$u^6 + u^5 - u^4 - 2u^3 + u + 1$
$c_8, c_9$	$u^6 - u^5 - u^4 + 2u^3 - u + 1$
$c_{10}$	$u^6 + 3u^5 + 5u^4 + 4u^3 + 2u^2 + u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y - 1)^6$
$c_3, c_7$	$y^6$
$c_5, c_{10}$	$y^6 + y^5 + 5y^4 + 6y^2 + 3y + 1$
$c_6, c_8, c_9$ $c_{11}$	$y^6 - 3y^5 + 5y^4 - 4y^3 + 2y^2 - y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.002190 + 0.295542I$ $a = -0.23185 - 1.65564I$ $b = 0.23185 + 1.65564I$	$0.245672 + 0.924305I$	$-6.22669 + 0.83820I$
$u = 1.002190 - 0.295542I$ $a = -0.23185 + 1.65564I$ $b = 0.23185 - 1.65564I$	$0.245672 - 0.924305I$	$-6.22669 - 0.83820I$
$u = -0.428243 + 0.664531I$ $a = -0.659772 + 0.298454I$ $b = 0.659772 - 0.298454I$	$-3.53554 + 0.92430I$	$-10.88169 - 1.11590I$
$u = -0.428243 - 0.664531I$ $a = -0.659772 - 0.298454I$ $b = 0.659772 + 0.298454I$	$-3.53554 - 0.92430I$	$-10.88169 + 1.11590I$
$u = -1.073950 + 0.558752I$ $a = -0.108378 + 0.818891I$ $b = 0.108378 - 0.818891I$	$-1.64493 - 5.69302I$	$-8.89162 + 7.09196I$
$u = -1.073950 - 0.558752I$ $a = -0.108378 - 0.818891I$ $b = 0.108378 + 0.818891I$	$-1.64493 + 5.69302I$	$-8.89162 - 7.09196I$

$$\text{III. } I_3^u = \langle b + 1, a^2 - a - 1, u - 1 \rangle$$

(i) Arc colorings

$$a_9 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a \\ -1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 2 \\ -a \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -a + 2 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ -a + 2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} a \\ -2a + 2 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0 \\ -a + 2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0 \\ -a + 2 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = 1

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_3$	$u^2 + u - 1$
$c_2$	$u^2 + 3u + 1$
$c_4, c_7$	$u^2 - u - 1$
$c_5$	$u^2 - 3u + 1$
$c_6, c_9$	$u^2$
$c_8, c_{10}$	$(u + 1)^2$
$c_{11}$	$(u - 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_3, c_4$ $c_7$	$y^2 - 3y + 1$
$c_2, c_5$	$y^2 - 7y + 1$
$c_6, c_9$	$y^2$
$c_8, c_{10}, c_{11}$	$(y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$ $a = -0.618034$ $b = -1.00000$	-7.23771	1.00000
$u = 1.00000$ $a = 1.61803$ $b = -1.00000$	0.657974	1.00000

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u-1)^6)(u^2+u-1)(u^{22}-8u^{21}+\dots-10u+1)$
$c_2$	$((u+1)^6)(u^2+3u+1)(u^{22}+36u^{21}+\dots+6u+1)$
$c_3$	$u^6(u^2+u-1)(u^{22}+2u^{21}+\dots+128u^2+64)$
$c_4$	$((u+1)^6)(u^2-u-1)(u^{22}-8u^{21}+\dots-10u+1)$
$c_5$	$(u^2-3u+1)(u^6-3u^5+5u^4-4u^3+2u^2-u+1)$ $\cdot (u^{22}-3u^{21}+\dots-u+1)$
$c_6$	$u^2(u^6+u^5+\dots+u+1)(u^{22}+2u^{21}+\dots+28u+4)$
$c_7$	$u^6(u^2-u-1)(u^{22}+2u^{21}+\dots+128u^2+64)$
$c_8$	$((u+1)^2)(u^6-u^5+\dots-u+1)(u^{22}+4u^{21}+\dots+8u+1)$
$c_9$	$u^2(u^6-u^5+\dots-u+1)(u^{22}+2u^{21}+\dots+28u+4)$
$c_{10}$	$(u+1)^2(u^6+3u^5+5u^4+4u^3+2u^2+u+1)$ $\cdot (u^{22}-8u^{21}+\dots-64u+1)$
$c_{11}$	$((u-1)^2)(u^6+u^5+\dots+u+1)(u^{22}+4u^{21}+\dots+8u+1)$



## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$((y - 1)^6)(y^2 - 3y + 1)(y^{22} - 36y^{21} + \dots - 6y + 1)$
$c_2$	$((y - 1)^6)(y^2 - 7y + 1)(y^{22} - 92y^{21} + \dots + 3898y + 1)$
$c_3, c_7$	$y^6(y^2 - 3y + 1)(y^{22} - 42y^{21} + \dots + 16384y + 4096)$
$c_5$	$(y^2 - 7y + 1)(y^6 + y^5 + \dots + 3y + 1)(y^{22} - 49y^{21} + \dots - 17y + 1)$
$c_6, c_9$	$y^2(y^6 - 3y^5 + \dots - y + 1)(y^{22} - 18y^{21} + \dots - 264y + 16)$
$c_8, c_{11}$	$(y - 1)^2(y^6 - 3y^5 + 5y^4 - 4y^3 + 2y^2 - y + 1)$ $\cdot (y^{22} - 8y^{21} + \dots - 64y + 1)$
$c_{10}$	$((y - 1)^2)(y^6 + y^5 + \dots + 3y + 1)(y^{22} + 16y^{21} + \dots - 3112y + 1)$