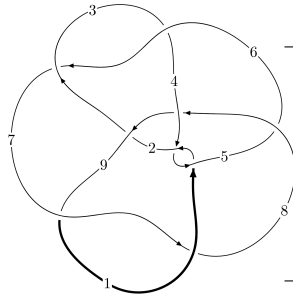
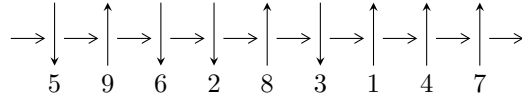


9₂₉ (K9a₃₁)

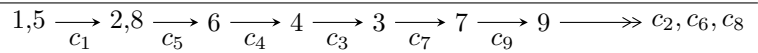


A knot diagram¹

Linearized knot diagram



Solving Sequence



Ideals for irreducible components² of X_{par}

$$\begin{aligned}
 I_1^u &= \langle u^9 - 2u^8 + 6u^7 - 9u^6 + 13u^5 - 19u^4 + 14u^3 - 12u^2 + 4b + 7u - 1, \\
 &\quad 3u^9 - 6u^8 + 18u^7 - 31u^6 + 47u^5 - 65u^4 + 58u^3 - 56u^2 + 8a + 29u - 11, \\
 &\quad u^{10} - u^9 + 4u^8 - 7u^7 + 8u^6 - 14u^5 + 11u^4 - 10u^3 + 7u^2 - 2u - 1 \rangle \\
 I_2^u &= \langle 3488u^{15} + 8516u^{14} + \dots + 887b + 5098, 5348u^{15} + 12394u^{14} + \dots + 887a + 7607, \\
 &\quad u^{16} + 3u^{15} + \dots + 2u + 1 \rangle \\
 I_3^u &= \langle b - 1, 2a - 1, u - 1 \rangle
 \end{aligned}$$

* 3 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 27 representations.

¹The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

²All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle u^9 - 2u^8 + \dots + 4b - 1, 3u^9 - 6u^8 + \dots + 8a - 11, u^{10} - u^9 + \dots - 2u - 1 \rangle \quad \text{I.}$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -\frac{3}{8}u^9 + \frac{3}{4}u^8 + \dots - \frac{29}{8}u + \frac{11}{8} \\ -\frac{1}{4}u^9 + \frac{1}{2}u^8 + \dots - \frac{7}{4}u + \frac{1}{4} \end{pmatrix}$$

$$a_6 = \begin{pmatrix} \frac{1}{16}u^9 - \frac{1}{8}u^8 + \dots + \frac{31}{16}u - \frac{17}{16} \\ \frac{1}{8}u^9 - \frac{1}{4}u^8 + \dots + \frac{15}{8}u - \frac{1}{8} \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} \frac{1}{16}u^9 - \frac{1}{8}u^8 + \dots + \frac{31}{16}u - \frac{1}{16} \\ \frac{1}{8}u^9 - \frac{1}{4}u^8 + \dots + \frac{7}{8}u - \frac{1}{8} \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -\frac{1}{8}u^9 + \frac{1}{4}u^8 + \dots - \frac{15}{8}u + \frac{9}{8} \\ -\frac{1}{4}u^9 + \frac{1}{2}u^8 + \dots - \frac{7}{4}u + \frac{1}{4} \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -\frac{7}{8}u^9 + \frac{3}{4}u^8 + \dots - \frac{17}{8}u + \frac{15}{8} \\ \frac{3}{4}u^9 - \frac{1}{2}u^8 + \dots - \frac{7}{4}u + \frac{1}{4} \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -\frac{7}{8}u^9 + \frac{3}{4}u^8 + \dots - \frac{17}{8}u + \frac{15}{8} \\ \frac{3}{4}u^9 - \frac{1}{2}u^8 + \dots - \frac{7}{4}u + \frac{1}{4} \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -\frac{37}{16}u^9 - \frac{3}{8}u^8 - \frac{55}{8}u^7 + \frac{145}{16}u^6 - \frac{41}{16}u^5 + \frac{351}{16}u^4 - \frac{35}{8}u^3 + \frac{19}{2}u^2 - \frac{219}{16}u + \frac{85}{16}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_3, c_4 c_6	$u^{10} + u^9 + 4u^8 + 7u^7 + 8u^6 + 14u^5 + 11u^4 + 10u^3 + 7u^2 + 2u - 1$
c_2, c_5	$2(2u^{10} + 3u^9 - 4u^8 - 8u^7 + 9u^6 + 7u^5 - 5u^4 - 2u^3 - u + 1)$
c_7, c_9	$u^{10} - 4u^8 + u^7 + 5u^6 - 3u^5 + 12u^4 + 18u^3 - 7u^2 - 11u - 4$
c_8	$u^{10} - 3u^9 + 3u^8 + 8u^7 - 7u^6 - 30u^5 + 80u^4 - 60u^3 + 41u^2 - 30u + 8$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_3, c_4 c_6	$y^{10} + 7y^9 + \dots - 18y + 1$
c_2, c_5	$4(4y^{10} - 25y^9 + \dots - y + 1)$
c_7, c_9	$y^{10} - 8y^9 + \dots - 65y + 16$
c_8	$y^{10} - 3y^9 + \dots - 244y + 64$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.642531 + 0.377867I$ $a = 0.425417 + 0.618053I$ $b = -0.388235 + 0.305929I$	$-1.18006 - 1.03831I$	$-4.73685 + 3.71172I$
$u = 0.642531 - 0.377867I$ $a = 0.425417 - 0.618053I$ $b = -0.388235 - 0.305929I$	$-1.18006 + 1.03831I$	$-4.73685 - 3.71172I$
$u = -0.296868 + 1.222110I$ $a = -0.265428 + 0.874553I$ $b = 0.310628 + 1.327070I$	$4.73127 + 5.96240I$	$6.55763 - 6.45237I$
$u = -0.296868 - 1.222110I$ $a = -0.265428 - 0.874553I$ $b = 0.310628 - 1.327070I$	$4.73127 - 5.96240I$	$6.55763 + 6.45237I$
$u = 0.090479 + 1.266340I$ $a = -0.180352 - 0.660546I$ $b = 1.72873 - 0.67558I$	$8.92450 - 2.36890I$	$11.53570 + 2.96432I$
$u = 0.090479 - 1.266340I$ $a = -0.180352 + 0.660546I$ $b = 1.72873 + 0.67558I$	$8.92450 + 2.36890I$	$11.53570 - 2.96432I$
$u = 1.36651$ $a = -0.570064$ $b = -1.16409$	0.587104	12.3230
$u = -0.50395 + 1.40837I$ $a = 0.204381 - 1.196050I$ $b = -1.50564 - 0.50027I$	$10.4508 + 12.2059I$	$7.05765 - 6.58910I$
$u = -0.50395 - 1.40837I$ $a = 0.204381 + 1.196050I$ $b = -1.50564 + 0.50027I$	$10.4508 - 12.2059I$	$7.05765 + 6.58910I$
$u = -0.230893$ $a = 2.70203$ $b = 0.873110$	1.26306	9.09880

$$\text{II. } I_2^u = \langle 3488u^{15} + 8516u^{14} + \dots + 887b + 5098, 5348u^{15} + 12394u^{14} + \dots + 887a + 7607, u^{16} + 3u^{15} + \dots + 2u + 1 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -6.02931u^{15} - 13.9729u^{14} + \dots - 2.86809u - 8.57610 \\ -3.93236u^{15} - 9.60090u^{14} + \dots - 2.30440u - 5.74746 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 7.33709u^{15} + 15.6888u^{14} + \dots + 5.98309u + 15.1251 \\ 3.02593u^{15} + 7.05299u^{14} + \dots + 3.38331u + 5.16347 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -4.12176u^{15} - 9.11838u^{14} + \dots + 4.54791u - 3.85457 \\ -0.676437u^{15} - 1.99098u^{14} + \dots + 2.04397u - 0.525366 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -2.09696u^{15} - 4.37204u^{14} + \dots - 0.563698u - 2.82864 \\ -3.93236u^{15} - 9.60090u^{14} + \dots - 2.30440u - 5.74746 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -4.00451u^{15} - 9.22661u^{14} + \dots - 1.97971u - 5.55017 \\ -1.15896u^{15} - 3.23788u^{14} + \dots - 0.784667u - 1.39346 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -4.00451u^{15} - 9.22661u^{14} + \dots - 1.97971u - 5.55017 \\ -1.15896u^{15} - 3.23788u^{14} + \dots - 0.784667u - 1.39346 \end{pmatrix}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = \frac{14664}{887}u^{15} + \frac{36136}{887}u^{14} + \dots + \frac{12068}{887}u + \frac{27426}{887}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_3, c_4 c_6	$u^{16} - 3u^{15} + \dots - 2u + 1$
c_2, c_5	$u^{16} - u^{15} + \dots + 136u + 47$
c_7, c_9	$(u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1)^2$
c_8	$(u^8 + u^7 - u^6 - 2u^5 + u^4 + 2u^3 - 2u - 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_3, c_4 c_6	$y^{16} + 11y^{15} + \dots + 20y^2 + 1$
c_2, c_5	$y^{16} - 9y^{15} + \dots - 13044y + 2209$
c_7, c_9	$(y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1)^2$
c_8	$(y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_2^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.181988 + 1.048500I$ $a = 1.25894 + 1.17937I$ $b = 0.463640$	4.13490	$7.89446 + 0.I$
$u = -0.181988 - 1.048500I$ $a = 1.25894 - 1.17937I$ $b = 0.463640$	4.13490	$7.89446 + 0.I$
$u = -1.142130 + 0.104845I$ $a = -0.895766 - 0.516597I$ $b = -1.334530 - 0.318930I$	$5.66955 + 6.44354I$	$5.42845 - 5.29417I$
$u = -1.142130 - 0.104845I$ $a = -0.895766 + 0.516597I$ $b = -1.334530 + 0.318930I$	$5.66955 - 6.44354I$	$5.42845 + 5.29417I$
$u = 0.309237 + 1.112330I$ $a = 0.034672 - 0.683601I$ $b = 0.108090 - 0.747508I$	$1.13045 - 2.57849I$	$0.27708 + 3.56796I$
$u = 0.309237 - 1.112330I$ $a = 0.034672 + 0.683601I$ $b = 0.108090 + 0.747508I$	$1.13045 + 2.57849I$	$0.27708 - 3.56796I$
$u = -0.072810 + 1.153150I$ $a = -1.02661 + 1.10040I$ $b = 1.180120 + 0.268597I$	$4.33052 + 1.13123I$	$3.41522 - 0.51079I$
$u = -0.072810 - 1.153150I$ $a = -1.02661 - 1.10040I$ $b = 1.180120 - 0.268597I$	$4.33052 - 1.13123I$	$3.41522 + 0.51079I$
$u = -0.597255 + 0.026660I$ $a = 1.20070 - 1.29659I$ $b = 0.108090 - 0.747508I$	$1.13045 - 2.57849I$	$0.27708 + 3.56796I$
$u = -0.597255 - 0.026660I$ $a = 1.20070 + 1.29659I$ $b = 0.108090 + 0.747508I$	$1.13045 + 2.57849I$	$0.27708 - 3.56796I$

Solutions to I_2^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.50715 + 1.45748I$		
$a = 0.219942 + 0.896459I$	$5.66955 - 6.44354I$	$5.42845 + 5.29417I$
$b = -1.334530 + 0.318930I$		
$u = 0.50715 - 1.45748I$		
$a = 0.219942 - 0.896459I$	$5.66955 + 6.44354I$	$5.42845 - 5.29417I$
$b = -1.334530 - 0.318930I$		
$u = -0.60300 + 1.44597I$		
$a = -0.091711 - 0.669730I$	9.79260	$9.86404 + 0.I$
$b = -1.37100$		
$u = -0.60300 - 1.44597I$		
$a = -0.091711 + 0.669730I$	9.79260	$9.86404 + 0.I$
$b = -1.37100$		
$u = 0.280801 + 0.318917I$		
$a = 3.29984 - 0.74872I$	$4.33052 - 1.13123I$	$3.41522 + 0.51079I$
$b = 1.180120 - 0.268597I$		
$u = 0.280801 - 0.318917I$		
$a = 3.29984 + 0.74872I$	$4.33052 + 1.13123I$	$3.41522 - 0.51079I$
$b = 1.180120 + 0.268597I$		

$$\text{III. } I_3^u = \langle b - 1, 2a - 1, u - 1 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0.5 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0.25 \\ 1.5 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.75 \\ 0.5 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -0.5 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0.5 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0.5 \\ 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -2.25

(iv) **u**-Polynomials at the component

Crossings	u -Polynomials at each crossing
c_1, c_3, c_9	$u - 1$
c_2	$2(2u - 1)$
c_4, c_6, c_7	$u + 1$
c_5	$2(2u + 1)$
c_8	u

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_3, c_4 c_6, c_7, c_9	$y - 1$
c_2, c_5	$4(4y - 1)$
c_8	y

(vi) Complex Volumes and Cusp Shapes

Solutions to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$		
$a = 0.500000$	0	-2.25000
$b = 1.00000$		

IV. u-Polynomials

Crossings	u-Polynomials at each crossing
c_1, c_3	$(u-1)(u^{10} + u^9 + \dots + 2u - 1)$ $\cdot (u^{16} - 3u^{15} + \dots - 2u + 1)$
c_2	$4(2u-1)(2u^{10} + 3u^9 - 4u^8 - 8u^7 + 9u^6 + 7u^5 - 5u^4 - 2u^3 - u + 1)$ $\cdot (u^{16} - u^{15} + \dots + 136u + 47)$
c_4, c_6	$(u+1)(u^{10} + u^9 + \dots + 2u - 1)$ $\cdot (u^{16} - 3u^{15} + \dots - 2u + 1)$
c_5	$4(2u+1)(2u^{10} + 3u^9 - 4u^8 - 8u^7 + 9u^6 + 7u^5 - 5u^4 - 2u^3 - u + 1)$ $\cdot (u^{16} - u^{15} + \dots + 136u + 47)$
c_7	$(u+1)(u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1)^2$ $\cdot (u^{10} - 4u^8 + u^7 + 5u^6 - 3u^5 + 12u^4 + 18u^3 - 7u^2 - 11u - 4)$
c_8	$u(u^8 + u^7 - u^6 - 2u^5 + u^4 + 2u^3 - 2u - 1)^2$ $\cdot (u^{10} - 3u^9 + 3u^8 + 8u^7 - 7u^6 - 30u^5 + 80u^4 - 60u^3 + 41u^2 - 30u + 8)$
c_9	$(u-1)(u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1)^2$ $\cdot (u^{10} - 4u^8 + u^7 + 5u^6 - 3u^5 + 12u^4 + 18u^3 - 7u^2 - 11u - 4)$

V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
c_1, c_3, c_4 c_6	$(y - 1)(y^{10} + 7y^9 + \dots - 18y + 1)(y^{16} + 11y^{15} + \dots + 20y^2 + 1)$
c_2, c_5	$16(4y - 1)(4y^{10} - 25y^9 + \dots - y + 1)$ $\cdot (y^{16} - 9y^{15} + \dots - 13044y + 2209)$
c_7, c_9	$(y - 1)(y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1)^2$ $\cdot (y^{10} - 8y^9 + \dots - 65y + 16)$
c_8	$y(y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1)^2$ $\cdot (y^{10} - 3y^9 + \dots - 244y + 64)$