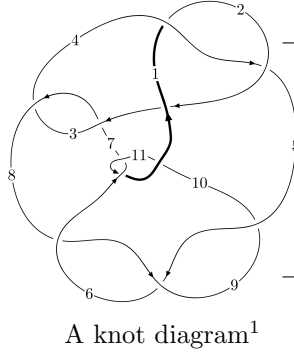
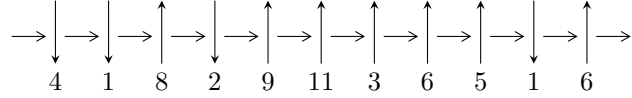


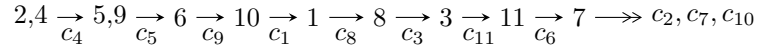
11n<sub>65</sub> (K11n<sub>65</sub>)



**Linearized knot diagram**



**Solving Sequence**



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$\begin{aligned}
 I_1^u &= \langle 2666u^{12} - 2811u^{11} + \dots + 9382b + 3250, -1563u^{12} + 2980u^{11} + \dots + 18764a + 7547, \\
 &\quad u^{13} - 2u^{12} - 2u^{11} + 7u^{10} - u^9 - 9u^8 + 16u^7 - 18u^6 - 2u^5 + 33u^4 - 24u^3 - 10u^2 + 17u - 4 \rangle \\
 I_2^u &= \langle -u^4 + u^3 + 2u^2 + b - a - u - 1, u^4 - 2u^2a - 2u^3 + a^2 + au + 2a + u + 2, u^5 - u^4 - 2u^3 + u^2 + u + 1 \rangle \\
 I_3^u &= \langle -u^2a + au - u^2 + b + u - 1, 2u^2a + a^2 - 4au + 3u^2 + 2a - 6u + 5, u^3 - u^2 + 1 \rangle \\
 I_4^u &= \langle 2b + 1, 2a - 1, u + 1 \rangle
 \end{aligned}$$

\* 4 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 30 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\mathbf{I. } I_1^u = \langle 2666u^{12} - 2811u^{11} + \dots + 9382b + 3250, -1563u^{12} + 2980u^{11} + \dots + 18764a + 7547, u^{13} - 2u^{12} + \dots + 17u - 4 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0.0832978u^{12} - 0.158815u^{11} + \dots - 2.36613u - 0.402206 \\ -0.284161u^{12} + 0.299616u^{11} + \dots + 3.21925u - 0.346408 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -0.0329887u^{12} + 0.0405031u^{11} + \dots + 0.801428u + 1.42640 \\ 0.0613942u^{12} - 0.0681091u^{11} + \dots - 0.833191u + 0.0125773 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0.00314432u^{12} + 0.0551055u^{11} + \dots + 0.652206u - 0.779738 \\ -0.0254743u^{12} + 0.129823u^{11} + \dots + 1.98721u - 0.131955 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -0.0853763u^{12} - 0.131848u^{11} + \dots - 2.30228u - 1.25187 \\ -0.481454u^{12} + 0.401300u^{11} + \dots + 4.61810u - 0.548284 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^3 \\ -u^3 + u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -0.168674u^{12} + 0.0269665u^{11} + \dots + 1.06385u - 0.849659 \\ -0.197293u^{12} + 0.101684u^{11} + \dots + 2.39885u - 0.201876 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0.334683u^{12} - 0.465039u^{11} + \dots - 4.08719u + 3.28384 \\ 0.169154u^{12} - 0.382967u^{11} + \dots - 3.37114u + 0.654445 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0.334683u^{12} - 0.465039u^{11} + \dots - 4.08719u + 3.28384 \\ 0.169154u^{12} - 0.382967u^{11} + \dots - 3.37114u + 0.654445 \end{pmatrix}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = \frac{34825}{18764}u^{12} - \frac{68759}{18764}u^{11} + \dots - \frac{496503}{18764}u + \frac{97945}{4691}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_4$	$u^{13} - 2u^{12} + \dots + 17u - 4$
$c_2$	$u^{13} + 8u^{12} + \dots + 209u + 16$
$c_3, c_7$	$u^{13} + 3u^{12} + \dots - 2u - 8$
$c_5, c_6, c_8$ $c_9, c_{11}$	$u^{13} - u^{12} + \dots + u - 1$
$c_{10}$	$u^{13} + 19u^{12} + \dots - 13u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$y^{13} - 8y^{12} + \dots + 209y - 16$
$c_2$	$y^{13} - 4y^{12} + \dots + 22817y - 256$
$c_3, c_7$	$y^{13} - 3y^{12} + \dots + 180y - 64$
$c_5, c_6, c_8$ $c_9, c_{11}$	$y^{13} + 19y^{12} + \dots - 13y - 1$
$c_{10}$	$y^{13} - 49y^{12} + \dots - 61y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.955186 + 0.433947I$ $a = -0.471198 - 0.436221I$ $b = 0.755123 - 0.031476I$	$0.903643 - 0.585016I$	$4.40140 + 1.35233I$
$u = 0.955186 - 0.433947I$ $a = -0.471198 + 0.436221I$ $b = 0.755123 + 0.031476I$	$0.903643 + 0.585016I$	$4.40140 - 1.35233I$
$u = -0.933504 + 0.177892I$ $a = -0.617558 + 0.567691I$ $b = -0.473971 + 0.403774I$	$-1.65953 + 0.62739I$	$-3.40176 - 1.52650I$
$u = -0.933504 - 0.177892I$ $a = -0.617558 - 0.567691I$ $b = -0.473971 - 0.403774I$	$-1.65953 - 0.62739I$	$-3.40176 + 1.52650I$
$u = 0.869334 + 0.624757I$ $a = 0.420041 + 0.925939I$ $b = -0.581132 + 0.140274I$	$1.00399 - 3.84064I$	$5.54977 + 8.01840I$
$u = 0.869334 - 0.624757I$ $a = 0.420041 - 0.925939I$ $b = -0.581132 - 0.140274I$	$1.00399 + 3.84064I$	$5.54977 - 8.01840I$
$u = -0.028967 + 1.273930I$ $a = -0.158932 + 0.197003I$ $b = 0.05120 - 2.05742I$	$-12.48260 + 4.81706I$	$-0.19074 - 2.27482I$
$u = -0.028967 - 1.273930I$ $a = -0.158932 - 0.197003I$ $b = 0.05120 + 2.05742I$	$-12.48260 - 4.81706I$	$-0.19074 + 2.27482I$
$u = 1.46956 + 0.59251I$ $a = -0.85770 - 1.47458I$ $b = 0.52077 - 2.38909I$	$-17.2166 - 11.4167I$	$-1.78764 + 5.02800I$
$u = 1.46956 - 0.59251I$ $a = -0.85770 + 1.47458I$ $b = 0.52077 + 2.38909I$	$-17.2166 + 11.4167I$	$-1.78764 - 5.02800I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.49484 + 0.63529I$ $a = 0.82284 - 1.22627I$ $b = -0.74959 - 1.95511I$	$-17.0497 + 2.0233I$	$-2.18781 - 0.87077I$
$u = -1.49484 - 0.63529I$ $a = 0.82284 + 1.22627I$ $b = -0.74959 + 1.95511I$	$-17.0497 - 2.0233I$	$-2.18781 + 0.87077I$
$u = 0.326480$ $a = -1.02499$ $b = 0.455205$	0.885241	11.4840

$$\text{II. } I_2^u = \langle -u^4 + u^3 + 2u^2 + b - a - u - 1, u^4 - 2u^2a - 2u^3 + a^2 + au + 2a + u + 2, u^5 - u^4 - 2u^3 + u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a \\ u^4 - u^3 - 2u^2 + a + u + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^4a - 2u^2a - 2u^2 + a + u + 2 \\ u^3a - u^4 + 2u^3 - 2au - 2u + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^4 - u^2a - u^3 - 2u^2 + 2a + u + 1 \\ -u^4a + u^4 + u^2a - u^3 - 2u^2 + a + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -2u^4 + 3u^2 + 1 \\ -u^4 + 2u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^3 \\ -u^3 + u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u^4a + u^3a + u^4 - 2u^2a - 2au - 2u^2 + a + u + 1 \\ u^3a + u^4 - 2au - 2u^2 + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 4u^4 - 6u^2 - 2u - 2 \\ 2u^4 - 4u^2 - u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 4u^4 - 6u^2 - 2u - 2 \\ 2u^4 - 4u^2 - u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $4u^3 - 8u - 2$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_4$	$(u^5 - u^4 - 2u^3 + u^2 + u + 1)^2$
$c_2$	$(u^5 + 5u^4 + 8u^3 + 3u^2 - u + 1)^2$
$c_3, c_7$	$(u^5 - u^4 + 2u^3 - u^2 + u - 1)^2$
$c_5, c_6, c_8$ $c_9, c_{11}$	$u^{10} + 3u^9 + \dots + 32u + 17$
$c_{10}$	$u^{10} + 11u^9 + \dots + 1016u + 289$



(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2$
$c_2$	$(y^5 - 9y^4 + 32y^3 - 35y^2 - 5y - 1)^2$
$c_3, c_7$	$(y^5 + 3y^4 + 4y^3 + y^2 - y - 1)^2$
$c_5, c_6, c_8$ $c_9, c_{11}$	$y^{10} + 11y^9 + \dots + 1016y + 289$
$c_{10}$	$y^{10} - 25y^9 + \dots + 78660y + 83521$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.21774$ $a = 1.09175 + 2.32396I$ $b = 1.91295 + 2.32396I$	-5.69095	0.518860
$u = -1.21774$ $a = 1.09175 - 2.32396I$ $b = 1.91295 - 2.32396I$	-5.69095	0.518860
$u = -0.309916 + 0.549911I$ $a = -0.653120 + 0.123189I$ $b = 0.12468 + 1.50332I$	$-3.61897 + 1.53058I$	$1.48489 - 4.43065I$
$u = -0.309916 + 0.549911I$ $a = -1.44967 - 1.35480I$ $b = -0.671868 + 0.025324I$	$-3.61897 + 1.53058I$	$1.48489 - 4.43065I$
$u = -0.309916 - 0.549911I$ $a = -0.653120 - 0.123189I$ $b = 0.12468 - 1.50332I$	$-3.61897 - 1.53058I$	$1.48489 + 4.43065I$
$u = -0.309916 - 0.549911I$ $a = -1.44967 + 1.35480I$ $b = -0.671868 - 0.025324I$	$-3.61897 - 1.53058I$	$1.48489 + 4.43065I$
$u = 1.41878 + 0.21917I$ $a = 0.171660 - 0.827142I$ $b = -0.516743 - 0.720802I$	$-9.16243 - 4.40083I$	$-2.74431 + 3.49859I$
$u = 1.41878 + 0.21917I$ $a = 0.33938 + 1.85177I$ $b = -0.34902 + 1.95811I$	$-9.16243 - 4.40083I$	$-2.74431 + 3.49859I$
$u = 1.41878 - 0.21917I$ $a = 0.171660 + 0.827142I$ $b = -0.516743 + 0.720802I$	$-9.16243 + 4.40083I$	$-2.74431 - 3.49859I$
$u = 1.41878 - 0.21917I$ $a = 0.33938 - 1.85177I$ $b = -0.34902 - 1.95811I$	$-9.16243 + 4.40083I$	$-2.74431 - 3.49859I$

III.

$$I_3^u = \langle -u^2a + au - u^2 + b + u - 1, 2u^2a + a^2 - 4au + 3u^2 + 2a - 6u + 5, u^3 - u^2 + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a \\ u^2a - au + u^2 - u + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^2a - au + 3u^2 + a - 5u + 4 \\ u^2a - au + 2u^2 - 3u + 3 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -au + u^2 + a - u + 1 \\ -au + 2u^2 + a - 2u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} au - u^2 + u - 1 \\ u^2a - u^2 - a + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^2 + 1 \\ -u^2 + u + 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -au + u^2 + a + 1 \\ -au + 2u^2 + a - u + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^2a + u - 1 \\ -u^2a + u - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^2a + u - 1 \\ -u^2a + u - 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $4u - 4$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$(u^3 + u^2 - 1)^2$
$c_2$	$(u^3 + u^2 + 2u + 1)^2$
$c_3, c_7$	$u^6 - 3u^4 + 2u^2 + 1$
$c_4$	$(u^3 - u^2 + 1)^2$
$c_5, c_6, c_8$ $c_9, c_{11}$	$(u^2 + 1)^3$
$c_{10}$	$(u - 1)^6$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$(y^3 - y^2 + 2y - 1)^2$
$c_2$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_3, c_7$	$(y^3 - 3y^2 + 2y + 1)^2$
$c_5, c_6, c_8$ $c_9, c_{11}$	$(y + 1)^6$
$c_{10}$	$(y - 1)^6$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.877439 + 0.744862I$		
$a = 1.102080 + 0.844941I$	$-0.26574 - 2.82812I$	$-0.49024 + 2.97945I$
$b = -0.867423 + 0.622301I$		
$u = 0.877439 + 0.744862I$		
$a = -0.022482 - 0.479777I$	$-0.26574 - 2.82812I$	$-0.49024 + 2.97945I$
$b = 0.622301 + 0.867423I$		
$u = 0.877439 - 0.744862I$		
$a = 1.102080 - 0.844941I$	$-0.26574 + 2.82812I$	$-0.49024 - 2.97945I$
$b = -0.867423 - 0.622301I$		
$u = 0.877439 - 0.744862I$		
$a = -0.022482 + 0.479777I$	$-0.26574 + 2.82812I$	$-0.49024 - 2.97945I$
$b = 0.622301 - 0.867423I$		
$u = -0.754878$		
$a = -3.07960 + 1.32472I$	$-4.40332$	$-7.01950$
$b = -1.75488 + 1.75488I$		
$u = -0.754878$		
$a = -3.07960 - 1.32472I$	$-4.40332$	$-7.01950$
$b = -1.75488 - 1.75488I$		

$$\text{IV. } I_4^u = \langle 2b + 1, 2a - 1, u + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -0.5 \\ -1.5 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -2.25

(iv)  $u$ -Polynomials at the component

Crossings	$u$ -Polynomials at each crossing
$c_1, c_8, c_9$ $c_{11}$	$u - 1$
$c_2, c_4, c_5$ $c_6, c_{10}$	$u + 1$
$c_3, c_7$	$u$



(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$ $c_5, c_6, c_8$ $c_9, c_{10}, c_{11}$	$y - 1$
$c_3, c_7$	$y$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_4^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.00000$ $a = 0.500000$ $b = -0.500000$	0	-2.25000

### V. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u-1)(u^3+u^2-1)^2(u^5-u^4-2u^3+u^2+u+1)^2$ $\cdot (u^{13}-2u^{12}+\dots+17u-4)$
$c_2$	$(u+1)(u^3+u^2+2u+1)^2(u^5+5u^4+8u^3+3u^2-u+1)^2$ $\cdot (u^{13}+8u^{12}+\dots+209u+16)$
$c_3, c_7$	$u(u^5-u^4+2u^3-u^2+u-1)^2(u^6-3u^4+2u^2+1)$ $\cdot (u^{13}+3u^{12}+\dots-2u-8)$
$c_4$	$(u+1)(u^3-u^2+1)^2(u^5-u^4-2u^3+u^2+u+1)^2$ $\cdot (u^{13}-2u^{12}+\dots+17u-4)$
$c_5, c_6$	$(u+1)(u^2+1)^3(u^{10}+3u^9+\dots+32u+17)(u^{13}-u^{12}+\dots+u-1)$
$c_8, c_9, c_{11}$	$(u-1)(u^2+1)^3(u^{10}+3u^9+\dots+32u+17)(u^{13}-u^{12}+\dots+u-1)$
$c_{10}$	$((u-1)^6)(u+1)(u^{10}+11u^9+\dots+1016u+289)$ $\cdot (u^{13}+19u^{12}+\dots-13u-1)$

## VI. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$(y-1)(y^3 - y^2 + 2y - 1)^2(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2$ $\cdot (y^{13} - 8y^{12} + \dots + 209y - 16)$
$c_2$	$(y-1)(y^3 + 3y^2 + 2y - 1)^2(y^5 - 9y^4 + 32y^3 - 35y^2 - 5y - 1)^2$ $\cdot (y^{13} - 4y^{12} + \dots + 22817y - 256)$
$c_3, c_7$	$y(y^3 - 3y^2 + 2y + 1)^2(y^5 + 3y^4 + 4y^3 + y^2 - y - 1)^2$ $\cdot (y^{13} - 3y^{12} + \dots + 180y - 64)$
$c_5, c_6, c_8$ $c_9, c_{11}$	$(y-1)(y+1)^6(y^{10} + 11y^9 + \dots + 1016y + 289)$ $\cdot (y^{13} + 19y^{12} + \dots - 13y - 1)$
$c_{10}$	$((y-1)^7)(y^{10} - 25y^9 + \dots + 78660y + 83521)$ $\cdot (y^{13} - 49y^{12} + \dots - 61y - 1)$