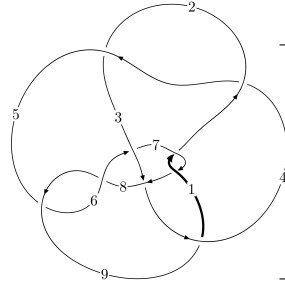
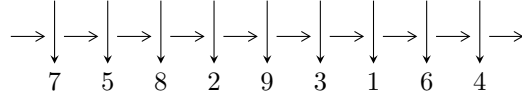


9<sub>38</sub> (K9a<sub>30</sub>)

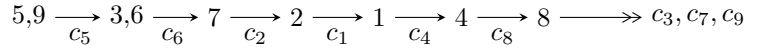


A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$\begin{aligned}
 I_1^u &= \langle -u^{10} - 2u^9 - 5u^8 - 11u^7 - 14u^6 - 20u^5 - 23u^4 - 16u^3 - 13u^2 + 4b - 4u + 1, \\
 &\quad u^{10} + 2u^9 + 5u^8 + 11u^7 + 10u^6 + 12u^5 + 15u^4 - 7u^2 + 8a - 4u - 9, \\
 &\quad u^{11} + u^{10} + 3u^9 + 6u^8 + 7u^7 + 10u^6 + 11u^5 + 9u^4 + 9u^3 + 3u^2 + 3u + 1 \rangle \\
 I_2^u &= \langle 20020u^{17} - 48508u^{16} + \dots + 11959b - 19142, -16736u^{17} + 49970u^{16} + \dots + 11959a - 645, \\
 &\quad u^{18} - 3u^{17} + \dots - 2u + 1 \rangle \\
 I_3^u &= \langle b + 1, 2a + 1, u - 1 \rangle
 \end{aligned}$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 30 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.

$$I_1^u = \langle -u^{10} - 2u^9 + \dots + 4b + 1, u^{10} + 2u^9 + \dots + 8a - 9, u^{11} + u^{10} + \dots + 3u + 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -\frac{1}{8}u^{10} - \frac{1}{4}u^9 + \dots + \frac{1}{2}u + \frac{9}{8} \\ \frac{1}{4}u^{10} + \frac{1}{2}u^9 + \dots + u - \frac{1}{4} \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -\frac{1}{16}u^{10} - \frac{1}{8}u^9 + \dots - \frac{7}{4}u + \frac{1}{16} \\ -\frac{1}{8}u^{10} - \frac{1}{4}u^9 + \dots - \frac{1}{2}u + \frac{1}{8} \end{pmatrix}$$

$$a_2 = \begin{pmatrix} \frac{1}{8}u^{10} + \frac{1}{4}u^9 + \dots + \frac{3}{2}u + \frac{7}{8} \\ \frac{1}{4}u^{10} + \frac{1}{2}u^9 + \dots + u - \frac{1}{4} \end{pmatrix}$$

$$a_1 = \begin{pmatrix} \frac{1}{16}u^{10} + \frac{1}{8}u^9 + \dots + \frac{7}{4}u + \frac{15}{16} \\ \frac{1}{8}u^{10} + \frac{1}{4}u^9 + \dots + \frac{3}{2}u - \frac{1}{8} \end{pmatrix}$$

$$a_4 = \begin{pmatrix} \frac{3}{8}u^{10} + \frac{3}{4}u^9 + \dots + \frac{5}{2}u + \frac{13}{8} \\ \frac{5}{4}u^{10} + \frac{3}{2}u^9 + \dots + u - \frac{1}{4} \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -\frac{27}{16}u^{10} + \frac{13}{8}u^9 - \frac{23}{16}u^8 - \frac{41}{16}u^7 + \frac{37}{8}u^6 + \frac{7}{4}u^5 - \frac{13}{16}u^4 + 7u^3 - \frac{3}{16}u^2 + \frac{31}{4}u - \frac{181}{16}$$

(iv) **u**-Polynomials at the component

Crossings	<b>u</b> -Polynomials at each crossing
$c_1, c_5, c_7$ $c_8$	$u^{11} + u^{10} + 3u^9 + 6u^8 + 7u^7 + 10u^6 + 11u^5 + 9u^4 + 9u^3 + 3u^2 + 3u + 1$
$c_2, c_4$	$u^{11} - 3u^9 + u^8 + 4u^7 - 2u^6 - u^5 + 10u^4 - 5u^3 - 16u^2 + 9u + 4$
$c_3$	$u^{11} - 3u^{10} + \dots - 6u + 8$
$c_6, c_9$	$2(2u^{11} + u^{10} - 3u^8 + 11u^7 + 2u^6 - 14u^5 + 7u^4 + 6u^3 - 5u^2 + 1)$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_5, c_7$ $c_8$	$y^{11} + 5y^{10} + 11y^9 + 8y^8 - 5y^7 + 47y^5 + 87y^4 + 73y^3 + 27y^2 + 3y - 1$
$c_2, c_4$	$y^{11} - 6y^{10} + \dots + 209y - 16$
$c_3$	$y^{11} + 3y^{10} + \dots + 52y - 64$
$c_6, c_9$	$4(4y^{11} - y^{10} + \dots + 10y - 1)$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.361784 + 0.962924I$ $a = 0.42885 - 1.90311I$ $b = -1.019420 + 0.904921I$	$0.53843 + 4.57539I$	$-8.21994 - 7.99945I$
$u = -0.361784 - 0.962924I$ $a = 0.42885 + 1.90311I$ $b = -1.019420 - 0.904921I$	$0.53843 - 4.57539I$	$-8.21994 + 7.99945I$
$u = -1.186630 + 0.355210I$ $a = 0.360998 + 0.003803I$ $b = 0.988348 + 0.222965I$	$-3.44203 - 0.72668I$	$-9.61068 + 7.91738I$
$u = -1.186630 - 0.355210I$ $a = 0.360998 - 0.003803I$ $b = 0.988348 - 0.222965I$	$-3.44203 + 0.72668I$	$-9.61068 - 7.91738I$
$u = 0.256965 + 0.681325I$ $a = 0.565680 + 0.993565I$ $b = -1.41820 - 0.12736I$	$-1.48009 - 1.36667I$	$-10.72983 + 4.40179I$
$u = 0.256965 - 0.681325I$ $a = 0.565680 - 0.993565I$ $b = -1.41820 + 0.12736I$	$-1.48009 + 1.36667I$	$-10.72983 - 4.40179I$
$u = 0.391610 + 1.210140I$ $a = -0.57189 + 1.31384I$ $b = 0.308687 - 1.224930I$	$6.41512 - 6.30680I$	$-3.61485 + 5.61897I$
$u = 0.391610 - 1.210140I$ $a = -0.57189 - 1.31384I$ $b = 0.308687 + 1.224930I$	$6.41512 + 6.30680I$	$-3.61485 - 5.61897I$
$u = 0.57851 + 1.29417I$ $a = -0.05089 - 1.59336I$ $b = 1.29294 + 0.67490I$	$3.25113 - 12.93290I$	$-6.73085 + 7.81031I$
$u = 0.57851 - 1.29417I$ $a = -0.05089 + 1.59336I$ $b = 1.29294 - 0.67490I$	$3.25113 + 12.93290I$	$-6.73085 - 7.81031I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.357337$		
$a = 1.03450$	$-0.695510$	$-14.4380$
$b = -0.304704$		

$$\text{II. } I_2^u = \langle 20020u^{17} - 48508u^{16} + \dots + 11959b - 19142, -16736u^{17} + 49970u^{16} + \dots + 11959a - 645, u^{18} - 3u^{17} + \dots - 2u + 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1.39945u^{17} - 4.17844u^{16} + \dots + 1.61753u + 0.0539343 \\ -1.67405u^{17} + 4.05619u^{16} + \dots - 1.50280u + 1.60064 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -0.146501u^{17} + 0.631491u^{16} + \dots - 2.98386u + 4.22619 \\ -0.318087u^{17} + 1.15194u^{16} + \dots - 0.807425u - 0.186972 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.274605u^{17} - 0.122251u^{16} + \dots + 0.114725u + 1.65457 \\ -1.67405u^{17} + 4.05619u^{16} + \dots - 1.50280u + 1.60064 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -3.02350u^{17} + 9.06932u^{16} + \dots - 5.85985u + 4.11447 \\ -1.51769u^{17} + 3.28171u^{16} + \dots + 1.19801u + 0.319257 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 2.04465u^{17} - 5.56234u^{16} + \dots + 1.85467u - 1.18196 \\ -0.318087u^{17} + 1.15194u^{16} + \dots - 0.807425u - 0.186972 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = \frac{35580}{11959}u^{17} - \frac{72112}{11959}u^{16} + \dots + \frac{57268}{11959}u - \frac{174278}{11959}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_5, c_7$ $c_8$	$u^{18} - 3u^{17} + \dots - 2u + 1$
$c_2, c_4$	$(u^9 - u^8 - 2u^7 + 3u^6 + u^5 - 3u^4 + 2u^3 - u + 1)^2$
$c_3$	$(u^9 + u^8 + 2u^7 + u^6 + 3u^5 + u^4 + 2u^3 + u - 1)^2$
$c_6, c_9$	$u^{18} - 3u^{17} + \dots + 4u + 11$



(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_5, c_7$ $c_8$	$y^{18} + 11y^{17} + \cdots + 14y^2 + 1$
$c_2, c_4$	$(y^9 - 5y^8 + 12y^7 - 15y^6 + 9y^5 + y^4 - 4y^3 + 2y^2 + y - 1)^2$
$c_3$	$(y^9 + 3y^8 + 8y^7 + 13y^6 + 17y^5 + 17y^4 + 12y^3 + 6y^2 + y - 1)^2$
$c_6, c_9$	$y^{18} + 7y^{17} + \cdots + 1260y + 121$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.131255 + 1.025520I$ $a = 2.21228 - 3.01855I$ $b = -0.825933$	2.09142	$-12.65235 + 0.I$
$u = -0.131255 - 1.025520I$ $a = 2.21228 + 3.01855I$ $b = -0.825933$	2.09142	$-12.65235 + 0.I$
$u = 1.068960 + 0.157811I$ $a = 0.330746 + 0.183937I$ $b = 1.172470 - 0.500383I$	$-0.30826 + 7.08493I$	$-9.57680 - 5.91335I$
$u = 1.068960 - 0.157811I$ $a = 0.330746 - 0.183937I$ $b = 1.172470 + 0.500383I$	$-0.30826 - 7.08493I$	$-9.57680 + 5.91335I$
$u = 0.255037 + 0.861194I$ $a = 0.31995 + 1.69908I$ $b = -1.173910 - 0.391555I$	$-1.08148 - 1.33617I$	$-11.28409 + 0.70175I$
$u = 0.255037 - 0.861194I$ $a = 0.31995 - 1.69908I$ $b = -1.173910 + 0.391555I$	$-1.08148 + 1.33617I$	$-11.28409 - 0.70175I$
$u = -0.287150 + 1.197360I$ $a = -0.077942 - 1.012210I$ $b = 0.141484 + 0.739668I$	$2.67293 + 2.45442I$	$-6.32792 - 2.91298I$
$u = -0.287150 - 1.197360I$ $a = -0.077942 + 1.012210I$ $b = 0.141484 - 0.739668I$	$2.67293 - 2.45442I$	$-6.32792 + 2.91298I$
$u = 0.605058 + 1.127080I$ $a = 0.639032 - 1.048120I$ $b = 0.772920 + 0.510351I$	$5.07330 - 2.09337I$	$-3.48501 + 4.16283I$
$u = 0.605058 - 1.127080I$ $a = 0.639032 + 1.048120I$ $b = 0.772920 - 0.510351I$	$5.07330 + 2.09337I$	$-3.48501 - 4.16283I$

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.658024 + 0.097431I$		
$a = 0.910679 + 0.215358I$	$2.67293 - 2.45442I$	$-6.32792 + 2.91298I$
$b = 0.141484 - 0.739668I$		
$u = 0.658024 - 0.097431I$		
$a = 0.910679 - 0.215358I$	$2.67293 + 2.45442I$	$-6.32792 - 2.91298I$
$b = 0.141484 + 0.739668I$		
$u = -0.62758 + 1.28014I$		
$a = 0.023182 + 1.259910I$	$-0.30826 + 7.08493I$	$-9.57680 - 5.91335I$
$b = 1.172470 - 0.500383I$		
$u = -0.62758 - 1.28014I$		
$a = 0.023182 - 1.259910I$	$-0.30826 - 7.08493I$	$-9.57680 + 5.91335I$
$b = 1.172470 + 0.500383I$		
$u = 0.31006 + 1.39846I$		
$a = -0.515395 + 0.355009I$	$5.07330 + 2.09337I$	$-3.48501 - 4.16283I$
$b = 0.772920 - 0.510351I$		
$u = 0.31006 - 1.39846I$		
$a = -0.515395 - 0.355009I$	$5.07330 - 2.09337I$	$-3.48501 + 4.16283I$
$b = 0.772920 + 0.510351I$		
$u = -0.351155 + 0.305986I$		
$a = 1.157480 - 0.200845I$	$-1.08148 - 1.33617I$	$-11.28409 + 0.70175I$
$b = -1.173910 - 0.391555I$		
$u = -0.351155 - 0.305986I$		
$a = 1.157480 + 0.200845I$	$-1.08148 + 1.33617I$	$-11.28409 - 0.70175I$
$b = -1.173910 + 0.391555I$		

$$\text{III. } I_3^u = \langle b + 1, 2a + 1, u - 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -0.5 \\ -1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1.25 \\ 1.5 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -1.5 \\ -1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -0.25 \\ 0.5 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -0.5 \\ -1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -9.75

(iv) **u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_4, c_8$	$u + 1$
$c_2, c_5, c_7$	$u - 1$
$c_3$	$u$
$c_6$	$2(2u + 1)$
$c_9$	$2(2u - 1)$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$ $c_5, c_7, c_8$	$y - 1$
$c_3$	$y$
$c_6, c_9$	$4(4y - 1)$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$		
$a = -0.500000$	-3.28987	-9.75000
$b = -1.00000$		

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_8$	$(u + 1)$ $\cdot (u^{11} + u^{10} + 3u^9 + 6u^8 + 7u^7 + 10u^6 + 11u^5 + 9u^4 + 9u^3 + 3u^2 + 3u + 1)$ $\cdot (u^{18} - 3u^{17} + \dots - 2u + 1)$
$c_2$	$(u - 1)(u^9 - u^8 - 2u^7 + 3u^6 + u^5 - 3u^4 + 2u^3 - u + 1)^2$ $\cdot (u^{11} - 3u^9 + u^8 + 4u^7 - 2u^6 - u^5 + 10u^4 - 5u^3 - 16u^2 + 9u + 4)$
$c_3$	$u(u^9 + u^8 + 2u^7 + u^6 + 3u^5 + u^4 + 2u^3 + u - 1)^2$ $\cdot (u^{11} - 3u^{10} + \dots - 6u + 8)$
$c_4$	$(u + 1)(u^9 - u^8 - 2u^7 + 3u^6 + u^5 - 3u^4 + 2u^3 - u + 1)^2$ $\cdot (u^{11} - 3u^9 + u^8 + 4u^7 - 2u^6 - u^5 + 10u^4 - 5u^3 - 16u^2 + 9u + 4)$
$c_5, c_7$	$(u - 1)$ $\cdot (u^{11} + u^{10} + 3u^9 + 6u^8 + 7u^7 + 10u^6 + 11u^5 + 9u^4 + 9u^3 + 3u^2 + 3u + 1)$ $\cdot (u^{18} - 3u^{17} + \dots - 2u + 1)$
$c_6$	$4(2u + 1)(2u^{11} + u^{10} + \dots - 5u^2 + 1)$ $\cdot (u^{18} - 3u^{17} + \dots + 4u + 11)$
$c_9$	$4(2u - 1)(2u^{11} + u^{10} + \dots - 5u^2 + 1)$ $\cdot (u^{18} - 3u^{17} + \dots + 4u + 11)$



## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_5, c_7$ $c_8$	$(y - 1)$ $\cdot (y^{11} + 5y^{10} + 11y^9 + 8y^8 - 5y^7 + 47y^5 + 87y^4 + 73y^3 + 27y^2 + 3y - 1)$ $\cdot (y^{18} + 11y^{17} + \dots + 14y^2 + 1)$
$c_2, c_4$	$(y - 1)(y^9 - 5y^8 + 12y^7 - 15y^6 + 9y^5 + y^4 - 4y^3 + 2y^2 + y - 1)^2$ $\cdot (y^{11} - 6y^{10} + \dots + 209y - 16)$
$c_3$	$y(y^9 + 3y^8 + 8y^7 + 13y^6 + 17y^5 + 17y^4 + 12y^3 + 6y^2 + y - 1)^2$ $\cdot (y^{11} + 3y^{10} + \dots + 52y - 64)$
$c_6, c_9$	$16(4y - 1)(4y^{11} - y^{10} + \dots + 10y - 1)(y^{18} + 7y^{17} + \dots + 1260y + 121)$