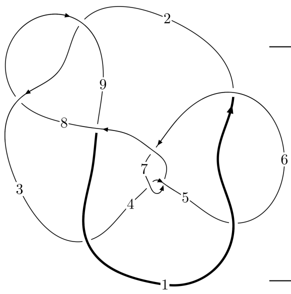
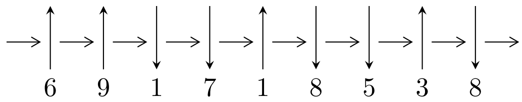
# $9_{44}$ $(K9n_1)$




A knot diagram[1]

**Linearized knot diagam**




**Solving Sequence**

$$5,8 \xrightarrow[c_7]{} 7 \xrightarrow[c_4]{} 1,4 \xrightarrow[c_3]{} 3 \xrightarrow[c_6]{} 6 \xrightarrow[c_1]{} 2 \xrightarrow[c_9]{} 9 \twoheadrightarrow c_2, c_5, c_8$$

**Ideals for irreducible components[2] of $X_{\mathbf{par}}$**

$$\begin{aligned}
I_1^u = \langle & -u^9 + 2u^8 - 5u^6 + 3u^5 + 3u^4 - 4u^3 - 2u^2 + 2b - u - 1, \\
& -u^9 + 3u^8 - 3u^7 - 3u^6 + 7u^5 - 3u^4 - 4u^3 + 2u^2 + a - u + 1, \\
& u^{10} - 3u^9 + 4u^8 + u^7 - 6u^6 + 6u^5 + u^4 - 2u^3 + 3u^2 - 2u + 1 \rangle \\
I_2^u = \langle & b^2 - b + 1,\ a,\ u + 1 \rangle
\end{aligned}$$

* 2 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 12 representations.

[1] The image of knot diagram is generated by the software "**Draw programme**" developed by Andrew Bartholomew(`http://www.layer8.co.uk/maths/draw/index.htm#Running-draw`), where we modified some parts for our purpose(`https://github.com/CATsTAILs/LinksPainter`).

[2] All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

**I.**

$$\boldsymbol{I_1^u = \langle -u^9+2u^8+\cdots+2b-1,\ -u^9+3u^8+\cdots+a+1,\ u^{10}-3u^9+\cdots-2u+1\rangle}$$

**(i) Arc colorings**

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^9 - 3u^8 + 3u^7 + 3u^6 - 7u^5 + 3u^4 + 4u^3 - 2u^2 + u - 1 \\ \frac{1}{2}u^9 - u^8 + \cdots + \frac{1}{2}u + \frac{1}{2} \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^4 - u^2 + 2u + 1 \\ -\frac{1}{2}u^9 + u^8 + \cdots - \frac{1}{2}u + \frac{1}{2} \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^2 + 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^8 - u^7 + u^6 + 2u^5 - u^4 + 3u^3 + 2u^2 - 2u + 1 \\ -\frac{1}{2}u^9 + 2u^8 + \cdots - \frac{3}{2}u + \frac{3}{2} \end{pmatrix}$$

$$a_9 = \begin{pmatrix} \frac{3}{2}u^9 - 4u^8 + \cdots + \frac{3}{2}u - \frac{1}{2} \\ \frac{1}{2}u^9 - u^8 + \cdots + \frac{1}{2}u + \frac{1}{2} \end{pmatrix}$$

$$a_9 = \begin{pmatrix} \frac{3}{2}u^9 - 4u^8 + \cdots + \frac{3}{2}u - \frac{1}{2} \\ \frac{1}{2}u^9 - u^8 + \cdots + \frac{1}{2}u + \frac{1}{2} \end{pmatrix}$$

**(ii) Obstruction class** $= -1$

**(iii) Cusp Shapes** $= 3u^9 - 7u^8 + 5u^7 + 12u^6 - 15u^5 + 2u^4 + 16u^3 - 2u^2 + 5u$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_5$ | $u^{10} + u^9 - 7u^8 - 8u^7 + 13u^6 + 14u^5 - 2u^4 + 2u^3 + 13u^2 + 12u + 4$ |
| $c_2, c_8$ | $u^{10} + 2u^9 + 3u^8 + 2u^7 + 4u^6 + 3u^5 + 3u^4 + 3u^2 + u + 1$ |
| $c_3$ | $u^{10} - 2u^9 + \cdots + 21u + 17$ |
| $c_4, c_7$ | $u^{10} - 3u^9 + 4u^8 + u^7 - 6u^6 + 6u^5 + u^4 - 2u^3 + 3u^2 - 2u + 1$ |
| $c_6$ | $u^{10} + u^9 + 10u^8 + 11u^7 + 26u^6 + 30u^5 + u^4 - 14u^3 + 3u^2 - 2u + 1$ |
| $c_9$ | $u^{10} + 2u^9 + 9u^8 + 14u^7 + 28u^6 + 31u^5 + 35u^4 + 20u^3 + 15u^2 + 5u + 1$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_5$ | $y^{10} - 15y^9 + \cdots - 40y + 16$ |
| $c_2, c_8$ | $y^{10} + 2y^9 + 9y^8 + 14y^7 + 28y^6 + 31y^5 + 35y^4 + 20y^3 + 15y^2 + 5y + 1$ |
| $c_3$ | $y^{10} + 26y^9 + \cdots + 2925y + 289$ |
| $c_4, c_7$ | $y^{10} - y^9 + 10y^8 - 11y^7 + 26y^6 - 30y^5 + y^4 + 14y^3 + 3y^2 + 2y + 1$ |
| $c_6$ | $y^{10} + 19y^9 + \cdots + 2y + 1$ |
| $c_9$ | $y^{10} + 14y^9 + \cdots + 5y + 1$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_1^u$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -1.016000 + 0.211624I$<br>$a = 0.493565 - 0.413430I$<br>$b = 0.230117 - 0.236010I$ | $-1.89922 + 0.79591I$ | $-4.77960 + 0.81155I$ |
| $u = -1.016000 - 0.211624I$<br>$a = 0.493565 + 0.413430I$<br>$b = 0.230117 + 0.236010I$ | $-1.89922 - 0.79591I$ | $-4.77960 - 0.81155I$ |
| $u = -0.076965 + 0.657059I$<br>$a = 1.01532 - 1.06291I$<br>$b = -0.110515 - 0.762837I$ | $1.14579 + 1.46073I$ | $2.65931 - 3.28644I$ |
| $u = -0.076965 - 0.657059I$<br>$a = 1.01532 + 1.06291I$<br>$b = -0.110515 + 0.762837I$ | $1.14579 - 1.46073I$ | $2.65931 + 3.28644I$ |
| $u = 0.482659 + 0.410726I$<br>$a = -1.077630 + 0.665030I$<br>$b = 0.826051 + 0.890915I$ | $-0.41291 - 2.81207I$ | $0.88002 + 4.64391I$ |
| $u = 0.482659 - 0.410726I$<br>$a = -1.077630 - 0.665030I$<br>$b = 0.826051 - 0.890915I$ | $-0.41291 + 2.81207I$ | $0.88002 - 4.64391I$ |
| $u = 0.98889 + 1.13481I$<br>$a = 0.766166 - 1.067440I$<br>$b = -0.18099 - 1.73332I$ | $9.86147 - 0.50253I$ | $1.49701 - 0.08773I$ |
| $u = 0.98889 - 1.13481I$<br>$a = 0.766166 + 1.067440I$<br>$b = -0.18099 + 1.73332I$ | $9.86147 + 0.50253I$ | $1.49701 + 0.08773I$ |
| $u = 1.12142 + 1.03617I$<br>$a = -0.697426 + 1.061500I$<br>$b = 0.23534 + 1.84389I$ | $9.39914 - 7.40677I$ | $0.74326 + 4.41038I$ |
| $u = 1.12142 - 1.03617I$<br>$a = -0.697426 - 1.061500I$<br>$b = 0.23534 - 1.84389I$ | $9.39914 + 7.40677I$ | $0.74326 - 4.41038I$ |

## II. $I_2^u = \langle b^2 - b + 1,\ a,\ u + 1 \rangle$

**(i) Arc colorings**

$$a_5 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} 0 \\ b \end{pmatrix}$$
$$a_4 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$
$$a_3 = \begin{pmatrix} -1 \\ b-1 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$
$$a_2 = \begin{pmatrix} 0 \\ b \end{pmatrix}$$
$$a_9 = \begin{pmatrix} b \\ b \end{pmatrix}$$
$$a_9 = \begin{pmatrix} b \\ b \end{pmatrix}$$

**(ii) Obstruction class** $= 1$

**(iii) Cusp Shapes** $= 4b - 5$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_5$ | $u^2$ |
| $c_2$ | $u^2 - u + 1$ |
| $c_3, c_8, c_9$ | $u^2 + u + 1$ |
| $c_4, c_6$ | $(u-1)^2$ |
| $c_7$ | $(u+1)^2$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_5$ | $y^2$ |
| $c_2, c_3, c_8$ $c_9$ | $y^2 + y + 1$ |
| $c_4, c_6, c_7$ | $(y-1)^2$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_2^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -1.00000$<br>$a = 0$<br>$b = 0.500000 + 0.866025I$ | $-1.64493 - 2.02988I$ | $-3.00000 + 3.46410I$ |
| $u = -1.00000$<br>$a = 0$<br>$b = 0.500000 - 0.866025I$ | $-1.64493 + 2.02988I$ | $-3.00000 - 3.46410I$ |

## III. u-Polynomials

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_5$ | $u^2(u^{10}+u^9-7u^8-8u^7+13u^6+14u^5-2u^4+2u^3+13u^2+12u+4)$ |
| $c_2$ | $(u^2-u+1)(u^{10}+2u^9+3u^8+2u^7+4u^6+3u^5+3u^4+3u^2+u+1)$ |
| $c_3$ | $(u^2+u+1)(u^{10}-2u^9+\cdots+21u+17)$ |
| $c_4$ | $(u-1)^2(u^{10}-3u^9+4u^8+u^7-6u^6+6u^5+u^4-2u^3+3u^2-2u+1)$ |
| $c_6$ | $(u-1)^2$ <br> $\cdot(u^{10}+u^9+10u^8+11u^7+26u^6+30u^5+u^4-14u^3+3u^2-2u+1)$ |
| $c_7$ | $(u+1)^2(u^{10}-3u^9+4u^8+u^7-6u^6+6u^5+u^4-2u^3+3u^2-2u+1)$ |
| $c_8$ | $(u^2+u+1)(u^{10}+2u^9+3u^8+2u^7+4u^6+3u^5+3u^4+3u^2+u+1)$ |
| $c_9$ | $(u^2+u+1)$ <br> $\cdot(u^{10}+2u^9+9u^8+14u^7+28u^6+31u^5+35u^4+20u^3+15u^2+5u+1)$ |

## IV. Riley Polynomials

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_5$ | $y^2(y^{10} - 15y^9 + \cdots - 40y + 16)$ |
| $c_2, c_8$ | $(y^2 + y + 1)$ <br> $\cdot (y^{10} + 2y^9 + 9y^8 + 14y^7 + 28y^6 + 31y^5 + 35y^4 + 20y^3 + 15y^2 + 5y + 1)$ |
| $c_3$ | $(y^2 + y + 1)(y^{10} + 26y^9 + \cdots + 2925y + 289)$ |
| $c_4, c_7$ | $(y - 1)^2$ <br> $\cdot (y^{10} - y^9 + 10y^8 - 11y^7 + 26y^6 - 30y^5 + y^4 + 14y^3 + 3y^2 + 2y + 1)$ |
| $c_6$ | $((y - 1)^2)(y^{10} + 19y^9 + \cdots + 2y + 1)$ |
| $c_9$ | $(y^2 + y + 1)(y^{10} + 14y^9 + \cdots + 5y + 1)$ |