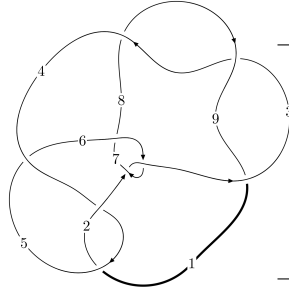
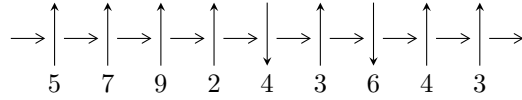


9<sub>48</sub> (K9n<sub>6</sub>)

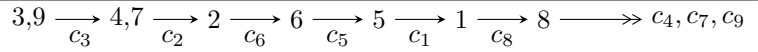


A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle b - u, -u^2 + a, u^3 - u^2 + u + 1 \rangle$$

$$I_2^u = \langle b - u, -u^3 + a + 1, u^4 + u^3 + u^2 + 1 \rangle$$

$$I_3^u = \langle u^2 + b + u, u^3 + 2u^2 + a + 2u, u^4 + u^3 + u^2 + 1 \rangle$$

$$I_4^u = \langle -u^3 + u^2 + b - u + 1, -u^3 + 2a + u - 1, u^4 - 2u^3 + 3u^2 - 3u + 2 \rangle$$

$$I_5^u = \langle b + u, a + 2u + 1, u^2 + 1 \rangle$$

\* 5 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 17 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle b - u, -u^2 + a, u^3 - u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u^2 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^2 - u \\ u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^2 - u \\ u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ -u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u \\ -u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^2 - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^2 - 1 \end{pmatrix}$$

(ii) Obstruction class =  $-1$

(iii) Cusp Shapes =  $-6u + 8$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_6, c_8$ $c_9$	$u^3 + u^2 + u - 1$
$c_5, c_7$	$u^3 + u^2 + 3u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_6, c_8$ $c_9$	$y^3 + y^2 + 3y - 1$
$c_5, c_7$	$y^3 + 5y^2 + 11y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.771845 + 1.115140I$ $a = -0.64780 + 1.72143I$ $b = 0.771845 + 1.115140I$	$2.02941 + 9.53188I$	$3.36893 - 6.69086I$
$u = 0.771845 - 1.115140I$ $a = -0.64780 - 1.72143I$ $b = 0.771845 - 1.115140I$	$2.02941 - 9.53188I$	$3.36893 + 6.69086I$
$u = -0.543689$ $a = 0.295598$ $b = -0.543689$	$0.875992$	$11.2620$

$$\text{II. } I_2^u = \langle b - u, -u^3 + a + 1, u^4 + u^3 + u^2 + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u^3 - 1 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u^3 - u^2 - u \\ u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^3 - u - 1 \\ u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u \\ -u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $-4u^2 - 4u + 2$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_4$	$u^4 + 2u^3 + 3u^2 + 3u + 2$
$c_2, c_3, c_6$ $c_8, c_9$	$u^4 - u^3 + u^2 + 1$
$c_5$	$u^4 + 2u^3 + u^2 + 3u + 4$
$c_7$	$u^4 + u^3 + 3u^2 + 2u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$y^4 + 2y^3 + y^2 + 3y + 4$
$c_2, c_3, c_6$ $c_8, c_9$	$y^4 + y^3 + 3y^2 + 2y + 1$
$c_5$	$y^4 - 2y^3 - 3y^2 - y + 16$
$c_7$	$y^4 + 5y^3 + 7y^2 + 2y + 1$



(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.351808 + 0.720342I$	$-3.50087 + 1.41510I$	$2.17326 - 4.90874I$
$a = -1.50411 - 0.10631I$		
$b = 0.351808 + 0.720342I$		
$u = 0.351808 - 0.720342I$	$-3.50087 - 1.41510I$	$2.17326 + 4.90874I$
$a = -1.50411 + 0.10631I$		
$b = 0.351808 - 0.720342I$		
$u = -0.851808 + 0.911292I$	$3.50087 - 3.16396I$	$5.82674 + 2.56480I$
$a = 0.504108 + 1.226850I$		
$b = -0.851808 + 0.911292I$		
$u = -0.851808 - 0.911292I$	$3.50087 + 3.16396I$	$5.82674 - 2.56480I$
$a = 0.504108 - 1.226850I$		
$b = -0.851808 - 0.911292I$		

$$\text{III. } I_3^u = \langle u^2 + b + u, u^3 + 2u^2 + a + 2u, u^4 + u^3 + u^2 + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^3 - 2u^2 - 2u \\ -u^2 - u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^3 - u - 1 \\ u^3 - 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^3 - u^2 - u \\ -u^2 - u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^3 - 2u^2 - u \\ -u^3 - 2u^2 - u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u \\ -u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $-4u^2 - 4u + 2$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_3, c_4$ $c_8, c_9$	$u^4 - u^3 + u^2 + 1$
$c_2, c_6$	$u^4 + 2u^3 + 3u^2 + 3u + 2$
$c_5$	$u^4 + u^3 + 3u^2 + 2u + 1$
$c_7$	$u^4 + 2u^3 + u^2 + 3u + 4$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_3, c_4$ $c_8, c_9$	$y^4 + y^3 + 3y^2 + 2y + 1$
$c_2, c_6$	$y^4 + 2y^3 + y^2 + 3y + 4$
$c_5$	$y^4 + 5y^3 + 7y^2 + 2y + 1$
$c_7$	$y^4 - 2y^3 - 3y^2 - y + 16$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.351808 + 0.720342I$	$-3.50087 + 1.41510I$	$2.17326 - 4.90874I$
$a = 0.59074 - 2.34806I$		
$b = 0.043315 - 1.227190I$		
$u = 0.351808 - 0.720342I$	$-3.50087 - 1.41510I$	$2.17326 + 4.90874I$
$a = 0.59074 + 2.34806I$		
$b = 0.043315 + 1.227190I$		
$u = -0.851808 + 0.911292I$	$3.50087 - 3.16396I$	$5.82674 + 2.56480I$
$a = 0.409261 + 0.055548I$		
$b = 0.956685 + 0.641200I$		
$u = -0.851808 - 0.911292I$	$3.50087 + 3.16396I$	$5.82674 - 2.56480I$
$a = 0.409261 - 0.055548I$		
$b = 0.956685 - 0.641200I$		

$$\text{IV. } I_4^u = \langle -u^3 + u^2 + b - u + 1, -u^3 + 2a + u - 1, u^4 - 2u^3 + 3u^2 - 3u + 2 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} \frac{1}{2}u^3 - \frac{1}{2}u + \frac{1}{2} \\ u^3 - u^2 + u - 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -\frac{1}{2}u^3 + u^2 - \frac{3}{2}u + \frac{3}{2} \\ -u^3 + u^2 - 2u + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -\frac{1}{2}u^3 + u^2 - \frac{3}{2}u + \frac{3}{2} \\ u^3 - u^2 + u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} \frac{1}{2}u^3 + \frac{1}{2}u + \frac{1}{2} \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u \\ -u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $4u + 2$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_4$ $c_6$	$u^4 - u^3 + u^2 + 1$
$c_3, c_8, c_9$	$u^4 + 2u^3 + 3u^2 + 3u + 2$
$c_5, c_7$	$u^4 + u^3 + 3u^2 + 2u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$ $c_6$	$y^4 + y^3 + 3y^2 + 2y + 1$
$c_3, c_8, c_9$	$y^4 + 2y^3 + y^2 + 3y + 4$
$c_5, c_7$	$y^4 + 5y^3 + 7y^2 + 2y + 1$



(vi) Complex Volumes and Cusp Shapes

Solutions to $I_4^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.956685 + 0.641200I$	$3.50087 - 3.16396I$	$5.82674 + 2.56480I$
$a = -0.130534 + 0.427872I$		
$b = -0.851808 + 0.911292I$		
$u = 0.956685 - 0.641200I$	$3.50087 + 3.16396I$	$5.82674 - 2.56480I$
$a = -0.130534 - 0.427872I$		
$b = -0.851808 - 0.911292I$		
$u = 0.043315 + 1.227190I$	$-3.50087 - 1.41510I$	$2.17326 + 4.90874I$
$a = 0.38053 - 1.53420I$		
$b = 0.351808 - 0.720342I$		
$u = 0.043315 - 1.227190I$	$-3.50087 + 1.41510I$	$2.17326 - 4.90874I$
$a = 0.38053 + 1.53420I$		
$b = 0.351808 + 0.720342I$		

$$\mathbf{V. } I_5^u = \langle b + u, a + 2u + 1, u^2 + 1 \rangle$$

**(i) Arc colorings**

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -2u - 1 \\ -u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u - 1 \\ -1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u - 1 \\ -u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ -u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ 0 \end{pmatrix}$$

**(ii) Obstruction class = 1**

**(iii) Cusp Shapes = -4**

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_6, c_8$ $c_9$	$u^2 + 1$
$c_5, c_7$	$(u + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_6, c_8$ $c_9$	$(y + 1)^2$
$c_5, c_7$	$(y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_5^u$		$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u =$	$1.000000I$		
$a =$	$-1.00000 - 2.00000I$	$-4.93480$	$-4.00000$
$b =$	$-1.000000I$		
$u =$	$-1.000000I$		
$a =$	$-1.00000 + 2.00000I$	$-4.93480$	$-4.00000$
$b =$	$1.000000I$		

## VI. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_6, c_8$ $c_9$	$(u^2 + 1)(u^3 + u^2 + u - 1)(u^4 - u^3 + u^2 + 1)^2(u^4 + 2u^3 + \dots + 3u + 2)$
$c_5, c_7$	$(u + 1)^2(u^3 + u^2 + 3u - 1)(u^4 + u^3 + 3u^2 + 2u + 1)^2$ $\cdot (u^4 + 2u^3 + u^2 + 3u + 4)$

## VII. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_6, c_8$ $c_9$	$(y + 1)^2(y^3 + y^2 + 3y - 1)(y^4 + y^3 + 3y^2 + 2y + 1)^2$ $\cdot (y^4 + 2y^3 + y^2 + 3y + 4)$
$c_5, c_7$	$(y - 1)^2(y^3 + 5y^2 + 11y - 1)(y^4 - 2y^3 - 3y^2 - y + 16)$ $\cdot (y^4 + 5y^3 + 7y^2 + 2y + 1)^2$