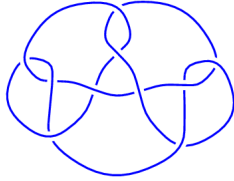
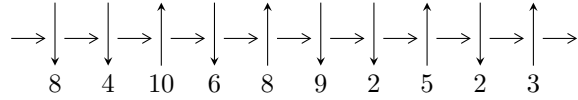


10<sub>138</sub> (K10n<sub>1</sub>)

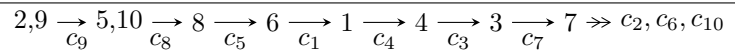


1

**Arc Sequences**



**Solving Sequence**



**Representation Ideals**

$$I = \bigcap_{i=1}^4 I_i^u$$

$$I_1^u = \langle b^2 + b + 1, b + a, u - 1 \rangle$$

$$I_2^u = \langle u^2 + u + 1, a + 1, b + u + 1 \rangle$$

$$I_3^u = \langle u^7 - u^6 + 9u^5 - 14u^4 + 23u^3 - 14u^2 + 8u - 1, \\ - 115u^6 + 86u^5 - 982u^4 + 1372u^3 - 2128u^2 + 277b + 859u - 542, \\ 285u^6 - 177u^5 + 2530u^4 - 3075u^3 + 5623u^2 + 277a - 2454u + 1837 \rangle$$

$$I_4^u = \langle u^{14} + u^{13} + \dots + 2u + 1, \\ - 1205138480001u^{13} + 3533853013940u^{12} + \dots + 51132550709402b + 88002408176841, \\ - 40546748429631u^{13} - 51692838289772u^{12} + \dots + 51132550709402a - 37861651496541 \rangle$$

There are 4 irreducible components with 25 representations.

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<sup>1</sup>The knot diagram image is adapter from “C. Livingston and A. H. Moore, KnotInfo: Table of Knot Invariants, <http://www.indiana.edu/~knotinfo>”

$$\mathbf{I. } I_1^u = \langle b^2 + b + 1, b + a, u - 1 \rangle$$

**(i) Arc colorings**

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -b \\ b \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -2b \\ b \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -b \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -b - 1 \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} b \\ -b \end{pmatrix}$$

$$a_3 = \begin{pmatrix} b + 2 \\ -b - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -b \\ 0 \end{pmatrix}$$

**(ii) Obstruction class = 1**

**(iii) Cusp Shapes = -3**

(iv) Complex Volumes and Cusp Shapes

Solution to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$		
$a = 0.500000 + 0.866025I$	0	-3.00000
$b = -0.500000 - 0.866025I$		
$u = 1.00000$		
$a = 0.500000 - 0.866025I$	0	-3.00000
$b = -0.500000 + 0.866025I$		

$$\text{II. } I_2^u = \langle u^2 + u + 1, a + 1, b + u + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ -u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ -u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u + 2 \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $8u + 4$

(iv) Complex Volumes and Cusp Shapes

Solution to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.500000 - 0.866025I$ $a = -1.00000$ $b = -0.500000 + 0.866025I$	4.05977I	- 6.92820I
$u = -0.500000 + 0.866025I$ $a = -1.00000$ $b = -0.500000 - 0.866025I$	- 4.05977I	6.92820I

$$\text{III. } I_3^u = \langle u^7 - u^6 + 9u^5 - 14u^4 + 23u^3 - 14u^2 + 8u - 1, -115u^6 + 86u^5 + \dots + 277b - 542, 285u^6 - 177u^5 + \dots + 277a + 1837 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_9 &= \begin{pmatrix} -1.02888u^6 + 0.638989u^5 + \dots + 8.85921u - 6.63177 \\ 0.415162u^6 - 0.310469u^5 + \dots - 3.10108u + 1.95668 \end{pmatrix} \\ a_5 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_{10} &= \begin{pmatrix} -1.44404u^6 + 0.949458u^5 + \dots + 11.9603u - 8.58845 \\ 0.415162u^6 - 0.310469u^5 + \dots - 3.10108u + 1.95668 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -1.02888u^6 + 0.638989u^5 + \dots + 8.85921u - 6.63177 \\ 0.151625u^6 - 0.104693u^5 + \dots - 1.01083u + 1.56679 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1.07942u^6 - 1.00722u^5 + \dots - 11.8628u + 6.48736 \\ -0.440433u^6 + 0.494585u^5 + \dots + 5.60289u - 1.88448 \end{pmatrix} \\ a_1 &= \begin{pmatrix} 1.88448u^6 - 1.44404u^5 + \dots - 17.5632u + 11.4729 \\ -0.541516u^6 + 0.231047u^5 + \dots + 3.61011u - 2.59567 \end{pmatrix} \\ a_4 &= \begin{pmatrix} 0.877256u^6 - 0.534296u^5 + \dots - 7.84838u + 5.06498 \\ -0.0722022u^6 + 0.0974729u^5 + \dots + 2.14801u - 1.07942 \end{pmatrix} \\ a_3 &= \begin{pmatrix} -0.617329u^6 + 0.783394u^5 + \dots + 8.11552u - 3.37906 \\ 0.342960u^6 - 0.212996u^5 + \dots - 1.95307u + 0.877256 \end{pmatrix} \\ a_7 &= \begin{pmatrix} -0.877256u^6 + 0.534296u^5 + \dots + 7.84838u - 5.06498 \\ 0.151625u^6 - 0.104693u^5 + \dots - 1.01083u + 1.56679 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = \frac{218}{277}u^6 + \frac{232}{277}u^5 + \frac{1770}{277}u^4 + \frac{622}{277}u^3 + \frac{946}{277}u^2 + \frac{2240}{277}u - \frac{702}{277}$$

(iv) Complex Volumes and Cusp Shapes

Solution to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.46632 - 2.74126I$		
$a = -0.422265 - 0.273313I$	$-11.0685 - 10.4672I$	$-6.45679 + 5.97165I$
$b = -2.01513 - 0.33917I$		
$u = -0.46632 + 2.74126I$		
$a = -0.422265 + 0.273313I$	$-11.0685 + 10.4672I$	$-6.45679 - 5.97165I$
$b = -2.01513 + 0.33917I$		
$u = 0.158515$		
$a = -5.69902$	$-3.61413$	$-1.15355$
$b = 1.64063$		
$u = 0.260920 - 0.655876I$		
$a = -0.614097 + 0.651143I$	$-0.184850 + 1.357363I$	$-2.08591 - 4.58406I$
$b = -0.023311 - 0.507189I$		
$u = 0.260920 + 0.655876I$		
$a = -0.614097 - 0.651143I$	$-0.184850 - 1.357363I$	$-2.08591 + 4.58406I$
$b = -0.023311 + 0.507189I$		
$u = 0.626141 - 1.116013I$		
$a = 0.885874 - 0.284512I$	$-4.21141 - 3.35522I$	$-7.88053 + 3.75965I$
$b = 0.718123 - 0.224799I$		
$u = 0.626141 + 1.116013I$		
$a = 0.885874 + 0.284512I$	$-4.21141 + 3.35522I$	$-7.88053 - 3.75965I$
$b = 0.718123 + 0.224799I$		

$$\text{IV. } I_4^u = \langle u^{14} + u^{13} + \dots + 2u + 1, -1.21 \times 10^{12}u^{13} + 3.53 \times 10^{12}u^{12} + \dots + 5.11 \times 10^{13}b + 8.80 \times 10^{13}, -4.05 \times 10^{13}u^{13} - 5.17 \times 10^{13}u^{12} + \dots + 5.11 \times 10^{13}a - 3.79 \times 10^{13} \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_9 &= \begin{pmatrix} 0.792973u^{13} + 1.01096u^{12} + \dots + 26.6378u + 0.740461 \\ 0.0235689u^{13} - 0.0691116u^{12} + \dots - 0.915315u - 1.72106 \end{pmatrix} \\ a_5 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_{10} &= \begin{pmatrix} 0.769404u^{13} + 1.08007u^{12} + \dots + 27.5531u + 2.46153 \\ 0.0235689u^{13} - 0.0691116u^{12} + \dots - 0.915315u - 1.72106 \end{pmatrix} \\ a_8 &= \begin{pmatrix} 0.792973u^{13} + 1.01096u^{12} + \dots + 26.6378u + 0.740461 \\ -0.166089u^{13} - 0.178205u^{12} + \dots - 2.14426u - 1.93905 \end{pmatrix} \\ a_6 &= \begin{pmatrix} -1.31222u^{13} - 1.03317u^{12} + \dots - 26.4352u - 0.994374 \\ -0.128536u^{13} + 0.0552761u^{12} + \dots + 3.66944u + 1.44202 \end{pmatrix} \\ a_1 &= \begin{pmatrix} 1.44202u^{13} + 1.57055u^{12} + \dots + 44.8024u + 1.21459 \\ -0.274408u^{13} - 0.288127u^{12} + \dots - 6.04501u - 3.23784 \end{pmatrix} \\ a_4 &= \begin{pmatrix} -0.626884u^{13} - 0.832752u^{12} + \dots - 24.4935u + 1.19859 \\ 0.184867u^{13} + 0.262200u^{12} + \dots + 6.69113u + 1.58682 \end{pmatrix} \\ a_3 &= \begin{pmatrix} -0.829049u^{13} - 0.665834u^{12} + \dots - 22.3564u + 8.44585 \\ 0.585885u^{13} + 0.660074u^{12} + \dots + 17.9742u + 1.02851 \end{pmatrix} \\ a_7 &= \begin{pmatrix} 0.626884u^{13} + 0.832752u^{12} + \dots + 24.4935u - 1.19859 \\ -0.166089u^{13} - 0.178205u^{12} + \dots - 2.14426u - 1.93905 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -\frac{3838300796474}{25566275354701}u^{13} + \frac{3851477719173}{25566275354701}u^{12} + \dots + \frac{391630963996580}{25566275354701}u - \frac{108530281491461}{25566275354701}$$



(iv) Complex Volumes and Cusp Shapes

Solution to $I_4^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.684697 - 0.025265I$ $a = -0.86141 + 1.42487I$ $b = -0.306521 - 0.195372I$	$-1.11654 - 3.28492I$	$-6.60141 + 2.44171I$
$u = -0.684697 + 0.025265I$ $a = -0.86141 - 1.42487I$ $b = -0.306521 + 0.195372I$	$-1.11654 + 3.28492I$	$-6.60141 - 2.44171I$
$u = -0.51639 - 2.58562I$ $a = -0.468436 - 0.254302I$ $b = -1.75993 - 0.38141I$	$-12.1121$	$-7.77053$
$u = -0.51639 + 2.58562I$ $a = -0.468436 + 0.254302I$ $b = -1.75993 + 0.38141I$	$-12.1121$	$-7.77053$
$u = -0.46039 - 1.77594I$ $a = 0.524911 + 0.031807I$ $b = 0.83720 - 1.20669I$	$-1.11654 + 3.28492I$	$-6.60141 - 2.44171I$
$u = -0.46039 + 1.77594I$ $a = 0.524911 - 0.031807I$ $b = 0.83720 + 1.20669I$	$-1.11654 - 3.28492I$	$-6.60141 + 2.44171I$
$u = -0.026394 - 0.197164I$ $a = 0.09255 - 5.01567I$ $b = -1.73502 + 0.09983I$	$-7.46645 + 4.93043I$	$-4.23989 - 2.98386I$
$u = -0.026394 + 0.197164I$ $a = 0.09255 + 5.01567I$ $b = -1.73502 - 0.09983I$	$-7.46645 - 4.93043I$	$-4.23989 + 2.98386I$
$u = 0.251357 - 0.560891I$ $a = -0.670116 + 1.116089I$ $b = -0.000704 - 0.384496I$	$-0.165382 + 1.372838I$	$-2.77344 - 4.48022I$
$u = 0.251357 + 0.560891I$ $a = -0.670116 - 1.116089I$ $b = -0.000704 + 0.384496I$	$-0.165382 - 1.372838I$	$-2.77344 + 4.48022I$

Solution to $I_4^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.43046 - 2.68133I$		
$a = 0.433855 - 0.248841I$	$-7.46645 + 4.93043I$	$-4.23989 - 2.98386I$
$b = 1.92690 - 0.44916I$		
$u = 0.43046 + 2.68133I$		
$a = 0.433855 + 0.248841I$	$-7.46645 - 4.93043I$	$-4.23989 + 2.98386I$
$b = 1.92690 + 0.44916I$		
$u = 0.506054 - 0.754738I$		
$a = -0.551357 + 0.355463I$	$-0.165382 + 1.372838I$	$-2.77344 - 4.48022I$
$b = 0.038076 - 0.785867I$		
$u = 0.506054 + 0.754738I$		
$a = -0.551357 - 0.355463I$	$-0.165382 - 1.372838I$	$-2.77344 + 4.48022I$
$b = 0.038076 + 0.785867I$		

### V. u-Polynomials

Crossings	u-Polynomials at each crossings
$c_1, c_7$	$u^4(u^7 - 2u^6 - 3u^5 + 8u^4 - 2u^3 - 2u^2 - u + 2)^2$ $(u^7 + 5u^6 + 10u^5 + 13u^4 + 18u^3 + 20u^2 + 12u + 4)$
$c_2, c_4$	$(u^2 - u + 1)^2(u^7 + 5u^6 + 11u^5 + 10u^4 - u^3 - 8u^2 - 4u - 1)$ $(u^{14} + 8u^{13} + \dots + 9u + 1)$
$c_3, c_8$	$(u^2 - u + 1)^2(u^7 + u^6 + 3u^5 + 2u^4 + 3u^3 + 2u^2 + 1)$ $(u^{14} + 2u^{13} + \dots + u + 1)$
$c_5, c_{10}$	$(u^2 + u + 1)^2(u^7 + u^6 + 3u^5 + 2u^4 + 3u^3 + 2u^2 + 1)$ $(u^{14} + 2u^{13} + \dots + u + 1)$
$c_6, c_9$	$(u^2 - u + 1)^2(u^7 + u^6 - 5u^5 - 2u^4 + 7u^3 - 4u^2 + 2u - 1)$ $(u^{14} + 2u^{13} + \dots + 5u + 1)$

## VI. Riley Polynomials

Crossings	Riley Polynomials at each crossings
$c_1, c_7$	$y^4(y^7 - 10y^6 + 37y^5 - 62y^4 + 50y^3 - 32y^2 + 9y - 4)^2$ $(y^7 - 5y^6 + 6y^5 + 15y^4 + 4y^3 - 72y^2 - 16y - 16)$
$c_2, c_4$	$(y^2 + y + 1)^2(y^7 - 3y^6 + 19y^5 - 50y^4 + 83y^3 - 36y^2 - 1)$ $(y^{14} - 4y^{13} + \dots - 15y + 1)$
$c_3, c_5, c_8$ $c_{10}$	$(y^2 + y + 1)^2(y^7 + 5y^6 + 11y^5 + 10y^4 - y^3 - 8y^2 - 4y - 1)$ $(y^{14} + 8y^{13} + \dots + 9y + 1)$
$c_6, c_9$	$(y^2 + y + 1)^2(y^7 - 11y^6 + 43y^5 - 62y^4 + 15y^3 + 8y^2 - 4y - 1)$ $(y^{14} - 16y^{13} + \dots + 9y + 1)$