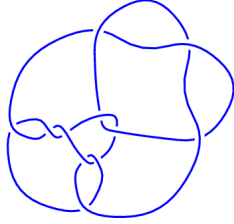
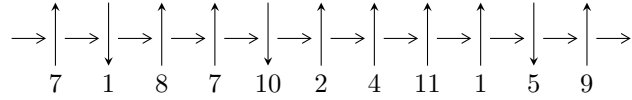


11n₉₉ (K11n₉₉)

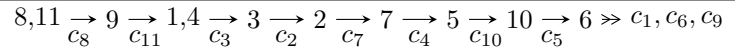


1

Arc Sequences



Solving Sequence



Representation Ideals

$$I = \bigcap_{i=1}^4 I_i^u$$

$$I_1^u = \langle u + 1, a + 1, 2b - 1 \rangle$$

$$I_2^u = \langle a^4 + 3a^2 + 1, a^3 + 2a + u, a^3 + a^2 + b + 3a + 2 \rangle$$

$$I_3^u = \langle u^{12} - u^{11} + 8u^{10} - 11u^9 + 26u^8 - 40u^7 + 45u^6 - 58u^5 + 38u^4 - 27u^3 + 3u^2 + u - 1, \\ 7u^{11} - 6u^{10} + 62u^9 - 75u^8 + 201u^7 - 297u^6 + 316u^5 - 434u^4 + 236u^3 - 169u^2 + 32b + 6u + 1, \\ -9u^{11} + 10u^{10} - 74u^9 + 109u^8 - 247u^7 + 399u^6 - 444u^5 + 582u^4 - 412u^3 + 271u^2 + 16a - 82u + 1 \rangle$$

$$I_4^u = \langle u^{12} + 3u^{11} + 10u^{10} + 22u^9 + 41u^8 + 63u^7 + 81u^6 + 85u^5 + 75u^4 + 53u^3 + 32u^2 + 12u + 9, \\ -5444u^{11} - 27924u^{10} + \dots + 35601b - 15318, 8542u^{11} + 46692u^{10} + \dots + 106803a + 142269 \rangle$$

There are 4 irreducible components with 29 representations.

¹The knot diagram image is adapter from “C. Livingston and A. H. Moore, KnotInfo: Table of Knot Invariants, <http://www.indiana.edu/~knotinfo>”

$$\mathbf{I. } I_1^u = \langle u + 1, a + 1, 2b - 1 \rangle$$

(i) Arc colorings

$$a_8 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -1 \\ \frac{1}{2} \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ -\frac{1}{2} \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -1 \\ \frac{1}{2} \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =unknown

(iv) Complex Volumes and Cusp Shapes

Solution to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.00000$		
$a = -1.00000$	3.28987	14.2500
$b = 0.500000$		

$$\text{II. } I_2^u = \langle a^4 + 3a^2 + 1, a^3 + 2a + u, a^3 + a^2 + b + 3a + 2 \rangle$$

(i) Arc colorings

$$a_8 = \begin{pmatrix} 0 \\ -a^3 - 2a \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} a \\ -a^3 - a^2 - 3a - 2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a^3 + a \\ -2a^3 - 4a - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} a^3 + a \\ -a^3 - a - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} a^3 + a + 1 \\ -a^3 - a - 2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} a^3 + 2a \\ -a^3 - 2a \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} a \\ -a^3 - a^2 - 4a - 2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -a^2 \\ a^3 + a^2 + 2a \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -a^2 \\ a^3 + a^2 + 2a \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = unknown

(iv) Complex Volumes and Cusp Shapes

Solution to I_2^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000I$		
$a = -0.618034I$	-2.30291	4.00000
$b = -1.61803 + 1.61803I$		
$u = -1.00000I$		
$a = 0.618034I$	-2.30291	4.00000
$b = -1.61803 - 1.61803I$		
$u = -1.00000I$		
$a = -1.61803I$	5.59278	4.00000
$b = 0.618034 + 0.618034I$		
$u = 1.00000I$		
$a = 1.61803I$	5.59278	4.00000
$b = 0.618034 - 0.618034I$		

$$\text{III. } I_3^u = \langle u^{12} - u^{11} + \dots + u - 1, 7u^{11} - 6u^{10} + \dots + 32b + 1, -9u^{11} + 10u^{10} + \dots + 16a + 1 \rangle$$

(i) Arc colorings

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} \frac{9}{16}u^{11} - \frac{5}{8}u^{10} + \dots + \frac{41}{8}u - \frac{1}{16} \\ -0.218750u^{11} + 0.187500u^{10} + \dots - 0.187500u - 0.0312500 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} \frac{1}{16}u^{11} - \frac{3}{8}u^{10} + \dots + \frac{47}{8}u - \frac{13}{16} \\ -0.0312500u^{11} + 0.0625000u^{10} + \dots - 0.0625000u + 0.0312500 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -\frac{1}{4}u^{10} + \frac{1}{4}u^9 + \dots - \frac{1}{4}u + \frac{5}{4} \\ \frac{1}{4}u^{10} - \frac{1}{4}u^9 + \dots + \frac{1}{4}u - \frac{1}{4} \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -\frac{1}{4}u^{10} + \frac{1}{4}u^9 + \dots - \frac{1}{4}u + \frac{5}{4} \\ \frac{1}{4}u^{10} - \frac{1}{4}u^9 + \dots + \frac{1}{4}u - \frac{1}{4} \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^2 + 1 \\ -u^2 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} \frac{5}{16}u^{11} - \frac{5}{8}u^{10} + \dots + \frac{41}{8}u - \frac{5}{16} \\ -0.343750u^{11} + 0.437500u^{10} + \dots + 0.0625000u + 0.0937500 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -\frac{1}{4}u^{11} + \frac{1}{4}u^{10} + \dots - \frac{1}{4}u^2 + \frac{9}{4}u \\ \frac{1}{4}u^{11} - \frac{1}{4}u^{10} + \dots + \frac{1}{4}u^2 - \frac{5}{4}u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -\frac{1}{4}u^{11} + \frac{1}{4}u^{10} + \dots - \frac{1}{4}u^2 + \frac{9}{4}u \\ \frac{1}{4}u^{11} - \frac{1}{4}u^{10} + \dots + \frac{1}{4}u^2 - \frac{5}{4}u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = unknown

(iv) Complex Volumes and Cusp Shapes

Solution to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.46036 - 1.59632I$ $a = 0.617140 - 0.709434I$ $b = -1.96352 + 0.61798I$	$-9.7228 + 11.6344I$	$3.05947 - 5.63312I$
$u = -0.46036 + 1.59632I$ $a = 0.617140 + 0.709434I$ $b = -1.96352 - 0.61798I$	$-9.7228 - 11.6344I$	$3.05947 + 5.63312I$
$u = -0.286848$ $a = -3.85090$ $b = 0.748460$	2.08000	2.69919
$u = -0.22221 - 1.69295I$ $a = -0.740223 - 0.119924I$ $b = 1.93980 - 0.00808I$	$-13.9790 + 4.8530I$	$0.50797 - 2.30086I$
$u = -0.22221 + 1.69295I$ $a = -0.740223 + 0.119924I$ $b = 1.93980 + 0.00808I$	$-13.9790 - 4.8530I$	$0.50797 + 2.30086I$
$u = 0.06595 - 1.61394I$ $a = 0.453344 + 0.915914I$ $b = -1.50627 - 0.49543I$	$-9.24108 - 2.07346I$	$2.05543 + 1.04459I$
$u = 0.06595 + 1.61394I$ $a = 0.453344 - 0.915914I$ $b = -1.50627 + 0.49543I$	$-9.24108 + 2.07346I$	$2.05543 - 1.04459I$
$u = 0.257303 - 0.306472I$ $a = 1.046990 + 0.141189I$ $b = -0.098944 - 0.484685I$	$0.367468 - 0.926038I$	$6.49064 + 7.55473I$
$u = 0.257303 + 0.306472I$ $a = 1.046990 - 0.141189I$ $b = -0.098944 + 0.484685I$	$0.367468 + 0.926038I$	$6.49064 - 7.55473I$
$u = 0.376488 - 0.828316I$ $a = -0.62795 - 1.54039I$ $b = 0.204419 + 0.641684I$	$6.26923 - 1.30619I$	$9.66269 + 5.18573I$

Solution to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.376488 + 0.828316I$	$6.26923 + 1.30619I$	$9.66269 - 5.18573I$
$a = -0.62795 + 1.54039I$		
$b = 0.204419 - 0.641684I$		
$u = 1.25251$	2.82934	-4.50161
$a = -0.647705$		
$b = 0.600571$		

$$\text{IV. } I_4^u = \langle u^{12} + 3u^{11} + \dots + 12u + 9, -5444u^{11} - 27924u^{10} + \dots + 35601b - 15318, 8542u^{11} + 46692u^{10} + \dots + 106803a + 142269 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_8 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_{11} &= \begin{pmatrix} -0.0799790u^{11} - 0.437179u^{10} + \dots - 3.87384u - 1.33207 \\ 0.152917u^{11} + 0.784360u^{10} + \dots + 1.96624u + 0.430269 \end{pmatrix} \\ a_9 &= \begin{pmatrix} -0.133517u^{11} - 0.508019u^{10} + \dots - 3.93721u - 1.26106 \\ 0.170388u^{11} + 0.741721u^{10} + \dots + 2.13137u + 0.590377 \end{pmatrix} \\ a_1 &= \begin{pmatrix} 0.0356638u^{11} + 0.0533693u^{10} + \dots - 0.249731u - 0.511334 \\ -1 \end{pmatrix} \\ a_4 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_3 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_2 &= \begin{pmatrix} 0.144116u^{11} + 0.338530u^{10} + \dots + 0.0727601u - 0.0287351 \\ -0.108452u^{11} - 0.285161u^{10} + \dots - 0.322491u - 1.48260 \end{pmatrix} \\ a_7 &= \begin{pmatrix} -u \\ u \end{pmatrix} \\ a_5 &= \begin{pmatrix} u^2 + 1 \\ -u^2 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} -0.152056u^{11} - 0.454201u^{10} + \dots - 4.64191u - 1.49740 \\ 0.367630u^{11} + 1.16710u^{10} + \dots + 2.50369u - 0.129435 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 0.151307u^{11} + 0.543917u^{10} + \dots + 3.37438u + 2.30940 \\ 0.0134266u^{11} - 0.158170u^{10} + \dots + 1.12047u - 0.655094 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 0.151307u^{11} + 0.543917u^{10} + \dots + 3.37438u + 2.30940 \\ 0.0134266u^{11} - 0.158170u^{10} + \dots + 1.12047u - 0.655094 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = unknown

(iv) Complex Volumes and Cusp Shapes

Solution to I_4^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.172057 - 0.407463I$ $a = 0.965356 + 0.141124I$ $b = -0.891259 + 0.143836I$	$-3.28987 + 5.69302I$	$4.00000 - 5.51057I$
$u = -1.172057 + 0.407463I$ $a = 0.965356 - 0.141124I$ $b = -0.891259 - 0.143836I$	$-3.28987 - 5.69302I$	$4.00000 + 5.51057I$
$u = -0.793458 - 0.920250I$ $a = -0.184052 - 0.624049I$ $b = -0.0718625 - 0.0733467I$	$-5.18047 + 0.92430I$	$0.283283 - 0.794226I$
$u = -0.793458 + 0.920250I$ $a = -0.184052 + 0.624049I$ $b = -0.0718625 + 0.0733467I$	$-5.18047 - 0.92430I$	$0.283283 + 0.794226I$
$u = -0.12359 - 1.51263I$ $a = 0.459386 - 0.245578I$ $b = -1.50589 - 0.10109I$	$-5.18047 - 0.92430I$	$0.283283 + 0.794226I$
$u = -0.12359 + 1.51263I$ $a = 0.459386 + 0.245578I$ $b = -1.50589 + 0.10109I$	$-5.18047 + 0.92430I$	$0.283283 - 0.794226I$
$u = 0.007520 - 1.191128I$ $a = -0.242797 + 0.842914I$ $b = 1.82424 - 1.33658I$	$-1.39926 + 0.92430I$	$7.71672 - 0.79423I$
$u = 0.007520 + 1.191128I$ $a = -0.242797 - 0.842914I$ $b = 1.82424 + 1.33658I$	$-1.39926 - 0.92430I$	$7.71672 + 0.79423I$
$u = 0.250689 - 0.621966I$ $a = 0.96746 + 1.22138I$ $b = -1.37089 + 1.38232I$	$-1.39926 - 0.92430I$	$7.71672 + 0.79423I$
$u = 0.250689 + 0.621966I$ $a = 0.96746 - 1.22138I$ $b = -1.37089 - 1.38232I$	$-1.39926 + 0.92430I$	$7.71672 - 0.79423I$

Solution to I_4^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.33089 - 1.60761I$	$-3.28987 - 5.69302I$	$4.00000 + 5.51057I$
$a = -0.465356 - 0.572259I$		
$b = 1.51567 + 0.58147I$		
$u = 0.33089 + 1.60761I$	$-3.28987 + 5.69302I$	$4.00000 - 5.51057I$
$a = -0.465356 + 0.572259I$		
$b = 1.51567 - 0.58147I$		

V. u-Polynomials

Crossings	u-Polynomials at each crossings
c_1, c_3, c_4	$(u + 1)(u^2 + 1)^2(u^{12} - u^{11} + \dots + u - 1)(u^{12} + 3u^{11} + \dots + 12u + 9)$
c_2	$(u - 1)(u + 1)^4(u^{12} + 11u^{11} + \dots + 432u + 81)$ $(u^{12} + 15u^{11} + \dots - 7u + 1)$
c_5, c_{10}	$(u)(u^4 + 3u^2 + 1)(1 - u + 2u^3 - u^4 - u^5 + u^6)^2(u^{12} + 3u^{11} + \dots + 30u + 8)$
c_6, c_7	$(u - 1)(u^2 + 1)^2(u^{12} - u^{11} + \dots + u - 1)(u^{12} + 3u^{11} + \dots + 12u + 9)$
c_8, c_9	$(u + 1)(u^2 - u - 1)^2(u^6 + u^5 - u^4 - 2u^3 + u + 1)^2$ $(u^{12} + 2u^{11} + \dots + 7u - 4)$
c_{11}	$(u - 1)(u^2 + u - 1)^2(u^6 + u^5 - u^4 - 2u^3 + u + 1)^2$ $(u^{12} + 2u^{11} + \dots + 7u - 4)$

VI. Riley Polynomials

Crossings	Riley Polynomials at each crossings
c_1, c_3, c_4 c_6, c_7	$(y - 1)(y + 1)^4(y^{12} + 11y^{11} + \dots + 432y + 81)$ $(y^{12} + 15y^{11} + \dots - 7y + 1)$
c_2	$(y - 1)^5(y^{12} - 37y^{11} + \dots - 75y + 1)$ $(y^{12} - 21y^{11} + \dots - 8100y + 6561)$
c_5, c_{10}	$y(y^2 + 3y + 1)^2(y^6 - 3y^5 + 5y^4 - 4y^3 + 2y^2 - y + 1)^2$ $(y^{12} + 3y^{11} + \dots - 180y + 64)$
c_8, c_9, c_{11}	$(y - 1)(y^2 - 3y + 1)^2(y^6 - 3y^5 + 5y^4 - 4y^3 + 2y^2 - y + 1)^2$ $(y^{12} - 10y^{11} + \dots - 65y + 16)$