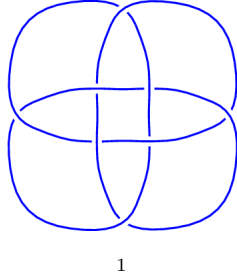
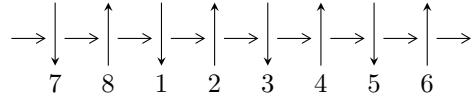


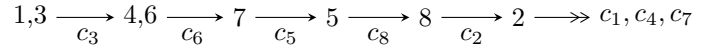
$\delta_{18} (K8a_{12})$



Arc Sequences



Solving Sequence



Representation Ideals

$$I = \bigcap_{i=1}^9 I_i^u \bigcap I_1^v$$

$$I_1^u = \langle u - 1, b, a + 1 \rangle$$

$$I_2^u = \langle u - 1, a + 1, b - 1 \rangle$$

$$I_3^u = \langle u + 1, a, b + 1 \rangle$$

$$I_4^u = \langle u^2 - u + 1, a + 1, b + u - 1 \rangle$$

$$I_5^u = \langle u^4 - u^3 + 2u + 1, u^3 - 2u^2 + a + 2u, u^3 - 2u^2 + b + u + 2 \rangle$$

$$I_6^u = \langle u^4 - u^3 + 2u + 1, a + 1, 2u^3 - 3u^2 + b + 2u + 2 \rangle$$

$$I_7^u = \langle u^4 + 2u^3 + u^2 - 2u - 1, a + 1, -u^3 - u^2 + b + 1 \rangle$$

$$I_8^u = \langle a^4 - 3a + 3, a^3 + 3a^2 + 5b + 4a - 1, 2a^3 + a^2 + 3a + 5u - 7 \rangle$$

$$I_9^u = \langle u^4 + 3u^3 + 6u^2 + 6u + 3, u^3 + 3a - 3, u^3 + 3u^2 + b + 5u + 4 \rangle$$

$$I_1^v = \langle b + 1, v - 1, a \rangle$$

There are 10 irreducible components with 26 representations.

¹The knot diagram image is adapter from “C. Livingston and A. H. Moore, KnotInfo: Table of Knot Invariants, <http://www.indiana.edu/~knotinfo>”

$$\text{I. } I_1^u = \langle u - 1, b, a + 1 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = -6

(iv) Complex Volumes and Cusp Shapes

Solution to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$		
$a = -1.00000$	-1.64493	-6.00000
$b = 0$		

$$\text{II. } I_2^u = \langle u - 1, a + 1, b - 1 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = -6

(iv) Complex Volumes and Cusp Shapes

Solution to I_2^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$		
$a = -1.00000$	-1.64493	-6.00000
$b = 1.00000$		

$$\text{III. } I_3^u = \langle u + 1, a, b + 1 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = 6

(iv) Complex Volumes and Cusp Shapes

Solution to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.00000$		
$a = 0$	1.64493	6.00000
$b = -1.00000$		

$$\text{IV. } I_4^u = \langle u^2 - u + 1, a + 1, b + u - 1 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ -u + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ -u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ u - 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = $8u - 4$

(iv) Complex Volumes and Cusp Shapes

Solution to I_4^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.500000 - 0.866025I$ $a = -1.00000$ $b = 0.500000 + 0.866025I$	$4.05977I$	$-6.92820I$
$u = 0.500000 + 0.866025I$ $a = -1.00000$ $b = 0.500000 - 0.866025I$	$-4.05977I$	$6.92820I$

$$\mathbf{V. } I_5^u = \langle u^4 - u^3 + 2u + 1, u^3 - 2u^2 + a + 2u, u^3 - 2u^2 + b + u + 2 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^3 + 2u^2 - 2u \\ -u^3 + 2u^2 - u - 2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -2u^3 + 4u^2 - 4u - 2 \\ u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^3 + 2u^2 - 2u \\ -1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -2u^3 + 3u^2 - u - 3 \\ u^3 - 2u^2 + u + 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 2u^3 - 4u^2 + 2u + 3 \\ u^2 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $-8u^3 + 16u^2 - 8u - 18$

(iv) Complex Volumes and Cusp Shapes

Solution to I_5^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.621964 - 0.187730I$		
$a = 2.12196 + 1.05376I$	$-1.64493 - 4.05977I$	$-6.00000 + 6.92820I$
$b = -0.500000 + 0.866025I$		
$u = -0.621964 + 0.187730I$		
$a = 2.12196 - 1.05376I$	$-1.64493 + 4.05977I$	$-6.00000 - 6.92820I$
$b = -0.500000 - 0.866025I$		
$u = 1.12196 - 1.05376I$		
$a = 0.378036 + 0.187730I$	$-1.64493 + 4.05977I$	$-6.00000 - 6.92820I$
$b = -0.500000 - 0.866025I$		
$u = 1.12196 + 1.05376I$		
$a = 0.378036 - 0.187730I$	$-1.64493 - 4.05977I$	$-6.00000 + 6.92820I$
$b = -0.500000 + 0.866025I$		

$$\text{VI. } I_6^u = \langle u^4 - u^3 + 2u + 1, a + 1, 2u^3 - 3u^2 + b + 2u + 2 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ -2u^3 + 3u^2 - 2u - 2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^3 + u^2 - 2 \\ -u^3 + 2u^2 - 2u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ -2u^3 + 4u^2 - 2u - 2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 2u^3 - 3u^2 + 2u + 3 \\ -3u^3 + 4u^2 - 2u - 5 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^3 - 2u^2 + 2u + 1 \\ -3u^3 + 5u^2 - 3u - 4 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $-8u^3 + 16u^2 - 8u - 18$

(iv) Complex Volumes and Cusp Shapes

Solution to I_6^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.621964 - 0.187730I$ $a = -1.00000$ $b = 0.64840 + 1.49853I$	$-1.64493 - 4.05977I$	$-6.00000 + 6.92820I$
$u = -0.621964 + 0.187730I$ $a = -1.00000$ $b = 0.64840 - 1.49853I$	$-1.64493 + 4.05977I$	$-6.00000 - 6.92820I$
$u = 1.12196 - 1.05376I$ $a = -1.00000$ $b = 0.851597 + 0.632502I$	$-1.64493 + 4.05977I$	$-6.00000 - 6.92820I$
$u = 1.12196 + 1.05376I$ $a = -1.00000$ $b = 0.851597 - 0.632502I$	$-1.64493 - 4.05977I$	$-6.00000 + 6.92820I$

$$\text{VII. } I_7^u = \langle u^4 + 2u^3 + u^2 - 2u - 1, a + 1, -u^3 - u^2 + b + 1 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ u^3 + u^2 - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u^3 + u^2 - u - 2 \\ u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ u^3 + 2u^2 - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^3 - u^2 + 2 \\ -u^2 + 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u^3 - 2u^2 + 2 \\ u^2 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $-4u^3 - 4u^2 - 4u + 4$

(iv) Complex Volumes and Cusp Shapes

Solution to I_7^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.20711 - 0.97832I$ $a = -1.00000$ $b = 1.20711 - 0.97832I$	$-12.3509I$	$7.82655I$
$u = -1.20711 + 0.97832I$ $a = -1.00000$ $b = 1.20711 + 0.97832I$	$12.3509I$	$-7.82655I$
$u = -0.468990$ $a = -1.00000$ $b = -0.883204$	1.71901	5.40877
$u = 0.883204$ $a = -1.00000$ $b = 0.468990$	-1.71901	-5.40877

$$\text{VIII. } I_8^u = \langle a^4 - 3a + 3, a^3 + 3a^2 + 5b + 4a - 1, 2a^3 + a^2 + 3a + 5u - 7 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ -\frac{2}{5}a^3 - \frac{1}{5}a^2 - \frac{3}{5}a + \frac{7}{5} \end{pmatrix}$$

$$a_4 = \begin{pmatrix} \frac{2}{5}a^3 + \frac{1}{5}a^2 + \frac{3}{5}a - \frac{7}{5} \\ -\frac{2}{5}a^3 - \frac{1}{5}a^2 - \frac{3}{5}a + \frac{7}{5} \end{pmatrix}$$

$$a_6 = \begin{pmatrix} a \\ -\frac{1}{5}a^3 - \frac{3}{5}a^2 - \frac{4}{5}a + \frac{1}{5} \end{pmatrix}$$

$$a_7 = \begin{pmatrix} \frac{2}{5}a^3 + \frac{1}{5}a^2 + \frac{3}{5}a - \frac{2}{5} \\ -\frac{3}{5}a^3 - \frac{4}{5}a^2 - \frac{2}{5}a + \frac{3}{5} \end{pmatrix}$$

$$a_5 = \begin{pmatrix} a \\ -1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -\frac{3}{5}a^3 - \frac{4}{5}a^2 - \frac{2}{5}a + \frac{8}{5} \\ a^3 + a^2 + a - 2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -\frac{4}{5}a^3 - \frac{2}{5}a^2 - \frac{1}{5}a + \frac{14}{5} \\ a^3 + a^2 - 3 \end{pmatrix}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = -\frac{16}{5}a^3 - \frac{8}{5}a^2 - \frac{24}{5}a + \frac{66}{5}$$

(iv) Complex Volumes and Cusp Shapes

Solution to I_g^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.500000 + 0.866025I$ $a = -0.97356 - 1.31080I$ $b = 0.621964 - 0.187730I$	$1.64493 - 4.05977I$	$6.00000 + 6.92820I$
$u = 0.500000 - 0.866025I$ $a = -0.97356 + 1.31080I$ $b = 0.621964 + 0.187730I$	$1.64493 + 4.05977I$	$6.00000 - 6.92820I$
$u = 0.500000 + 0.866025I$ $a = 0.973561 - 0.421254I$ $b = -1.12196 + 1.05376I$	$1.64493 - 4.05977I$	$6.00000 + 6.92820I$
$u = 0.500000 - 0.866025I$ $a = 0.973561 + 0.421254I$ $b = -1.12196 - 1.05376I$	$1.64493 + 4.05977I$	$6.00000 - 6.92820I$

$$\text{IX. } I_9^u = \langle u^4 + 3u^3 + 6u^2 + 6u + 3, u^3 + 3a - 3, u^3 + 3u^2 + b + 5u + 4 \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -\frac{1}{3}u^3 + 1 \\ -u^3 - 3u^2 - 5u - 4 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -\frac{1}{3}u^3 - u^2 - 2u - 2 \\ -u^3 - 2u^2 - 3u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -\frac{1}{3}u^3 + 1 \\ -1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -\frac{2}{3}u^3 - 2u^2 - 3u - 2 \\ 2u^3 + 4u^2 + 7u + 4 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} \frac{1}{3}u^3 \\ -u^3 - 2u^2 - 3u - 2 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $-8u^3 - 16u^2 - 24u - 6$

(iv) Complex Volumes and Cusp Shapes

Solution to I_9^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.851597 - 0.632502I$		
$a = 0.865175 + 0.374356I$	$1.64493 - 4.05977I$	$6.00000 + 6.92820I$
$b = -1.12196 + 1.05376I$		
$u = -0.851597 + 0.632502I$		
$a = 0.865175 - 0.374356I$	$1.64493 + 4.05977I$	$6.00000 - 6.92820I$
$b = -1.12196 - 1.05376I$		
$u = -0.64840 - 1.49853I$		
$a = -0.365175 - 0.491670I$	$1.64493 + 4.05977I$	$6.00000 - 6.92820I$
$b = 0.621964 + 0.187730I$		
$u = -0.64840 + 1.49853I$		
$a = -0.365175 + 0.491670I$	$1.64493 - 4.05977I$	$6.00000 + 6.92820I$
$b = 0.621964 - 0.187730I$		

$$\mathbf{X. } I_1^v = \langle b + 1, v - 1, a \rangle$$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = 6

(iv) Complex Volumes and Cusp Shapes

Solution to I_1^v	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$v = 1.00000$		
$a = 0$	1.64493	6.00000
$b = -1.00000$		

XI. u-Polynomials

Crossings	u-Polynomials at each crossings
c_1, c_3, c_5 c_7	$u(u-1)^2(u+1)(u^2-u+1)^2(u^2+u+1)(u^4-u^3+2u+1)^2$ $(u^4+2u^3+u^2-2u-1)(u^4+3u^3+6u^2+6u+3)$
c_2, c_4, c_6 c_8	$u(u-1)^2(u+1)(u^2-u+1)^3(u^4-u^3+2u+1)^2$ $(u^4+2u^3+u^2-2u-1)(u^4+3u^3+6u^2+6u+3)$

XII. Riley Polynomials

Crossings	Riley Polynomials at each crossings
c_1, c_2, c_3 c_4, c_5, c_6 c_7, c_8	$y(y-1)^3(y^2+y+1)^3(y^4-2y^3+7y^2-6y+1)$ $(y^4-y^3+6y^2-4y+1)^2(y^4+3y^3+6y^2+9)$