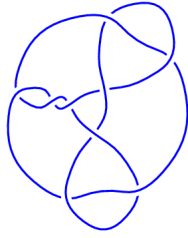
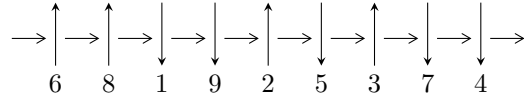


9₃₇ (K9a₁₈)

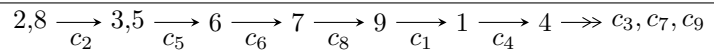


1

Arc Sequences



Solving Sequence



Representation Ideals

$$I = \bigcap_{i=1}^6 I_i^u$$

$$I_1^u = \langle u^2 + 1, b + u, a - u - 1 \rangle$$

$$I_2^u = \langle u^4 - u^3 + 2u^2 - 2u + 1, -u^3 + b - u + 1, -u^3 + u^2 + a - 2u + 2 \rangle$$

$$I_3^u = \langle u^6 + u^4 + 2u^3 + u^2 + u + 2, -u^3 + b - 1, -u^5 - u^3 - 2u^2 + 2a - u - 1 \rangle$$

$$I_4^u = \langle u^8 - u^7 + 2u^6 - 2u^5 + 4u^4 - 3u^3 + 2u^2 + 1, b - u, u^7 + 2u^3 + u^2 + 2a - 3u + 1 \rangle$$

$$I_5^u = \langle u^2 + u + 1, a + 2, b - u \rangle$$

$$I_6^u = \langle b^4 - b^3 + 2b^2 - 2b + 1, -b^3 - b + a, -b^3 - b + u + 1 \rangle$$

There are 6 irreducible components with 26 representations.

¹The knot diagram image is adapter from “C. Livingston and A. H. Moore, KnotInfo: Table of Knot Invariants, <http://www.indiana.edu/~knotinfo>”

$$\text{I. } I_1^u = \langle u^2 + 1, b + u, a - u - 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u + 1 \\ -u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u \\ 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 1 \\ -u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ u + 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ u + 1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = -4

(iv) Complex Volumes and Cusp Shapes

Solution to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.00000I$		
$a = 1.00000 - 1.00000I$	-1.64493	-4.00000
$b = 1.00000I$		
$u = 1.00000I$		
$a = 1.00000 + 1.00000I$	-1.64493	-4.00000
$b = -1.00000I$		

$$\text{II. } I_2^u = \langle u^4 - u^3 + 2u^2 - 2u + 1, -u^3 + b - u + 1, -u^3 + u^2 + a - 2u + 2 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^3 - u^2 + 2u - 2 \\ u^3 + u - 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^3 + 2u \\ u^3 + u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^2 + 1 \\ u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $4u^3 + 4u - 2$

(iv) Complex Volumes and Cusp Shapes

Solution to I_2^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.121744 - 1.306622I$ $a = 0.070696 - 0.758745I$ $b = -0.500000 + 0.866025I$	$-2.02988I$	$3.46410I$
$u = -0.121744 + 1.306622I$ $a = 0.070696 + 0.758745I$ $b = -0.500000 - 0.866025I$	$2.02988I$	$-3.46410I$
$u = 0.621744 - 0.440597I$ $a = -1.070696 - 0.758745I$ $b = -0.500000 - 0.866025I$	$2.02988I$	$-3.46410I$
$u = 0.621744 + 0.440597I$ $a = -1.070696 + 0.758745I$ $b = -0.500000 + 0.866025I$	$-2.02988I$	$3.46410I$

III.

$$I_3^u = \langle u^6 + u^4 + 2u^3 + u^2 + u + 2, -u^3 + b - 1, -u^5 - u^3 - 2u^2 + 2a - u - 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_8 &= \begin{pmatrix} \frac{1}{2}u^5 + \frac{1}{2}u^3 + u^2 + \frac{1}{2}u + \frac{1}{2} \\ u^3 + 1 \end{pmatrix} \\ a_3 &= \begin{pmatrix} -\frac{1}{2}u^5 - \frac{1}{2}u^3 - \frac{1}{2}u + \frac{1}{2} \\ u^4 + u^2 + u + 1 \end{pmatrix} \\ a_5 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_6 &= \begin{pmatrix} u \\ u \end{pmatrix} \\ a_7 &= \begin{pmatrix} u \\ u^3 + u \end{pmatrix} \\ a_9 &= \begin{pmatrix} -\frac{1}{2}u^5 - \frac{1}{2}u^3 - \frac{1}{2}u + \frac{1}{2} \\ -u^5 + u^4 + u + 1 \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^2 + 1 \\ u^2 \end{pmatrix} \\ a_4 &= \begin{pmatrix} \frac{1}{2}u^5 + \frac{3}{2}u^3 + u^2 + \frac{1}{2}u + \frac{3}{2} \\ u^5 + u^4 + u^3 + 2u^2 + u + 1 \end{pmatrix} \\ a_4 &= \begin{pmatrix} \frac{1}{2}u^5 + \frac{3}{2}u^3 + u^2 + \frac{1}{2}u + \frac{3}{2} \\ u^5 + u^4 + u^3 + 2u^2 + u + 1 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $-4u^4 - 4u^3 - 8u - 6$

(iv) Complex Volumes and Cusp Shapes

Solution to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.931903 - 0.428993I$		
$a = 0.885437 - 0.407603I$	$6.15087 - 5.13794I$	$3.31793 + 3.20902I$
$b = 0.705204 - 1.038718I$		
$u = -0.931903 + 0.428993I$		
$a = 0.885437 + 0.407603I$	$6.15087 + 5.13794I$	$3.31793 - 3.20902I$
$b = 0.705204 + 1.038718I$		
$u = 0.226699 - 1.074326I$		
$a = -0.188043 - 0.891136I$	-4.07707	-8.63587
$b = 0.226699 + 1.074326I$		
$u = 0.226699 + 1.074326I$		
$a = -0.188043 + 0.891136I$	-4.07707	-8.63587
$b = 0.226699 - 1.074326I$		
$u = 0.705204 - 1.038718I$		
$a = -0.447394 - 0.658981I$	$6.15087 - 5.13794I$	$3.31793 + 3.20902I$
$b = -0.931903 - 0.428993I$		
$u = 0.705204 + 1.038718I$		
$a = -0.447394 + 0.658981I$	$6.15087 + 5.13794I$	$3.31793 - 3.20902I$
$b = -0.931903 + 0.428993I$		

IV.

$$I_4^u = \langle u^8 - u^7 + 2u^6 - 2u^5 + 4u^4 - 3u^3 + 2u^2 + 1, b - u, u^7 + 2u^3 + u^2 + 2a - 3u + 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -\frac{1}{2}u^7 - u^3 - \frac{1}{2}u^2 + \frac{3}{2}u - \frac{1}{2} \\ u \end{pmatrix} \\ a_3 &= \begin{pmatrix} \frac{1}{2}u^7 - u^6 + \dots + \frac{1}{2}u + \frac{1}{2} \\ -u^2 \end{pmatrix} \\ a_5 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_6 &= \begin{pmatrix} u \\ u \end{pmatrix} \\ a_7 &= \begin{pmatrix} u \\ u^3 + u \end{pmatrix} \\ a_9 &= \begin{pmatrix} -\frac{1}{2}u^7 - \frac{1}{2}u^2 + \frac{3}{2}u - \frac{1}{2} \\ u^5 + u^3 + u \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^2 + 1 \\ u^2 \end{pmatrix} \\ a_4 &= \begin{pmatrix} \frac{1}{2}u^7 + u^5 + \dots + \frac{1}{2}u + \frac{1}{2} \\ \frac{1}{2}u^7 + u^5 + \dots + \frac{1}{2}u - \frac{1}{2} \end{pmatrix} \\ a_4 &= \begin{pmatrix} \frac{1}{2}u^7 + u^5 + \dots + \frac{1}{2}u + \frac{1}{2} \\ \frac{1}{2}u^7 + u^5 + \dots + \frac{1}{2}u - \frac{1}{2} \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $-4u^6 + 2u^5 - 4u^4 + 6u^3 - 12u^2 + 6u$

(iv) Complex Volumes and Cusp Shapes

Solution to I_4^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.666851 - 1.155534I$ $a = -1.55320 + 0.75511I$ $b = -0.666851 - 1.155534I$	$3.94193 + 10.98945I$	$0.47099 - 7.14773I$
$u = -0.666851 + 1.155534I$ $a = -1.55320 - 0.75511I$ $b = -0.666851 + 1.155534I$	$3.94193 - 10.98945I$	$0.47099 + 7.14773I$
$u = -0.273948 - 0.520074I$ $a = -1.011913 - 0.934421I$ $b = -0.273948 - 0.520074I$	$0.221012 + 1.276802I$	$1.83102 - 5.88514I$
$u = -0.273948 + 0.520074I$ $a = -1.011913 + 0.934421I$ $b = -0.273948 + 0.520074I$	$0.221012 - 1.276802I$	$1.83102 + 5.88514I$
$u = 0.578102 - 1.055334I$ $a = 1.84091 + 0.61494I$ $b = 0.578102 - 1.055334I$	$-1.73404 - 6.79402I$	$-3.11839 + 7.09473I$
$u = 0.578102 + 1.055334I$ $a = 1.84091 - 0.61494I$ $b = 0.578102 + 1.055334I$	$-1.73404 + 6.79402I$	$-3.11839 - 7.09473I$
$u = 0.862697 - 0.615401I$ $a = 1.224206 + 0.050581I$ $b = 0.862697 - 0.615401I$	$7.44069 - 0.66722I$	$4.81639 + 2.10627I$
$u = 0.862697 + 0.615401I$ $a = 1.224206 - 0.050581I$ $b = 0.862697 + 0.615401I$	$7.44069 + 0.66722I$	$4.81639 - 2.10627I$

$$\mathbf{V. } I_5^u = \langle u^2 + u + 1, a + 2, b - u \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -2 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 2u + 1 \\ u + 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ u + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u \\ -u - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $4u + 2$

(iv) Complex Volumes and Cusp Shapes

Solution to I_5^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.500000 - 0.866025I$ $a = -2.00000$ $b = -0.500000 - 0.866025I$	$2.02988I$	$-3.46410I$
$u = -0.500000 + 0.866025I$ $a = -2.00000$ $b = -0.500000 + 0.866025I$	$-2.02988I$	$3.46410I$

$$\text{VI. } I_6^u = \langle b^4 - b^3 + 2b^2 - 2b + 1, -b^3 - b + a, -b^3 - b + u + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} b^3 + b \\ b \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -b^3 + b^2 - 2b + 2 \\ -b^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ b^3 + b - 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} b^3 + b - 1 \\ b^3 + b - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} b^3 + b - 1 \\ b^3 + b \end{pmatrix}$$

$$a_9 = \begin{pmatrix} b^3 - b^2 + 2b - 1 \\ 2b - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -b^3 - b + 1 \\ -b^3 - b \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -b + 1 \\ b^3 - 2b^2 + 2b - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -b + 1 \\ b^3 - 2b^2 + 2b - 1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $4b^3 + 4b - 2$

(iv) Complex Volumes and Cusp Shapes

Solution to I_6^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.500000 + 0.866025I$		
$a = 0.500000 + 0.866025I$	$-2.02988I$	$3.46410I$
$b = -0.121744 - 1.306622I$		
$u = -0.500000 - 0.866025I$		
$a = 0.500000 - 0.866025I$	$2.02988I$	$-3.46410I$
$b = -0.121744 + 1.306622I$		
$u = -0.500000 - 0.866025I$		
$a = 0.500000 - 0.866025I$	$2.02988I$	$-3.46410I$
$b = 0.621744 - 0.440597I$		
$u = -0.500000 + 0.866025I$		
$a = 0.500000 + 0.866025I$	$-2.02988I$	$3.46410I$
$b = 0.621744 + 0.440597I$		

VII. u-Polynomials

Crossings	u-Polynomials at each crossings
c_1, c_2, c_5 c_7	$(u^2 + 1)(u^2 + u + 1)^3(u^4 - u^3 + \dots - 2u + 1)(u^6 + u^4 + \dots + u + 2)$ $(u^8 - u^7 + 2u^6 - 2u^5 + 4u^4 - 3u^3 + 2u^2 + 1)$
c_3, c_4, c_9	$(u^2 + 1)(u^2 + u + 1)(u^3 + 2u + 1)^2(u^4 - u^3 + 2u^2 - 2u + 1)^2$ $(u^8 + 2u^7 + 6u^6 + 8u^5 + 10u^4 + 9u^3 + 5u^2 + 3u + 2)$
c_6, c_8	$(u + 1)^2(u^2 + u + 1)^3(u^4 + 3u^3 + 2u^2 + 1)$ $(u^6 + 2u^5 + 3u^4 + 2u^3 + u^2 + 3u + 4)$ $(u^8 + 3u^7 + 8u^6 + 10u^5 + 14u^4 + 11u^3 + 12u^2 + 4u + 1)$

VIII. Riley Polynomials

Crossings	Riley Polynomials at each crossings
c_1, c_2, c_5 c_7	$(y + 1)^2(y^2 + y + 1)^3(y^4 + 3y^3 + 2y^2 + 1)$ $(y^6 + 2y^5 + 3y^4 + 2y^3 + y^2 + 3y + 4)$ $(y^8 + 3y^7 + 8y^6 + 10y^5 + 14y^4 + 11y^3 + 12y^2 + 4y + 1)$
c_3, c_4, c_9	$(y + 1)^2(y^2 + y + 1)(y^3 + 4y^2 + 4y - 1)^2(y^4 + 3y^3 + 2y^2 + 1)^2$ $(y^8 + 8y^7 + 24y^6 + 30y^5 + 8y^4 - 5y^3 + 11y^2 + 11y + 4)$
c_6, c_8	$(y - 1)^2(y^2 + y + 1)^3(y^4 - 5y^3 + 6y^2 + 4y + 1)$ $(y^6 + 2y^5 + 3y^4 - 2y^3 + 13y^2 - y + 16)$ $(y^8 + 7y^7 + 32y^6 + 82y^5 + 146y^4 + 151y^3 + 84y^2 + 8y + 1)$