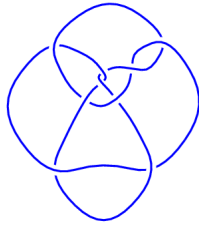
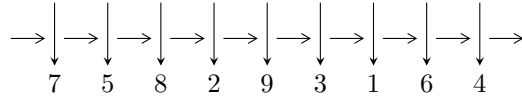


9<sub>38</sub> (K9a<sub>30</sub>)

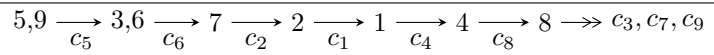


1

**Arc Sequences**



**Solving Sequence**



**Representation Ideals**

$$I = \bigcap_{i=1}^3 I_i^u$$

$$I_1^u = \langle u - 1, a + 1, 2b + 1 \rangle$$

$$I_2^u = \langle u^{11} - 3u^9 - u^8 + 4u^7 + 2u^6 - u^5 - 10u^4 - 5u^3 + 16u^2 + 9u - 4, \\ - 3321u^{10} + 1809u^9 + \dots + 62998b + 34730, -22281u^{10} + 9832u^9 + \dots + 125996a - 41833 \rangle$$

$$I_3^u = \langle a^{18} - 4a^{17} + \dots - 4a + 1, \\ - 200500518a^{17} + 12776149583u + \dots + 24671263349a - 19999446477, \\ 3625488013a^{17} + 12776149583b + \dots + 34592584052a - 2877667804 \rangle$$

There are 3 irreducible components with 30 representations.

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<sup>1</sup>The knot diagram image is adapter from “C. Livingston and A. H. Moore, KnotInfo: Table of Knot Invariants, <http://www.indiana.edu/~knotinfo>”

$$\text{I. } I_1^u = \langle u - 1, a + 1, 2b + 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ -\frac{1}{2} \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ \frac{1}{2} \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -\frac{3}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -9.75

(iv) Complex Volumes and Cusp Shapes

| Solution to $I_1^u$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---------------------|---------------------------------------|------------|
| $u = 1.00000$       |                                       |            |
| $a = -1.00000$      | -3.28987                              | -9.75000   |
| $b = -0.500000$     |                                       |            |

$$\text{II. } I_2^u = \langle u^{11} - 3u^9 + \cdots + 9u - 4, -3321u^{10} + 1809u^9 + \cdots + 62998b + 34730, -22281u^{10} + 9832u^9 + \cdots + 125996a - 41833 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_5 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_9 &= \begin{pmatrix} 0.176839u^{10} - 0.0780342u^9 + \cdots + 2.93914u + 0.332018 \\ 0.0527160u^{10} - 0.0287152u^9 + \cdots + 0.554700u - 0.551287 \end{pmatrix} \\ a_3 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_6 &= \begin{pmatrix} -0.221332u^{10} + 0.155116u^9 + \cdots - 2.53484u + 0.315605 \\ -0.00547636u^{10} + 0.0517635u^9 + \cdots + 0.534065u + 0.293946 \end{pmatrix} \\ a_7 &= \begin{pmatrix} -0.215856u^{10} + 0.103352u^9 + \cdots - 3.06891u + 0.0216594 \\ -0.00547636u^{10} + 0.0517635u^9 + \cdots + 0.534065u + 0.293946 \end{pmatrix} \\ a_2 &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_1 &= \begin{pmatrix} 0.228602u^{10} - 0.228182u^9 + \cdots + 3.28237u + 0.310113 \\ -0.00169847u^{10} + 0.0334455u^9 + \cdots - 0.660450u + 0.0273977 \end{pmatrix} \\ a_4 &= \begin{pmatrix} u \\ -u^3 + u \end{pmatrix} \\ a_8 &= \begin{pmatrix} 0.403743u^{10} - 0.272771u^9 + \cdots + 5.56107u - 0.330471 \\ 0.0510175u^{10} + 0.00473031u^9 + \cdots - 0.105749u - 0.523890 \end{pmatrix} \\ a_8 &= \begin{pmatrix} 0.403743u^{10} - 0.272771u^9 + \cdots + 5.56107u - 0.330471 \\ 0.0510175u^{10} + 0.00473031u^9 + \cdots - 0.105749u - 0.523890 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = \frac{87169}{125996}u^{10} - \frac{83847}{125996}u^9 - \frac{3869}{4846}u^8 + \frac{159181}{125996}u^7 + \frac{4589}{9692}u^6 - \frac{267389}{125996}u^5 + \frac{132445}{62998}u^4 - \frac{176912}{31499}u^3 + \frac{560139}{125996}u^2 + \frac{537971}{125996}u - \frac{504165}{31499}$$

(iv) Complex Volumes and Cusp Shapes

| Solution to $I_2^u$         | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape             |
|-----------------------------|---------------------------------------|------------------------|
| $u = -1.29294 - 0.67490I$   |                                       |                        |
| $a = 0.762233 + 0.603071I$  | $3.25113 - 12.93286I$                 | $-6.73085 + 7.81031I$  |
| $b = 2.03263 - 0.98763I$    |                                       |                        |
| $u = -1.29294 + 0.67490I$   |                                       |                        |
| $a = 0.762233 - 0.603071I$  | $3.25113 + 12.93286I$                 | $-6.73085 - 7.81031I$  |
| $b = 2.03263 + 0.98763I$    |                                       |                        |
| $u = -0.988348 - 0.222965I$ |                                       |                        |
| $a = -1.065329 + 0.599728I$ | $-3.44203 - 0.72668I$                 | $-9.61068 + 7.91738I$  |
| $b = -0.429723 + 0.123718I$ |                                       |                        |
| $u = -0.988348 + 0.222965I$ |                                       |                        |
| $a = -1.065329 - 0.599728I$ | $-3.44203 + 0.72668I$                 | $-9.61068 - 7.91738I$  |
| $b = -0.429723 - 0.123718I$ |                                       |                        |
| $u = -0.308687 - 1.224932I$ |                                       |                        |
| $a = -0.853176 - 0.534702I$ | $6.41512 + 6.30680I$                  | $-3.61485 - 5.61897I$  |
| $b = -1.81389 + 0.17756I$   |                                       |                        |
| $u = -0.308687 + 1.224932I$ |                                       |                        |
| $a = -0.853176 + 0.534702I$ | $6.41512 - 6.30680I$                  | $-3.61485 + 5.61897I$  |
| $b = -1.81389 - 0.17756I$   |                                       |                        |
| $u = 0.304704$              |                                       |                        |
| $a = 1.17274$               | $-0.695510$                           | $-14.4377$             |
| $b = -0.369666$             |                                       |                        |
| $u = 1.019424 - 0.904921I$  |                                       |                        |
| $a = 0.667443 - 0.352101I$  | $0.53843 + 4.57539I$                  | $-8.21994 - 7.99945I$  |
| $b = 1.67740 + 1.10147I$    |                                       |                        |
| $u = 1.019424 + 0.904921I$  |                                       |                        |
| $a = 0.667443 + 0.352101I$  | $0.53843 - 4.57539I$                  | $-8.21994 + 7.99945I$  |
| $b = 1.67740 - 1.10147I$    |                                       |                        |
| $u = 1.41820 - 0.12736I$    |                                       |                        |
| $a = -0.222540 + 0.460430I$ | $-1.48009 + 1.36667I$                 | $-10.72983 - 4.40179I$ |
| $b = -0.531581 - 0.640723I$ |                                       |                        |
|                             |                                       |                        |
| Solution to $I_2^u$         | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape             |
| $u = 1.41820 + 0.12736I$    |                                       |                        |
| $a = -0.222540 - 0.460430I$ | $-1.48009 - 1.36667I$                 | $-10.72983 + 4.40179I$ |
| $b = -0.531581 + 0.640723I$ |                                       |                        |

III.

$$I_3^u = \langle a^{18} - 4a^{17} + \dots - 4a + 1, 1.28 \times 10^{10}u - 2.01 \times 10^8 a^{17} + \dots + 2.47 \times 10^{10}a - 2.00 \times 10^{10}, 1.28 \times 10^{10}b + 3.63 \times 10^9 a^{17} + \dots + 3.46 \times 10^{10}a - 2.88 \times 10^9 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 0 \\ 0.0156933a^{17} + 0.277494a^{16} + \dots - 1.93104a + 1.56537 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a \\ -0.283770a^{17} + 1.00134a^{16} + \dots - 2.70759a + 0.225237 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -0.307406a^{17} + 1.15986a^{16} + \dots - 1.34538a + 0.340267 \\ 0.0252331a^{17} + 0.241224a^{16} + \dots - 1.08111a + 1.37542 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -0.332639a^{17} + 0.918640a^{16} + \dots - 0.264267a - 1.03515 \\ 0.0252331a^{17} + 0.241224a^{16} + \dots - 1.08111a + 1.37542 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ 0.379495a^{17} - 1.53831a^{16} + \dots + 5.06323a - 2.22442 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0.433484a^{17} - 1.61396a^{16} + \dots + 4.63079a - 0.134331 \\ 0.0635000a^{17} - 0.322077a^{16} + \dots + 0.744103a - 1.29532 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.0156933a^{17} + 0.277494a^{16} + \dots - 1.93104a + 1.56537 \\ 0.631124a^{17} - 2.35612a^{16} + \dots + 6.60223a - 1.38641 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -0.166444a^{17} + 0.552507a^{16} + \dots + 0.838909a + 0.0351905 \\ -0.490809a^{17} + 1.79742a^{16} + \dots - 3.82633a + 0.0146775 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -0.166444a^{17} + 0.552507a^{16} + \dots + 0.838909a + 0.0351905 \\ -0.490809a^{17} + 1.79742a^{16} + \dots - 3.82633a + 0.0146775 \end{pmatrix}$$

(ii) Obstruction class = -1

$$(iii) \text{ Cusp Shapes} = -\frac{5920287080}{12776149583}a^{17} + \frac{13955345304}{12776149583}a^{16} + \dots - \frac{7614395024}{12776149583}a - \frac{149319179942}{12776149583}$$

(iv) Complex Volumes and Cusp Shapes

| Solution to $I_3^u$   | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape             |
|---|---------------------------------------|------------------------|
| $u = -1.172473 - 0.500383I$<br>$a = -0.846962 - 0.730365I$<br>$b = -1.62741 + 0.76102I$ | $-0.30826 - 7.08493I$                 | $-9.57680 + 5.91335I$  |
| $u = -1.172473 + 0.500383I$<br>$a = -0.846962 + 0.730365I$<br>$b = -1.62741 - 0.76102I$ | $-0.30826 + 7.08493I$                 | $-9.57680 - 5.91335I$  |
| $u = -0.772920 - 0.510351I$<br>$a = -0.55260 - 1.44445I$<br>$b = -0.656269 + 0.610688I$ | $5.07330 - 2.09337I$                  | $-3.48501 + 4.16283I$  |
| $u = -0.772920 + 0.510351I$<br>$a = -0.55260 + 1.44445I$<br>$b = -0.656269 - 0.610688I$ | $5.07330 + 2.09337I$                  | $-3.48501 - 4.16283I$  |
| $u = 1.173911 + 0.391555I$<br>$a = -0.415700 - 0.594955I$<br>$b = -1.38164 + 0.70886I$  | $-1.08148 - 1.33617I$                 | $-11.28409 + 0.70175I$ |
| $u = 1.173911 - 0.391555I$<br>$a = -0.415700 + 0.594955I$<br>$b = -1.38164 - 0.70886I$  | $-1.08148 + 1.33617I$                 | $-11.28409 - 0.70175I$ |
| $u = -0.141484 - 0.739668I$<br>$a = 0.037087 - 0.882527I$<br>$b = 0.578266 - 0.230439I$ | $2.67293 + 2.45442I$                  | $-6.32792 - 2.91298I$  |
| $u = -0.141484 + 0.739668I$<br>$a = 0.037087 + 0.882527I$<br>$b = 0.578266 + 0.230439I$ | $2.67293 - 2.45442I$                  | $-6.32792 + 2.91298I$  |
| $u = 0.825933$<br>$a = 0.158918 - 1.241647I$<br>$b = 2.80520 + 2.66493I$                | 2.09142                               | -12.6523               |
| $u = 0.825933$<br>$a = 0.158918 + 1.241647I$<br>$b = 2.80520 - 2.66493I$                | 2.09142                               | -12.6523               |

| Solution to $I_3^u$   | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape             |
|---|---------------------------------------|------------------------|
| $u = 1.173911 + 0.391555I$<br>$a = 0.190948 - 0.324346I$<br>$b = -0.344997 + 0.424699I$ | $-1.08148 - 1.33617I$                 | $-11.28409 + 0.70175I$ |
| $u = 1.173911 - 0.391555I$<br>$a = 0.190948 + 0.324346I$<br>$b = -0.344997 - 0.424699I$ | $-1.08148 + 1.33617I$                 | $-11.28409 - 0.70175I$ |
| $u = -1.172473 - 0.500383I$<br>$a = 0.722652 - 0.443007I$<br>$b = 0.324529 - 0.248817I$ | $-0.30826 - 7.08493I$                 | $-9.57680 + 5.91335I$  |
| $u = -1.172473 + 0.500383I$<br>$a = 0.722652 + 0.443007I$<br>$b = 0.324529 + 0.248817I$ | $-0.30826 + 7.08493I$                 | $-9.57680 - 5.91335I$  |
| $u = -0.772920 + 0.510351I$<br>$a = 1.215657 - 0.655526I$<br>$b = 1.56797 - 0.08607I$   | $5.07330 + 2.09337I$                  | $-3.48501 - 4.16283I$  |
| $u = -0.772920 - 0.510351I$<br>$a = 1.215657 + 0.655526I$<br>$b = 1.56797 + 0.08607I$   | $5.07330 - 2.09337I$                  | $-3.48501 + 4.16283I$  |
| $u = -0.141484 + 0.739668I$<br>$a = 1.49000 - 0.67322I$<br>$b = 1.234355 - 0.197331I$   | $2.67293 - 2.45442I$                  | $-6.32792 + 2.91298I$  |
| $u = -0.141484 - 0.739668I$<br>$a = 1.49000 + 0.67322I$<br>$b = 1.234355 + 0.197331I$   | $2.67293 + 2.45442I$                  | $-6.32792 - 2.91298I$  |



#### IV. u-Polynomials

| Crossings  | u-Polynomials at each crossings  |
|------------|--|
| $c_1, c_8$ | $(u + 1)$ $(u^{11} - u^{10} + 3u^9 - 6u^8 + 7u^7 - 10u^6 + 11u^5 - 9u^4 + 9u^3 - 3u^2 + 3u - 1)$ $(u^{18} + 3u^{17} + \dots + 2u + 1)$       |
| $c_2$      | $(u - 1)(u^9 + u^8 - 2u^7 - 3u^6 + u^5 + 3u^4 + 2u^3 - u - 1)^2$ $(u^{11} - 3u^9 - u^8 + 4u^7 + 2u^6 - u^5 - 10u^4 - 5u^3 + 16u^2 + 9u - 4)$ |
| $c_3$      | $u(u^9 - u^8 + 2u^7 - u^6 + 3u^5 - u^4 + 2u^3 + u + 1)^2$ $(u^{11} + 3u^{10} + \dots - 6u - 8)$  |
| $c_4$      | $(u + 1)(u^9 + u^8 - 2u^7 - 3u^6 + u^5 + 3u^4 + 2u^3 - u - 1)^2$ $(u^{11} - 3u^9 - u^8 + 4u^7 + 2u^6 - u^5 - 10u^4 - 5u^3 + 16u^2 + 9u - 4)$ |
| $c_5, c_7$ | $(u - 1)$ $(u^{11} - u^{10} + 3u^9 - 6u^8 + 7u^7 - 10u^6 + 11u^5 - 9u^4 + 9u^3 - 3u^2 + 3u - 1)$ $(u^{18} + 3u^{17} + \dots + 2u + 1)$       |
| $c_6$      | $(2u + 1)(2u^{11} - u^{10} + \dots + 5u^2 - 1)$ $(u^{18} + 3u^{17} + \dots - 4u + 11)$   |
| $c_9$      | $(2u - 1)(2u^{11} - u^{10} + \dots + 5u^2 - 1)$ $(u^{18} + 3u^{17} + \dots - 4u + 11)$   |

## V. Riley Polynomials

| Crossings                | Riley Polynomials at each crossings   |
|--------------------------|---|
| $c_1, c_5, c_7$<br>$c_8$ | $(y - 1)$<br>$(y^{11} + 5y^{10} + 11y^9 + 8y^8 - 5y^7 + 47y^5 + 87y^4 + 73y^3 + 27y^2 + 3y - 1)$<br>$(y^{18} + 11y^{17} + \dots + 14y^2 + 1)$ |
| $c_2, c_4$               | $(y - 1)(y^9 - 5y^8 + 12y^7 - 15y^6 + 9y^5 + y^4 - 4y^3 + 2y^2 + y - 1)^2$<br>$(y^{11} - 6y^{10} + \dots + 209y - 16)$                        |
| $c_3$                    | $y(y^9 + 3y^8 + 8y^7 + 13y^6 + 17y^5 + 17y^4 + 12y^3 + 6y^2 + y - 1)^2$<br>$(y^{11} + 3y^{10} + \dots + 52y - 64)$                            |
| $c_6, c_9$               | $(4y - 1)(4y^{11} - y^{10} + \dots + 10y - 1)(y^{18} + 7y^{17} + \dots + 1260y + 121)$  |