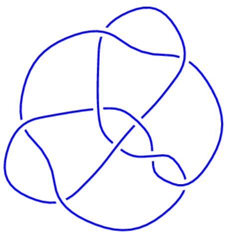
9_{39} ($K9a_{32}$)



1

Arc Sequences

$\to \uparrow \to \downarrow \to \uparrow \to \uparrow \to \uparrow \to \uparrow \to \uparrow \to \downarrow \to \uparrow \to$

7 9 6 1 8 2 4 3 5

Solving Sequence

$$1{,}7 \xrightarrow[c_1]{} 2{,}5 \xrightarrow[c_4]{} 4 \xrightarrow[c_7]{} 8 \xrightarrow[c_6]{} 6 \xrightarrow[c_3]{} 3 \xrightarrow[c_9]{} 9 \twoheadrightarrow c_2, c_5, c_8$$

Representation Ideals

$$I = \bigcap_{i=1}^{3} I_i^u$$

$$\begin{aligned}
I_1^u &= \langle u^4 + 2u^2 + u + 1,\ b + u,\ u^3 - u^2 + a + u \rangle \\
I_2^u &= \langle u^{11} + 4u^9 - u^8 + 7u^7 - 3u^6 + 4u^5 - 3u^4 + u^3 - u^2 + u + 1,\ b - u, \\
&\quad 17u^{10} + 18u^9 + 65u^8 + 43u^7 + 91u^6 + 38u^5 + 20u^4 - 21u^3 + 8u^2 + 25a + 15u + 27 \rangle \\
I_3^u &= \langle u^{20} + u^{19} + \cdots - 8u + 7,\ -63269332u^{19} - 195765489u^{18} + \cdots + 599392561b - 259471427, \\
&\quad 5187188287u^{19} + 10884118778u^{18} + \cdots + 20978739635a + 61432716820 \rangle
\end{aligned}$$

There are 3 irreducible components with 35 representations.

[1]The knot diagram image is adapter from "C. Livingston and A. H. Moore, KnotInfo: Table of Knot Invariants, http://www.indiana.edu/ knotinfo"

I. $I_1^u = \langle u^4 + 2u^2 + u + 1,\ b + u,\ u^3 - u^2 + a + u \rangle$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} -u^3 + u^2 - u \\ -u \end{pmatrix}$$
$$a_2 = \begin{pmatrix} u^3 + u^2 + u + 2 \\ -u^2 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_4 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -u^3 - u - 1 \\ u^2 - u + 1 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} -1 \\ -u^3 - u \end{pmatrix}$$
$$a_3 = \begin{pmatrix} u^2 - u + 1 \\ -u^2 + u \end{pmatrix}$$
$$a_9 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

(ii) Obstruction class $= 1$

(iii) Cusp Shapes $= 7u^3 - 2u^2 + 11u + 8$

(iv) Complex Volumes and Cusp Shapes

Solution to I_1^u	$\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.343815 - 0.625358I$ $a = -0.291777 + 1.032580I$ $b = 0.343815 + 0.625358I$	$1.13814 + 3.38562I$	$7.30286 - 7.57942I$
$u = -0.343815 + 0.625358I$ $a = -0.291777 - 1.032580I$ $b = 0.343815 - 0.625358I$	$1.13814 - 3.38562I$	$7.30286 + 7.57942I$
$u = 0.343815 - 1.358435I$ $a = -0.20822 - 1.60071I$ $b = -0.343815 + 1.358435I$	$-4.42801 - 2.37936I$	$2.19714 + 1.10073I$
$u = 0.343815 + 1.358435I$ $a = -0.20822 + 1.60071I$ $b = -0.343815 - 1.358435I$	$-4.42801 + 2.37936I$	$2.19714 - 1.10073I$

II. $I_2^u = \langle u^{11} + 4u^9 + \cdots + u + 1,\ b - u,\ 17u^{10} + 18u^9 + \cdots + 25a + 27\rangle$

(i) Arc colorings

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} -0.680000u^{10} - 0.720000u^9 + \cdots - 0.600000u - 1.08000 \\ u \end{pmatrix}$$
$$a_2 = \begin{pmatrix} \frac{18}{25}u^{10} - \frac{3}{25}u^9 + \cdots + \frac{2}{5}u + \frac{8}{25} \\ -u^2 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_4 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -0.560000u^{10} - 0.240000u^9 + \cdots + 0.800000u - 0.360000 \\ -0.120000u^{10} - 0.480000u^9 + \cdots - 0.400000u - 0.720000 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} -0.560000u^{10} - 0.240000u^9 + \cdots - 0.200000u - 0.360000 \\ u^3 + u \end{pmatrix}$$
$$a_3 = \begin{pmatrix} \frac{6}{25}u^{10} - \frac{1}{25}u^9 + \cdots - \frac{1}{5}u + \frac{11}{25} \\ \frac{8}{25}u^{10} + \frac{7}{25}u^9 + \cdots + \frac{2}{5}u - \frac{2}{25} \end{pmatrix}$$
$$a_9 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes
$= \frac{31}{25}u^{10} - \frac{51}{25}u^9 + \frac{24}{5}u^8 - \frac{201}{25}u^7 + \frac{238}{25}u^6 - \frac{341}{25}u^5 + \frac{47}{5}u^4 - \frac{103}{25}u^3 + \frac{144}{25}u^2 - \frac{6}{5}u + \frac{211}{25}$

(iv) Complex Volumes and Cusp Shapes

Solution to I_2^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.56939 - 1.41435I$ $a = -0.51246 + 1.61981I$ $b = -0.56939 - 1.41435I$	$-3.64137 + 12.81031I$	$2.99547 - 7.42806I$
$u = -0.56939 + 1.41435I$ $a = -0.51246 - 1.61981I$ $b = -0.56939 + 1.41435I$	$-3.64137 - 12.81031I$	$2.99547 + 7.42806I$
$u = -0.483399 - 0.706724I$ $a = -0.621005 + 0.007669I$ $b = -0.483399 - 0.706724I$	$1.11176 + 2.13095I$	$7.34122 - 2.95650I$
$u = -0.483399 + 0.706724I$ $a = -0.621005 - 0.007669I$ $b = -0.483399 + 0.706724I$	$1.11176 - 2.13095I$	$7.34122 + 2.95650I$
$u = -0.450687$ $a = -0.984340$ $b = -0.450687$	0.895812	11.2897
$u = 0.127465 - 1.057022I$ $a = -0.39223 + 2.89250I$ $b = 0.127465 - 1.057022I$	$-0.55548 - 3.69188I$	$3.27466 + 4.59532I$
$u = 0.127465 + 1.057022I$ $a = -0.39223 - 2.89250I$ $b = 0.127465 + 1.057022I$	$-0.55548 + 3.69188I$	$3.27466 - 4.59532I$
$u = 0.424463 - 1.293844I$ $a = 0.90958 + 1.56370I$ $b = 0.424463 - 1.293844I$	$-6.69869 - 6.38540I$	$0.12486 + 5.46357I$
$u = 0.424463 + 1.293844I$ $a = 0.90958 - 1.56370I$ $b = 0.424463 + 1.293844I$	$-6.69869 + 6.38540I$	$0.12486 - 5.46357I$
$u = 0.726207 - 0.303425I$ $a = 0.108291 + 1.300027I$ $b = 0.726207 - 0.303425I$	$3.57861 + 2.27941I$	$10.11894 - 1.15857I$

Solution to I_2^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.726207 + 0.303425I$ $a = 0.108291 - 1.300027I$ $b = 0.726207 + 0.303425I$	$3.57861 - 2.27941I$	$10.11894 + 1.15857I$

III.

$\boldsymbol{I^u_3 = \langle u^{20}+u^{19}+\cdots-8u+7,\ -6.33\times 10^7 u^{19}-1.96\times 10^8 u^{18}+\cdots+5.99\times 10^8 b-2.59\times 10^8,\ 5.19\times 10^9 u^{19}+1.09\times 10^{10} u^{18}+\cdots+2.10\times 10^{10} a+6.14\times 10^{10}\rangle}$

(i) Arc colorings

$$a_1 = \begin{pmatrix}1\\0\end{pmatrix}$$

$$a_7 = \begin{pmatrix}-0.247259u^{19} - 0.518817u^{18} + \cdots + 0.335520u - 2.92833\\ 0.105556u^{19} + 0.326606u^{18} + \cdots + 1.69754u + 0.432891\end{pmatrix}$$

$$a_2 = \begin{pmatrix}-0.349552u^{19} - 0.0796380u^{18} + \cdots - 1.25873u + 1.73518\\ 0.551898u^{19} + 0.446663u^{18} + \cdots + 6.28274u - 0.316957\end{pmatrix}$$

$$a_5 = \begin{pmatrix}0\\u\end{pmatrix}$$

$$a_4 = \begin{pmatrix}-u\\u\end{pmatrix}$$

$$a_8 = \begin{pmatrix}-0.0543867u^{19} - 0.364665u^{18} + \cdots + 0.923392u - 2.57479\\ -0.0873168u^{19} + 0.172455u^{18} + \cdots + 1.10967u + 0.0793444\end{pmatrix}$$

$$a_6 = \begin{pmatrix}0.120650u^{19} + 0.527823u^{18} + \cdots - 1.39873u + 2.23494\\ 0.135717u^{19} - 0.241519u^{18} + \cdots + 6.24336u - 2.65232\end{pmatrix}$$

$$a_3 = \begin{pmatrix}-0.392610u^{19} - 0.353345u^{18} + \cdots - 2.93530u + 0.162741\\ 0.478230u^{19} + 0.643378u^{18} + \cdots + 4.73897u + 1.29758\end{pmatrix}$$

$$a_9 = \begin{pmatrix}1\\u^2\end{pmatrix}$$

$$a_9 = \begin{pmatrix}1\\u^2\end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= -\frac{1402773924}{2996962805}u^{19} - \frac{100220208}{2996962805}u^{18} + \cdots - \frac{22381283924}{2996962805}u + \frac{26005127826}{2996962805}$

(iv) Complex Volumes and Cusp Shapes

Solution to I_3^u	$\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.286373 - 0.028870I$ $a = 0.016417 - 0.483818I$ $b = 0.387179 + 1.147990I$	$0.93776 + 6.43072I$	$4.74431 - 6.96269I$
$u = -1.286373 + 0.028870I$ $a = 0.016417 + 0.483818I$ $b = 0.387179 - 1.147990I$	$0.93776 - 6.43072I$	$4.74431 + 6.96269I$
$u = -0.590252 - 0.825819I$ $a = -0.478334 - 0.384375I$ $b = 0.070663 - 0.512466I$	$0.93776 + 2.37095I$	$4.74431 - 0.03448I$
$u = -0.590252 + 0.825819I$ $a = -0.478334 + 0.384375I$ $b = 0.070663 + 0.512466I$	$0.93776 - 2.37095I$	$4.74431 + 0.03448I$
$u = -0.38849 - 1.61565I$ $a = 0.509858 - 1.243014I$ $b = 0.067213 + 1.072302I$	$-4.60570 + 0.49930I$	$0.515115 + 0.966547I$
$u = -0.38849 + 1.61565I$ $a = 0.509858 + 1.243014I$ $b = 0.067213 - 1.072302I$	$-4.60570 - 0.49930I$	$0.515115 - 0.966547I$
$u = -0.133857 - 1.341626I$ $a = -0.08909 - 1.65343I$ $b = 0.76505 + 1.34819I$	$-4.60570 + 3.56046I$	$0.51511 - 7.89475I$
$u = -0.133857 + 1.341626I$ $a = -0.08909 + 1.65343I$ $b = 0.76505 - 1.34819I$	$-4.60570 - 3.56046I$	$0.51511 + 7.89475I$
$u = -0.130820 - 1.153334I$ $a = 0.050211 - 0.442673I$ $b = 0.739688 + 0.098744I$	$-2.53372 + 2.02988I$	$1.48114 - 3.46410I$
$u = -0.130820 + 1.153334I$ $a = 0.050211 + 0.442673I$ $b = 0.739688 - 0.098744I$	$-2.53372 - 2.02988I$	$1.48114 + 3.46410I$

Solution to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.067213 - 1.072302I$ $a = -0.44509 - 2.02969I$ $b = -0.38849 + 1.61565I$	$-4.60570 - 0.49930I$	$0.515115 - 0.966547I$
$u = 0.067213 + 1.072302I$ $a = -0.44509 + 2.02969I$ $b = -0.38849 - 1.61565I$	$-4.60570 + 0.49930I$	$0.515115 + 0.966547I$
$u = 0.070663 - 0.512466I$ $a = -1.200159 + 0.097022I$ $b = -0.590252 - 0.825819I$	$0.93776 + 2.37095I$	$4.74431 - 0.03448I$
$u = 0.070663 + 0.512466I$ $a = -1.200159 - 0.097022I$ $b = -0.590252 + 0.825819I$	$0.93776 - 2.37095I$	$4.74431 + 0.03448I$
$u = 0.387179 - 1.147990I$ $a = 0.477144 - 0.191488I$ $b = -1.286373 + 0.028870I$	$0.93776 - 6.43072I$	$4.74431 + 6.96269I$
$u = 0.387179 + 1.147990I$ $a = 0.477144 + 0.191488I$ $b = -1.286373 - 0.028870I$	$0.93776 + 6.43072I$	$4.74431 - 6.96269I$
$u = 0.739688 - 0.098744I$ $a = -0.686864 - 0.091692I$ $b = -0.130820 + 1.153334I$	$-2.53372 - 2.02988I$	$1.48114 + 3.46410I$
$u = 0.739688 + 0.098744I$ $a = -0.686864 + 0.091692I$ $b = -0.130820 - 1.153334I$	$-2.53372 + 2.02988I$	$1.48114 - 3.46410I$
$u = 0.76505 - 1.34819I$ $a = -0.51123 - 1.34643I$ $b = -0.133857 + 1.341626I$	$-4.60570 - 3.56046I$	$0.51511 + 7.89475I$
$u = 0.76505 + 1.34819I$ $a = -0.51123 + 1.34643I$ $b = -0.133857 - 1.341626I$	$-4.60570 + 3.56046I$	$0.51511 - 7.89475I$

IV. u-Polynomials

Crossings	u-Polynomials at each crossings
c_1, c_4	(u^4+2u^2-u+1) $(u^{11}+4u^9-u^8+7u^7-3u^6+4u^5-3u^4+u^3-u^2+u+1)$ $(u^{20}+u^{19}+\cdots-8u+7)$
c_2	$(u^4+u^3+2u^2+1)(u^5-u^4+2u^3-u^2+u-1)^4$ $(u^{11}+6u^{10}+\cdots+26u+4)$
c_3, c_5	$(u^4+u+1)(u^{11}-2u^9+\cdots-u-1)$ $(u^{20}+5u^{19}+\cdots+2u+1)$
c_6, c_9	(u^4+2u^2+u+1) $(u^{11}+4u^9-u^8+7u^7-3u^6+4u^5-3u^4+u^3-u^2+u+1)$ $(u^{20}+u^{19}+\cdots-8u+7)$
c_7	$(u^2-u+1)^{10}(u^4-u^3+1)(u^{11}+10u^{10}+\cdots+176u+32)$
c_8	$(u^4-u^3+2u^2+1)(u^5-u^4+2u^3-u^2+u-1)^4$ $(u^{11}+6u^{10}+\cdots+26u+4)$

V. Riley Polynomials

Crossings	Riley Polynomials at each crossings
c_1, c_4, c_6 c_9	$(y^4+4y^3+\cdots+3y+1)(y^{11}+8y^{10}+\cdots+3y-1)$ $(y^{20}+15y^{19}+\cdots+468y+49)$
c_2, c_8	$(y^4+3y^3+6y^2+4y+1)(y^5+3y^4+4y^3+y^2-y-1)^4$ $(y^{11}+6y^{10}+\cdots+124y-16)$
c_3, c_5	$(y^4+2y^2-y+1)(y^{11}-4y^{10}+\cdots+11y-1)$ $(y^{20}+3y^{19}+\cdots+12y+1)$
c_7	$(y^2+y+1)^{10}(y^4-y^3+2y^2+1)(y^{11}+2y^9+\cdots+1792y-1024)$