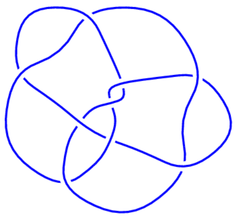
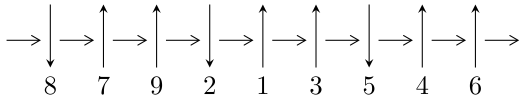
9_{41} $(K9a_{29})$



[1]

Arc Sequences



Solving Sequence

$$3{,}9 \xrightarrow[c_3]{} 4{,}6 \xrightarrow[c_6]{} 7 \xrightarrow[c_9]{} 1 \xrightarrow[c_2]{} 2 \xrightarrow[c_5]{} 5 \xrightarrow[c_8]{} 8 \twoheadrightarrow c_1, c_4, c_7$$

Representation Ideals

$$I = \bigcap_{i=1}^{6} I_i^u$$

$$I_1^u = \langle u^4 + 2u^2 + u + 1,\ a + 1,\ b + u\rangle$$
$$I_2^u = \langle u^4 + 2u^3 + 4u^2 + 3u + 1,\ a + 1,\ b - u\rangle$$
$$I_3^u = \langle u^6 + 5u^5 + 14u^4 + 25u^3 + 28u^2 + 20u + 8,\ 3u^5 + 9u^4 + 20u^3 + 23u^2 + 8a + 14u + 4,$$
$$3u^5 + 11u^4 + 26u^3 + 35u^2 + 4b + 28u + 12\rangle$$
$$I_4^u = \langle b^6 + 5b^5 + 14b^4 + 25b^3 + 28b^2 + 20b + 8,\ a + 1,\ 3b^5 + 11b^4 + 26b^3 + 35b^2 + 28b + 4u + 12\rangle$$
$$I_5^u = \langle a^6 - 5a^5 + 14a^4 - 25a^3 + 28a^2 - 20a + 8,\ a^5 - 4a^4 + 11a^3 - 17a^2 + 2b + 17a - 10,$$
$$a^5 - 4a^4 + 11a^3 - 17a^2 + 17a + 2u - 10\rangle$$
$$I_6^u = \langle a^6 - a^5 + 5a^4 - 9a^3 + 5a^2 - a + 1,\ -5a^5 + 3a^4 - 26a^3 + 35a^2 - 21a + 7u + 4,$$
$$3a^5 - 4a^4 + 16a^3 - 31a^2 + 7b + 20a - 6\rangle$$

There are 6 irreducible components with 32 representations.

[1]The knot diagram image is adapter from "C. Livingston and A. H. Moore, KnotInfo: Table of Knot Invariants, http://www.indiana.edu/ knotinfo"

I. $I_1^u = \langle u^4 + 2u^2 + u + 1,\ a + 1,\ b + u \rangle$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ -u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u + 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ u^2 - u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^2 + 1 \\ u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ u^3 - u^2 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^2 + u - 1 \\ -u^3 - u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^2 + u - 1 \\ -u^3 - u \end{pmatrix}$$

(ii) Obstruction class $= 1$

(iii) Cusp Shapes $= 3u^3 - 3u^2 + 6u - 3$

(iv) Complex Volumes and Cusp Shapes

Solution to I_1^u	$\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.343815 - 0.625358I$ $a = -1.00000$ $b = 0.343815 + 0.625358I$	$-2.15173 + 3.38562I$	$-3.15611 - 4.97381I$
$u = -0.343815 + 0.625358I$ $a = -1.00000$ $b = 0.343815 - 0.625358I$	$-2.15173 - 3.38562I$	$-3.15611 + 4.97381I$
$u = 0.343815 - 1.358435I$ $a = -1.00000$ $b = -0.343815 + 1.358435I$	$-7.71788 - 2.37936I$	$-1.34389 + 0.72682I$
$u = 0.343815 + 1.358435I$ $a = -1.00000$ $b = -0.343815 - 1.358435I$	$-7.71788 + 2.37936I$	$-1.34389 - 0.72682I$

II. $\boldsymbol{I_2^u = \langle u^4 + 2u^3 + 4u^2 + 3u + 1,\ a + 1,\ b - u\rangle}$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -1 \\ u \end{pmatrix}$$
$$a_4 = \begin{pmatrix} u+1 \\ -u^2 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_7 = \begin{pmatrix} u \\ u \end{pmatrix}$$
$$a_1 = \begin{pmatrix} -1 \\ u^2+u \end{pmatrix}$$
$$a_2 = \begin{pmatrix} u^2+1 \\ u^2 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} -u \\ u^3+u^2+u \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -u^2-u-1 \\ u^3+u \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -u^2-u-1 \\ u^3+u \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= 3u^3 + 9u^2 + 18u + 15$

(iv) Complex Volumes and Cusp Shapes

Solution to I_2^u	$\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.50000 - 1.53884I$ $a = -1.00000$ $b = -0.50000 - 1.53884I$	$-10.8566 + 12.0989I$	$-2.78115 - 6.37988I$
$u = -0.50000 + 1.53884I$ $a = -1.00000$ $b = -0.50000 + 1.53884I$	$-10.8566 - 12.0989I$	$-2.78115 + 6.37988I$
$u = -0.500000 - 0.363271I$ $a = -1.00000$ $b = -0.500000 - 0.363271I$	$0.986960 + 0.735995I$	$7.28115 - 3.94298I$
$u = -0.500000 + 0.363271I$ $a = -1.00000$ $b = -0.500000 + 0.363271I$	$0.986960 - 0.735995I$	$7.28115 + 3.94298I$

III. $\boldsymbol{I^u_3 = \langle u^6 + 5u^5 + 14u^4 + 25u^3 + 28u^2 + 20u + 8,\ 3u^5 + 9u^4 + 20u^3 + 23u^2 + 8a + 14u + 4,\ 3u^5 + 11u^4 + 26u^3 + 35u^2 + 4b + 28u + 12\rangle}$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -\frac{3}{8}u^5 - \frac{9}{8}u^4 + \cdots - \frac{7}{4}u - \frac{1}{2} \\ -\frac{3}{4}u^5 - \frac{11}{4}u^4 + \cdots - 7u - 3 \end{pmatrix}$$
$$a_4 = \begin{pmatrix} \frac{5}{8}u^5 + \frac{19}{8}u^4 + \cdots + \frac{23}{4}u + 4 \\ \frac{3}{4}u^5 + \frac{13}{4}u^4 + \cdots + \frac{19}{2}u + 5 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_7 = \begin{pmatrix} u \\ u \end{pmatrix}$$
$$a_1 = \begin{pmatrix} -\frac{3}{8}u^5 - \frac{9}{8}u^4 + \cdots - \frac{7}{4}u - \frac{1}{2} \\ \frac{1}{4}u^5 + \frac{5}{4}u^4 + \cdots + 5u + 3 \end{pmatrix}$$
$$a_2 = \begin{pmatrix} u^2 + 1 \\ u^2 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} -\frac{5}{8}u^5 - \frac{19}{8}u^4 + \cdots - \frac{23}{4}u - 3 \\ -\frac{1}{4}u^5 - \frac{3}{4}u^4 + \cdots + \frac{3}{2}u + 1 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} \frac{5}{8}u^5 + \frac{17}{8}u^4 + \cdots + \frac{13}{2}u + 3 \\ -\frac{1}{2}u^4 - \frac{3}{2}u^3 - 3u^2 - \frac{5}{2}u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} \frac{5}{8}u^5 + \frac{17}{8}u^4 + \cdots + \frac{13}{2}u + 3 \\ -\frac{1}{2}u^4 - \frac{3}{2}u^3 - 3u^2 - \frac{5}{2}u - 1 \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= u^5 + u^4 + 2u^3 + u^2 + 4u + 6$

(iv) Complex Volumes and Cusp Shapes

Solution to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.37744 - 0.32041I$ $a = 0.357540 + 0.865797I$ $b = 0.215080 + 1.307141I$	$-4.93480 + 5.65624I$	$-2.00000 - 5.95889I$
$u = -1.37744 + 0.32041I$ $a = 0.357540 - 0.865797I$ $b = 0.215080 - 1.307141I$	$-4.93480 - 5.65624I$	$-2.00000 + 5.95889I$
$u = -0.71508 - 1.73159I$ $a = 0.688719 + 0.160205I$ $b = 0.215080 + 1.307141I$	$-9.07239 + 2.82812I$	$-8.52927 - 2.97945I$
$u = -0.71508 + 1.73159I$ $a = 0.688719 - 0.160205I$ $b = 0.215080 - 1.307141I$	$-9.07239 - 2.82812I$	$-8.52927 + 2.97945I$
$u = -0.407481 - 0.986732I$ $a = 0.203741 - 0.493366I$ $b = 0.569840$	$-0.79722 + 2.82812I$	$4.52927 - 2.97945I$
$u = -0.407481 + 0.986732I$ $a = 0.203741 + 0.493366I$ $b = 0.569840$	$-0.79722 - 2.82812I$	$4.52927 + 2.97945I$

IV. $\boldsymbol{I_4^u = \langle b^6 + 5b^5 + 14b^4 + 25b^3 + 28b^2 + 20b + 8,\ a + 1,\ 3b^5 + 11b^4 + 26b^3 + 35b^2 + 28b + 4u + 12\rangle}$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ b \end{pmatrix}$$

$$a_4 = \begin{pmatrix} b+1 \\ -b^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0 \\ -\frac{3}{4}b^5 - \frac{11}{4}b^4 + \cdots - 7b - 3 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -\frac{3}{4}b^5 - \frac{11}{4}b^4 + \cdots - 7b - 3 \\ -\frac{3}{4}b^5 - \frac{11}{4}b^4 + \cdots - 7b - 3 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ -\frac{3}{4}b^5 - \frac{13}{4}b^4 + \cdots - \frac{17}{2}b - 5 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -\frac{3}{4}b^5 - \frac{13}{4}b^4 + \cdots - \frac{19}{2}b - 4 \\ -\frac{3}{4}b^5 - \frac{13}{4}b^4 + \cdots - \frac{19}{2}b - 5 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} \frac{3}{4}b^5 + \frac{11}{4}b^4 + \cdots + 7b + 3 \\ b^5 + \frac{7}{2}b^4 + \frac{17}{2}b^3 + 11b^2 + \frac{19}{2}b + 5 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -b^2 - b - 1 \\ b^3 + b \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -b^2 - b - 1 \\ b^3 + b \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= b^5 + b^4 + 2b^3 + b^2 + 4b + 6$

(iv) Complex Volumes and Cusp Shapes

Solution to I_4^u	$\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$	Cusp shape
$u = 0.215080 + 1.307141I$ $a = -1.00000$ $b = -1.37744 - 0.32041I$	$-4.93480 + 5.65624I$	$-2.00000 - 5.95889I$
$u = 0.215080 - 1.307141I$ $a = -1.00000$ $b = -1.37744 + 0.32041I$	$-4.93480 - 5.65624I$	$-2.00000 + 5.95889I$
$u = 0.215080 + 1.307141I$ $a = -1.00000$ $b = -0.71508 - 1.73159I$	$-9.07239 + 2.82812I$	$-8.52927 - 2.97945I$
$u = 0.215080 - 1.307141I$ $a = -1.00000$ $b = -0.71508 + 1.73159I$	$-9.07239 - 2.82812I$	$-8.52927 + 2.97945I$
$u = 0.569840$ $a = -1.00000$ $b = -0.407481 - 0.986732I$	$-0.79722 + 2.82812I$	$4.52927 - 2.97945I$
$u = 0.569840$ $a = -1.00000$ $b = -0.407481 + 0.986732I$	$-0.79722 - 2.82812I$	$4.52927 + 2.97945I$

V. $\boldsymbol{I^u_5 = \langle a^6 - 5a^5 + 14a^4 - 25a^3 + 28a^2 - 20a + 8,\ a^5 - 4a^4 + 11a^3 - 17a^2 + 2b + 17a - 10,\ a^5 - 4a^4 + 11a^3 - 17a^2 + 17a + 2u - 10\rangle}$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} a \\ -\frac{1}{2}a^5 + 2a^4 + \cdots - \frac{17}{2}a + 5 \end{pmatrix}$$
$$a_4 = \begin{pmatrix} \frac{1}{2}a^5 - \frac{3}{2}a^4 + 4a^3 - \frac{11}{2}a^2 + 5a - 3 \\ \frac{1}{4}a^5 - \frac{5}{4}a^4 + \cdots + 5a - 1 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 0 \\ -\frac{1}{2}a^5 + 2a^4 + \cdots - \frac{17}{2}a + 5 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} -\frac{1}{2}a^5 + 2a^4 + \cdots - \frac{17}{2}a + 5 \\ -\frac{1}{2}a^5 + 2a^4 + \cdots - \frac{17}{2}a + 5 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} a \\ -\frac{1}{2}a^5 + 2a^4 + \cdots - \frac{9}{2}a + 3 \end{pmatrix}$$
$$a_2 = \begin{pmatrix} -\frac{1}{4}a^5 + \frac{5}{4}a^4 + \cdots - 5a + 2 \\ -\frac{1}{4}a^5 + \frac{5}{4}a^4 + \cdots - 5a + 1 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} a^5 - 3a^4 + 7a^3 - 9a^2 + 6a - 4 \\ \frac{3}{2}a^5 - 5a^4 + \cdots + \frac{19}{2}a - 5 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} \frac{1}{2}a^5 - 2a^4 + \cdots + \frac{11}{2}a - 3 \\ \frac{1}{4}a^5 - \frac{3}{4}a^4 + \cdots + \frac{7}{2}a - 3 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} \frac{1}{2}a^5 - 2a^4 + \cdots + \frac{11}{2}a - 3 \\ \frac{1}{4}a^5 - \frac{3}{4}a^4 + \cdots + \frac{7}{2}a - 3 \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= 2a^5 - 8a^4 + 22a^3 - 38a^2 + 38a - 26$

(iv) Complex Volumes and Cusp Shapes

Solution to I_5^u	$\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$	Cusp shape
$u = 0.215080 + 1.307141I$ $a = 0.407481 - 0.986732I$ $b = 0.215080 + 1.307141I$	$-4.93480 + 5.65624I$	$-2.00000 - 5.95889I$
$u = 0.215080 - 1.307141I$ $a = 0.407481 + 0.986732I$ $b = 0.215080 - 1.307141I$	$-4.93480 - 5.65624I$	$-2.00000 + 5.95889I$
$u = 0.569840$ $a = 0.71508 - 1.73159I$ $b = 0.569840$	$-0.79722 - 2.82812I$	$4.52927 + 2.97945I$
$u = 0.569840$ $a = 0.71508 + 1.73159I$ $b = 0.569840$	$-0.79722 + 2.82812I$	$4.52927 - 2.97945I$
$u = 0.215080 + 1.307141I$ $a = 1.37744 - 0.32041I$ $b = 0.215080 + 1.307141I$	$-9.07239 + 2.82812I$	$-8.52927 - 2.97945I$
$u = 0.215080 - 1.307141I$ $a = 1.37744 + 0.32041I$ $b = 0.215080 - 1.307141I$	$-9.07239 - 2.82812I$	$-8.52927 + 2.97945I$

VI. $\boldsymbol{I^u_6 = \langle a^6 - a^5 + 5a^4 - 9a^3 + 5a^2 - a + 1,\ -5a^5 + 3a^4 - 26a^3 + 35a^2 - 21a + 7u + 4,\ 3a^5 - 4a^4 + 16a^3 - 31a^2 + 7b + 20a - 6\rangle}$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} a \\ -\frac{3}{7}a^5 + \frac{4}{7}a^4 + \cdots - \frac{20}{7}a + \frac{6}{7} \end{pmatrix}$$
$$a_4 = \begin{pmatrix} -\frac{1}{7}a^5 + \frac{1}{7}a^4 + \cdots - \frac{3}{7}a + \frac{4}{7} \\ \frac{4}{7}a^5 - \frac{3}{7}a^4 + \cdots + \frac{8}{7}a + \frac{6}{7} \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 0 \\ \frac{5}{7}a^5 - \frac{3}{7}a^4 + \cdots + 3a - \frac{4}{7} \end{pmatrix}$$
$$a_7 = \begin{pmatrix} \frac{5}{7}a^5 - \frac{3}{7}a^4 + \cdots + 3a - \frac{4}{7} \\ \frac{5}{7}a^5 - \frac{3}{7}a^4 + \cdots + 3a - \frac{4}{7} \end{pmatrix}$$
$$a_1 = \begin{pmatrix} a \\ -\frac{4}{7}a^5 + \frac{3}{7}a^4 + \cdots - \frac{8}{7}a + \frac{8}{7} \end{pmatrix}$$
$$a_2 = \begin{pmatrix} \frac{2}{7}a^5 - \frac{1}{7}a^4 + \cdots + \frac{9}{7}a - 1 \\ \frac{2}{7}a^5 - \frac{1}{7}a^4 + \cdots + \frac{9}{7}a - 2 \end{pmatrix}$$
$$a_5 = \begin{pmatrix} -\frac{3}{7}a^5 - 2a^3 + \cdots + \frac{3}{7}a + \frac{2}{7} \\ -\frac{2}{7}a^5 - \frac{3}{7}a^4 + \cdots + 2a + \frac{3}{7} \end{pmatrix}$$
$$a_8 = \begin{pmatrix} \frac{2}{7}a^5 - \frac{3}{7}a^4 + \cdots + \frac{17}{7}a - \frac{2}{7} \\ -\frac{1}{7}a^5 - \frac{1}{7}a^4 + \cdots + \frac{12}{7}a - \frac{5}{7} \end{pmatrix}$$
$$a_8 = \begin{pmatrix} \frac{2}{7}a^5 - \frac{3}{7}a^4 + \cdots + \frac{17}{7}a - \frac{2}{7} \\ -\frac{1}{7}a^5 - \frac{1}{7}a^4 + \cdots + \frac{12}{7}a - \frac{5}{7} \end{pmatrix}$$

(ii) Obstruction class $= -1$

(iii) Cusp Shapes $= -2$

(iv) Complex Volumes and Cusp Shapes

Solution to I_6^u	$\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$	Cusp shape
$u = 0.569840$ $a = -0.37744 - 2.29387I$ $b = 0.215080 - 1.307141I$	-4.93480	-2.00000
$u = 0.569840$ $a = -0.37744 + 2.29387I$ $b = 0.215080 + 1.307141I$	-4.93480	-2.00000
$u = 0.215080 - 1.307141I$ $a = -0.069840 - 0.424452I$ $b = 0.215080 + 1.307141I$	-4.93480	-2.00000
$u = 0.215080 + 1.307141I$ $a = -0.069840 + 0.424452I$ $b = 0.215080 - 1.307141I$	-4.93480	-2.00000
$u = 0.215080 - 1.307141I$ $a = 0.947279 - 0.320410I$ $b = 0.569840$	-4.93480	-2.00000
$u = 0.215080 + 1.307141I$ $a = 0.947279 + 0.320410I$ $b = 0.569840$	-4.93480	-2.00000

VII. u-Polynomials

Crossings	u-Polynomials at each crossings
c_1, c_4, c_7	$(u^3 - u^2 + 1)^6(u^4 + u^3 + 1)(u^4 + 3u^3 + 4u^2 + 2u + 1)$ $(u^6 + 7u^5 + 24u^4 + 47u^3 + 54u^2 + 32u + 8)$
c_2, c_5, c_8	$(u^3 - u^2 + 2u - 1)^6(u^4 + 2u^2 + u + 1)(u^4 + 2u^3 + 4u^2 + 3u + 1)$ $(u^6 + 5u^5 + 14u^4 + 25u^3 + 28u^2 + 20u + 8)$
c_3, c_6, c_9	$(u^3 - u^2 + 2u - 1)^6(u^4 + 2u^2 - u + 1)(u^4 + 2u^3 + 4u^2 + 3u + 1)$ $(u^6 + 5u^5 + 14u^4 + 25u^3 + 28u^2 + 20u + 8)$

VIII. Riley Polynomials

Crossings	Riley Polynomials at each crossings
c_1, c_4, c_7	$(y^3 - y^2 + 2y - 1)^6(y^4 - y^3 + 2y^2 + 1)(y^4 - y^3 + 6y^2 + 4y + 1)$ $(y^6 - y^5 + 26y^4 - 49y^3 + 292y^2 - 160y + 64)$
c_2, c_3, c_5 c_6, c_8, c_9	$(-1 + 2y + 3y^2 + y^3)^6(y^4 + 4y^3 + \cdots - y + 1)(y^4 + 4y^3 + \cdots + 3y + 1)$ $(y^6 + 3y^5 + 2y^4 - 25y^3 + 8y^2 + 48y + 64)$