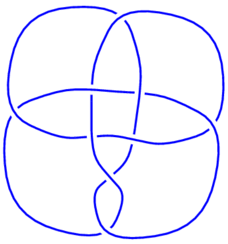
# $9_{47}$ $(K9n_7)$



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### Arc Sequences

$$\to \downarrow_{8} \to \uparrow_{9} \to \downarrow_{8} \to \downarrow_{2} \to \uparrow_{3} \to \downarrow_{4} \to \downarrow_{2} \to \uparrow_{6} \to \downarrow_{5} \to$$

### Solving Sequence

$$3,9 \xrightarrow[c_2]{} 2,6 \xrightarrow[c_5]{} 5 \xrightarrow[c_4]{} 4 \xrightarrow[c_6]{} 7 \xrightarrow[c_8]{} 8 \xrightarrow[c_1]{} 1 \twoheadrightarrow c_3, c_7, c_9$$

### Representation Ideals

$$I = \bigcap_{i=1}^{5} I_i^u$$

$$I_1^u = \langle u^3 + u^2 - 1,\ a + 1,\ b + u\rangle$$
$$I_2^u = \langle u^4 - 4u^3 + 7u^2 - 5u + 2,\ u^3 - 2u^2 + 2a + u + 3,\ -u^3 + 3u^2 + b - 4u + 1\rangle$$
$$I_3^u = \langle u^4 - 2u^3 + 3u^2 - u + 1,\ b + u,\ a - 1\rangle$$
$$I_4^u = \langle u^4 + u^3 + u^2 + 1,\ b + u,\ u^3 + 2u^2 + a + 2u + 1\rangle$$
$$I_5^u = \langle u^4 + u^3 + u^2 + 1,\ a - 1,\ -u^3 - u^2 + b - u + 1\rangle$$

There are 5 irreducible components with 19 representations.

[1] The knot diagram image is adapter from "C. Livingston and A. H. Moore, KnotInfo: Table of Knot Invariants, http://www.indiana.edu/ knotinfo"

## I. $I_1^u = \langle u^3 + u^2 - 1,\ a + 1,\ b + u \rangle$

**(i) Arc colorings**

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -1 \\ -u \end{pmatrix}$$
$$a_2 = \begin{pmatrix} u + 1 \\ u^2 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_5 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_4 = \begin{pmatrix} u^2 + u + 1 \\ u + 1 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} -u^2 - 2u - 2 \\ -u^2 - u - 1 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -1 \\ -u^2 - u \end{pmatrix}$$
$$a_1 = \begin{pmatrix} 0 \\ -u - 1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} 0 \\ -u - 1 \end{pmatrix}$$

**(ii) Obstruction class** $= 1$

**(iii) Cusp Shapes** $= -6u^2 - 3u$

**(iv) Complex Volumes and Cusp Shapes**

| Solution to $I_1^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -0.877439 - 0.744862I$<br>$a = -1.00000$<br>$b = \ \ 0.877439 + 0.744862I$ | $1.45094 + 3.77083I$ | $1.34184 - 5.60826I$ |
| $u = -0.877439 + 0.744862I$<br>$a = -1.00000$<br>$b = \ \ 0.877439 - 0.744862I$ | $1.45094 - 3.77083I$ | $1.34184 + 5.60826I$ |
| $u = \ \ 0.754878$<br>$a = -1.00000$<br>$b = -0.754878$ | $-6.19175$ | $-5.68367$ |

**II.**

$$\boldsymbol{I_2^u = \langle u^4 - 4u^3 + 7u^2 - 5u + 2,\ u^3 - 2u^2 + 2a + u + 3,\ -u^3 + 3u^2 + b - 4u + 1\rangle}$$

**(i) Arc colorings**

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -\frac{1}{2}u^3 + u^2 - \frac{1}{2}u - \frac{3}{2} \\ u^3 - 3u^2 + 4u - 1 \end{pmatrix}$$
$$a_2 = \begin{pmatrix} -\frac{3}{2}u^3 + 5u^2 - \frac{13}{2}u + \frac{5}{2} \\ u^3 - 4u^2 + 6u - 3 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_5 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_4 = \begin{pmatrix} -\frac{1}{2}u^3 + u^2 - \frac{1}{2}u - \frac{1}{2} \\ 1 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} \frac{1}{2}u^3 - u^2 + \frac{1}{2}u - \frac{1}{2} \\ u^3 - 3u^2 + 4u - 1 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -\frac{1}{2}u^3 + u^2 - \frac{1}{2}u - \frac{3}{2} \\ 1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} -\frac{1}{2}u^3 + u^2 + \frac{1}{2}u - \frac{3}{2} \\ u^3 - 3u^2 + 3u - 1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} -\frac{1}{2}u^3 + u^2 + \frac{1}{2}u - \frac{3}{2} \\ u^3 - 3u^2 + 3u - 1 \end{pmatrix}$$

**(ii) Obstruction class** $= -1$

**(iii) Cusp Shapes** $= 4u^2 - 8u - 2$

**(iv) Complex Volumes and Cusp Shapes**

| Solution to $I_2^u$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 0.452576 - 0.585652I$<br>$a = -1.67796 - 0.15778I$<br>$b = 0.851808 - 0.911292I$ | $0.21101 - 3.16396I$ | $-6.17326 + 2.56480I$ |
| $u = 0.452576 + 0.585652I$<br>$a = -1.67796 + 0.15778I$<br>$b = 0.851808 + 0.911292I$ | $0.21101 + 3.16396I$ | $-6.17326 - 2.56480I$ |
| $u = 1.54742 - 1.12087I$<br>$a = -0.072042 + 0.413327I$<br>$b = -0.351808 - 0.720342I$ | $-6.79074 + 1.41510I$ | $-9.82674 - 4.90874I$ |
| $u = 1.54742 + 1.12087I$<br>$a = -0.072042 - 0.413327I$<br>$b = -0.351808 + 0.720342I$ | $-6.79074 - 1.41510I$ | $-9.82674 + 4.90874I$ |

**III.** $\boldsymbol{I^u_3 = \langle u^4 - 2u^3 + 3u^2 - u + 1,\ b + u,\ a - 1\rangle}$

**(i) Arc colorings**

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} 1 \\ -u \end{pmatrix}$$
$$a_2 = \begin{pmatrix} -u+1 \\ u^2 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_5 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_4 = \begin{pmatrix} u^2-u+1 \\ -2u^2+u-1 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} u^2 \\ -3u^2+u-1 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} 1 \\ u^2-u \end{pmatrix}$$
$$a_1 = \begin{pmatrix} u^3-u^2+1 \\ -u^3+u^2-u \end{pmatrix}$$
$$a_1 = \begin{pmatrix} u^3-u^2+1 \\ -u^3+u^2-u \end{pmatrix}$$

**(ii) Obstruction class** $= -1$

**(iii) Cusp Shapes** $= 3u^3 - 9u^2 + 9u - 9$

**(iv) Complex Volumes and Cusp Shapes**

| Solution to $I_3^u$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 0.043315 - 0.641200I$<br>$a = 1.00000$<br>$b = -0.043315 + 0.641200I$ | $-0.858683 + 1.068331I$ | $-5.08685 - 4.49083I$ |
| $u = 0.043315 + 0.641200I$<br>$a = 1.00000$<br>$b = -0.043315 - 0.641200I$ | $-0.858683 - 1.068331I$ | $-5.08685 + 4.49083I$ |
| $u = 0.95668 - 1.22719I$<br>$a = 1.00000$<br>$b = -0.95668 + 1.22719I$ | $-8.18845 - 10.04996I$ | $-5.41315 + 5.52365I$ |
| $u = 0.95668 + 1.22719I$<br>$a = 1.00000$<br>$b = -0.95668 - 1.22719I$ | $-8.18845 + 10.04996I$ | $-5.41315 - 5.52365I$ |

**IV. $I^u_4 = \langle u^4 + u^3 + u^2 + 1,\ b + u,\ u^3 + 2u^2 + a + 2u + 1\rangle$**

**(i) Arc colorings**

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} -u^3 - 2u^2 - 2u - 1 \\ -u \end{pmatrix}$$
$$a_2 = \begin{pmatrix} u^3 + u^2 + u \\ u^2 \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_5 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_4 = \begin{pmatrix} -u^3 - 2u^2 - 2u \\ 1 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} 2u^3 + 4u^2 + 4u + 1 \\ u^3 + u^2 + u - 1 \end{pmatrix}$$
$$a_8 = \begin{pmatrix} -u^3 - 2u^2 - 2u - 1 \\ 1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} -2u^2 - 3u - 2 \\ -u^3 + 1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} -2u^2 - 3u - 2 \\ -u^3 + 1 \end{pmatrix}$$

**(ii) Obstruction class** $= -1$

**(iii) Cusp Shapes** $= -4u^2 - 4u - 10$

**(iv) Complex Volumes and Cusp Shapes**

| Solution to $I_4^u$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -0.851808 - 0.911292I$<br>$a = -0.590739 - 0.055548I$<br>$b = 0.851808 + 0.911292I$ | $0.21101 + 3.16396I$ | $-6.17326 - 2.56480I$ |
| $u = -0.851808 + 0.911292I$<br>$a = -0.590739 + 0.055548I$<br>$b = 0.851808 - 0.911292I$ | $0.21101 - 3.16396I$ | $-6.17326 + 2.56480I$ |
| $u = 0.351808 - 0.720342I$<br>$a = -0.40926 + 2.34806I$<br>$b = -0.351808 + 0.720342I$ | $-6.79074 - 1.41510I$ | $-9.82674 + 4.90874I$ |
| $u = 0.351808 + 0.720342I$<br>$a = -0.40926 - 2.34806I$<br>$b = -0.351808 - 0.720342I$ | $-6.79074 + 1.41510I$ | $-9.82674 - 4.90874I$ |

**V.** $\boldsymbol{I_5^u = \langle u^4 + u^3 + u^2 + 1,\ a - 1,\ -u^3 - u^2 + b - u + 1\rangle}$

**(i) Arc colorings**

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$a_9 = \begin{pmatrix} 1 \\ u^3 + u^2 + u - 1 \end{pmatrix}$$
$$a_2 = \begin{pmatrix} u^3 + u^2 + u \\ -2u^3 - 3u^2 - 3u \end{pmatrix}$$
$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$
$$a_5 = \begin{pmatrix} -u \\ u \end{pmatrix}$$
$$a_4 = \begin{pmatrix} u^3 + 2u^2 + u \\ -3u^3 - 6u^2 - 5u - 1 \end{pmatrix}$$
$$a_7 = \begin{pmatrix} u^3 + 2u^2 + 2u - 1 \\ -4u^3 - 8u^2 - 7u \end{pmatrix}$$
$$a_8 = \begin{pmatrix} 1 \\ u^3 + 2u^2 + u - 1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} u + 1 \\ u^3 + u^2 - 1 \end{pmatrix}$$
$$a_1 = \begin{pmatrix} u + 1 \\ u^3 + u^2 - 1 \end{pmatrix}$$

**(ii) Obstruction class** $= -1$

**(iii) Cusp Shapes** $= -4u^2 - 4u - 10$

**(iv) Complex Volumes and Cusp Shapes**

| Solution to $I_5^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -0.851808 - 0.911292I$<br>$a = \quad 1.00000$<br>$b = -0.452576 - 0.585652I$ | $0.21101 + 3.16396I$ | $-6.17326 - 2.56480I$ |
| $u = -0.851808 + 0.911292I$<br>$a = \quad 1.00000$<br>$b = -0.452576 + 0.585652I$ | $0.21101 - 3.16396I$ | $-6.17326 + 2.56480I$ |
| $u = \quad 0.351808 - 0.720342I$<br>$a = \quad 1.00000$<br>$b = -1.54742 - 1.12087I$ | $-6.79074 - 1.41510I$ | $-9.82674 + 4.90874I$ |
| $u = \quad 0.351808 + 0.720342I$<br>$a = \quad 1.00000$<br>$b = -1.54742 + 1.12087I$ | $-6.79074 + 1.41510I$ | $-9.82674 - 4.90874I$ |

## VI. u-Polynomials

| Crossings | u-Polynomials at each crossings |
|---|---|
| $c_1, c_4, c_6$ | $(u^3+2u^2+u+1)(u^4-3u^3+u^2+2u+1)(u^4-2u^3+u^2-3u+4)$ <br> $(u^4+3u^3+u^2-2u+1)^2$ |
| $c_2, c_5, c_8$ | $(u^3-u^2+1)(u^4-u^3+u^2+1)^2(u^4+2u^3+3u^2+u+1)$ <br> $(u^4+4u^3+7u^2+5u+2)$ |
| $c_3, c_9$ | $(u-1)^8(u^3-u+1)(u^4+4u^3+\cdots+5u+2)(u^4+5u^3+\cdots+12u+8)$ |
| $c_7$ | $(u^3-2u^2+u-1)(u^4-3u^3+u^2+2u+1)(u^4-2u^3+u^2-3u+4)$ <br> $(u^4+3u^3+u^2-2u+1)^2$ |

## VII. Riley Polynomials

| Crossings | Riley Polynomials at each crossings |
|---|---|
| $c_1, c_4, c_6$<br>$c_7$ | $(y^3-2y^2-3y-1)(1-2y+15y^2-7y^3+y^4)^3(y^4-2y^3+\cdots-y+16)$ |
| $c_2, c_5, c_8$ | $(y^3-y^2+2y-1)(y^4-2y^3+\cdots+3y+4)(1+2y+3y^2+y^3+y^4)^2$<br>$(y^4+2y^3+7y^2+5y+1)$ |
| $c_3, c_9$ | $(y-1)^8(y^3-2y^2+y-1)(y^4-2y^3+13y^2+3y+4)$<br>$(y^4-y^3+40y^2+48y+64)$ |