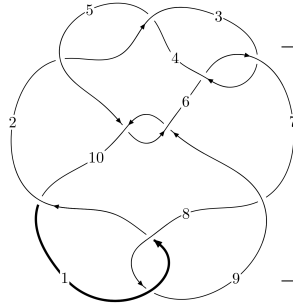
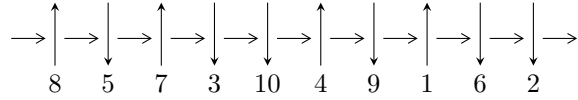


10<sub>58</sub> (K10a<sub>20</sub>)

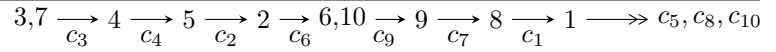


A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle u^7 + u^5 + 2u^3 + b + u, -u^6 - u^4 - 2u^2 + a - 1, u^{10} - u^9 + 2u^8 - u^7 + 4u^6 - 2u^5 + 4u^4 - u^3 + 3u^2 + u + 1 \rangle$$

$$I_2^u = \langle u^{25} + 2u^{24} + \dots + b + 3, 3u^{25} - 7u^{24} + \dots + a - 6, u^{26} - 2u^{25} + \dots - u + 1 \rangle$$

$$I_3^u = \langle b + u + 1, a - u, u^2 + u + 1 \rangle$$

$$I_4^u = \langle b - u, a - 1, u^2 + u + 1 \rangle$$

\* 4 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 40 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\mathbf{I. } I_1^u = \langle u^7 + u^5 + 2u^3 + b + u, -u^6 - u^4 - 2u^2 + a - 1, u^{10} - u^9 + \dots + u + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^2 + 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^4 + u^2 + 1 \\ -u^4 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^6 + u^4 + 2u^2 + 1 \\ -u^7 - u^5 - 2u^3 - u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u \\ u^8 - u^7 + 2u^6 - u^5 + 3u^4 - u^3 + 3u^2 + 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^3 \\ -u^6 + u^5 - u^4 + u^3 - 2u^2 - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^2 + 1 \\ -u^7 + u^6 - u^5 - 2u^3 - u \end{pmatrix}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = 4u^9 - 6u^8 + 6u^7 - 4u^6 + 14u^5 - 14u^4 + 10u^3 - 6u^2 + 10u$$

(iv) u-Polynomials at the component

| Crossings                   | u-Polynomials at each crossing  |
|-----------------------------|---|
| $c_1, c_3, c_6$<br>$c_8$    | $u^{10} + u^9 + 2u^8 + u^7 + 4u^6 + 2u^5 + 4u^4 + u^3 + 3u^2 - u + 1$ |
| $c_2, c_4, c_7$<br>$c_{10}$ | $u^{10} + 3u^9 + \dots + 5u + 1$                                      |
| $c_5, c_9$                  | $u^{10} - 5u^9 + \dots - 8u + 4$                                      |

(v) Riley Polynomials at the component

| Crossings                   | Riley Polynomials at each crossing  |
|-----------------------------|-------------------------------------|
| $c_1, c_3, c_6$<br>$c_8$    | $y^{10} + 3y^9 + \cdots + 5y + 1$   |
| $c_2, c_4, c_7$<br>$c_{10}$ | $y^{10} + 11y^9 + \cdots + 13y + 1$ |
| $c_5, c_9$                  | $y^{10} + 5y^9 + \cdots + 32y + 16$ |

(vi) Complex Volumes and Cusp Shapes

| Solutions to $I_1^u$        | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape            |
|-----------------------------|---------------------------------------|-----------------------|
| $u = -0.100577 + 0.954526I$ |                                       |                       |
| $a = -0.658857 - 0.498555I$ | $-3.48123 - 2.16643I$                 | $-9.00466 + 4.21901I$ |
| $b = -0.542150 + 0.578753I$ |                                       |                       |
| $u = -0.100577 - 0.954526I$ |                                       |                       |
| $a = -0.658857 + 0.498555I$ | $-3.48123 + 2.16643I$                 | $-9.00466 - 4.21901I$ |
| $b = -0.542150 - 0.578753I$ |                                       |                       |
| $u = 0.900362 + 0.768734I$  |                                       |                       |
| $a = -1.68093 + 0.92466I$   | $10.21950 - 0.19532I$                 | $4.14143 - 1.59060I$  |
| $b = 2.22427 + 0.45966I$    |                                       |                       |
| $u = 0.900362 - 0.768734I$  |                                       |                       |
| $a = -1.68093 - 0.92466I$   | $10.21950 + 0.19532I$                 | $4.14143 + 1.59060I$  |
| $b = 2.22427 - 0.45966I$    |                                       |                       |
| $u = -0.774061 + 0.907730I$ |                                       |                       |
| $a = -0.053403 + 0.383357I$ | $4.50100 - 5.87397I$                  | $1.27770 + 5.35715I$  |
| $b = 0.306648 + 0.345217I$  |                                       |                       |
| $u = -0.774061 - 0.907730I$ |                                       |                       |
| $a = -0.053403 - 0.383357I$ | $4.50100 + 5.87397I$                  | $1.27770 - 5.35715I$  |
| $b = 0.306648 - 0.345217I$  |                                       |                       |
| $u = 0.782324 + 1.035710I$  |                                       |                       |
| $a = 1.19625 - 1.47594I$    | $8.4959 + 12.7213I$                   | $1.50029 - 7.98966I$  |
| $b = -2.46450 - 0.08430I$   |                                       |                       |
| $u = 0.782324 - 1.035710I$  |                                       |                       |
| $a = 1.19625 + 1.47594I$    | $8.4959 - 12.7213I$                   | $1.50029 + 7.98966I$  |
| $b = -2.46450 + 0.08430I$   |                                       |                       |
| $u = -0.308049 + 0.477623I$ |                                       |                       |
| $a = 0.696944 - 0.500305I$  | $0.004061 - 1.246020I$                | $0.08524 + 5.02615I$  |
| $b = -0.024265 - 0.486995I$ |                                       |                       |
| $u = -0.308049 - 0.477623I$ |                                       |                       |
| $a = 0.696944 + 0.500305I$  | $0.004061 + 1.246020I$                | $0.08524 - 5.02615I$  |
| $b = -0.024265 + 0.486995I$ |                                       |                       |

**II.**

$$I_2^u = \langle u^{25} + 2u^{24} + \dots + b + 3, 3u^{25} - 7u^{24} + \dots + a - 6, u^{26} - 2u^{25} + \dots - u + 1 \rangle$$

**(i) Arc colorings**

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^2 + 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^4 + u^2 + 1 \\ -u^4 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -3u^{25} + 7u^{24} + \dots - 4u + 6 \\ -u^{25} - 2u^{24} + \dots - 3u - 3 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^{25} + 3u^{24} + \dots - 2u + 4 \\ -u^{22} - 3u^{20} + \dots - 3u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^{25} + u^{24} + \dots - 3u - 1 \\ u^{25} - 2u^{24} + \dots + 3u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^{25} + 3u^{24} + \dots - u + 5 \\ -u^{25} - 4u^{23} + \dots - 3u - 2 \end{pmatrix}$$

**(ii) Obstruction class = -1**

**(iii) Cusp Shapes** =  $5u^{25} - 12u^{24} + 25u^{23} - 43u^{22} + 75u^{21} - 109u^{20} + 126u^{19} - 175u^{18} + 168u^{17} - 200u^{16} + 98u^{15} - 164u^{14} - 6u^{13} - 49u^{12} - 144u^{11} - 10u^{10} - 179u^9 + 26u^8 - 112u^7 - 28u^6 - 47u^5 - 26u^4 + 9u^3 - 16u^2 + 15u - 5$

(iv) u-Polynomials at the component

| Crossings                   | u-Polynomials at each crossing          |
|-----------------------------|---|
| $c_1, c_3, c_6$<br>$c_8$    | $u^{26} + 2u^{25} + \dots + u + 1$      |
| $c_2, c_4, c_7$<br>$c_{10}$ | $u^{26} + 8u^{25} + \dots + 13u + 1$    |
| $c_5, c_9$                  | $(u^{13} + 2u^{12} + \dots + 3u + 2)^2$ |

(v) Riley Polynomials at the component

| Crossings                   | Riley Polynomials at each crossing       |
|-----------------------------|--|
| $c_1, c_3, c_6$<br>$c_8$    | $y^{26} + 8y^{25} + \dots + 13y + 1$     |
| $c_2, c_4, c_7$<br>$c_{10}$ | $y^{26} + 20y^{25} + \dots - 11y + 1$    |
| $c_5, c_9$                  | $(y^{13} + 10y^{12} + \dots - 7y - 4)^2$ |



(vi) Complex Volumes and Cusp Shapes

| Solutions to $I_2^u$   | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape             |
|--|---------------------------------------|------------------------|
| $u = 0.752045 + 0.803934I$<br>$a = 1.97258 - 1.23710I$<br>$b = -2.47801 - 0.65547I$      | $1.88524 - 0.96841I$                  | $0.413632 + 1.140295I$ |
| $u = 0.752045 - 0.803934I$<br>$a = 1.97258 + 1.23710I$<br>$b = -2.47801 + 0.65547I$      | $1.88524 + 0.96841I$                  | $0.413632 - 1.140295I$ |
| $u = -0.578645 + 0.950081I$<br>$a = 0.030039 + 0.285319I$<br>$b = 0.288458 + 0.136559I$  | $-0.80957 - 3.02973I$                 | $-5.16840 + 1.62282I$  |
| $u = -0.578645 - 0.950081I$<br>$a = 0.030039 - 0.285319I$<br>$b = 0.288458 - 0.136559I$  | $-0.80957 + 3.02973I$                 | $-5.16840 - 1.62282I$  |
| $u = -0.496478 + 0.720203I$<br>$a = 0.299461 - 0.224790I$<br>$b = -0.013218 - 0.327276I$ | $0.00150 - 1.41503I$                  | $-1.90513 + 4.60201I$  |
| $u = -0.496478 - 0.720203I$<br>$a = 0.299461 + 0.224790I$<br>$b = -0.013218 + 0.327276I$ | $0.00150 + 1.41503I$                  | $-1.90513 - 4.60201I$  |
| $u = -0.335785 + 1.109920I$<br>$a = 0.267955 + 0.444237I$<br>$b = 0.583042 - 0.148240I$  | $1.88524 - 0.96841I$                  | $0.413632 + 1.140295I$ |
| $u = -0.335785 - 1.109920I$<br>$a = 0.267955 - 0.444237I$<br>$b = 0.583042 + 0.148240I$  | $1.88524 + 0.96841I$                  | $0.413632 - 1.140295I$ |
| $u = 0.905446 + 0.730041I$<br>$a = 1.71372 - 0.85399I$<br>$b = -2.17513 - 0.47784I$      | $9.45063 - 6.48172I$                  | $3.04187 + 3.27257I$   |
| $u = 0.905446 - 0.730041I$<br>$a = 1.71372 + 0.85399I$<br>$b = -2.17513 + 0.47784I$      | $9.45063 + 6.48172I$                  | $3.04187 - 3.27257I$   |

| Solutions to $I_2^u$  | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape            |
|---|---------------------------------------|-----------------------|
| $u = -0.269616 + 1.131670I$<br>$a = -0.308288 - 0.503667I$<br>$b = -0.653102 + 0.213082I$ | $1.46125 - 6.61332I$                  | $-1.15142 + 6.72912I$ |
| $u = -0.269616 - 1.131670I$<br>$a = -0.308288 + 0.503667I$<br>$b = -0.653102 - 0.213082I$ | $1.46125 + 6.61332I$                  | $-1.15142 - 6.72912I$ |
| $u = -0.786233 + 0.860060I$<br>$a = 0.072545 - 0.387289I$<br>$b = -0.276054 - 0.366893I$  | $4.64840$                             | $1.75564 + 0.I$       |
| $u = -0.786233 - 0.860060I$<br>$a = 0.072545 + 0.387289I$<br>$b = -0.276054 + 0.366893I$  | $4.64840$                             | $1.75564 + 0.I$       |
| $u = -0.819468 + 0.042718I$<br>$a = 0.021039 - 0.644673I$<br>$b = -0.010298 - 0.529188I$  | $5.42596 - 2.97283I$                  | $4.39163 + 2.88376I$  |
| $u = -0.819468 - 0.042718I$<br>$a = 0.021039 + 0.644673I$<br>$b = -0.010298 + 0.529188I$  | $5.42596 + 2.97283I$                  | $4.39163 - 2.88376I$  |
| $u = 0.791857 + 0.886903I$<br>$a = -1.65384 + 1.34745I$<br>$b = 2.50466 + 0.39980I$       | $5.42596 + 2.97283I$                  | $4.39163 - 2.88376I$  |
| $u = 0.791857 - 0.886903I$<br>$a = -1.65384 - 1.34745I$<br>$b = 2.50466 - 0.39980I$       | $5.42596 - 2.97283I$                  | $4.39163 + 2.88376I$  |
| $u = 0.732196 + 0.941652I$<br>$a = 1.54369 - 1.64448I$<br>$b = -2.67881 - 0.24953I$       | $1.46125 + 6.61332I$                  | $-1.15142 - 6.72912I$ |
| $u = 0.732196 - 0.941652I$<br>$a = 1.54369 + 1.64448I$<br>$b = -2.67881 + 0.24953I$       | $1.46125 - 6.61332I$                  | $-1.15142 + 6.72912I$ |

| Solutions to $I_2^u$   | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape            |
|--|---------------------------------------|-----------------------|
| $u = 0.156803 + 0.747604I$<br>$a = -1.50536 - 0.52470I$<br>$b = -0.156223 + 1.207680I$ | $-0.80957 + 3.02973I$                 | $-5.16840 - 1.62282I$ |
| $u = 0.156803 - 0.747604I$<br>$a = -1.50536 + 0.52470I$<br>$b = -0.156223 - 1.207680I$ | $-0.80957 - 3.02973I$                 | $-5.16840 + 1.62282I$ |
| $u = 0.799863 + 1.014160I$<br>$a = -1.26195 + 1.42943I$<br>$b = 2.45906 + 0.13647I$    | $9.45063 + 6.48172I$                  | $3.04187 - 3.27257I$  |
| $u = 0.799863 - 1.014160I$<br>$a = -1.26195 - 1.42943I$<br>$b = 2.45906 - 0.13647I$    | $9.45063 - 6.48172I$                  | $3.04187 + 3.27257I$  |
| $u = 0.148015 + 0.419312I$<br>$a = 1.80840 - 0.30217I$<br>$b = -0.394374 - 0.713561I$  | $0.00150 - 1.41503I$                  | $-1.90513 + 4.60201I$ |
| $u = 0.148015 - 0.419312I$<br>$a = 1.80840 + 0.30217I$<br>$b = -0.394374 + 0.713561I$  | $0.00150 + 1.41503I$                  | $-1.90513 - 4.60201I$ |

$$\text{III. } I_3^u = \langle b + u + 1, a - u, u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ u + 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u \\ u + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ -u - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u \\ -u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ -u \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $8u + 4$

(iv) **u**-Polynomials at the component

| Crossings                        | <b>u</b> -Polynomials at each crossing |
|----------------------------------|--|
| $c_1, c_2, c_6$<br>$c_7, c_{10}$ | $u^2 - u + 1$                          |
| $c_3, c_4, c_8$                  | $u^2 + u + 1$                          |
| $c_5, c_9$                       | $u^2$                                  |

(v) Riley Polynomials at the component

| Crossings   | Riley Polynomials at each crossing |
|---|------------------------------------|
| $c_1, c_2, c_3$<br>$c_4, c_6, c_7$<br>$c_8, c_{10}$ | $y^2 + y + 1$                      |
| $c_5, c_9$  | $y^2$                              |

(vi) Complex Volumes and Cusp Shapes

| Solutions to $I_3^u$        | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape      |
|-----------------------------|---------------------------------------|-----------------|
| $u = -0.500000 + 0.866025I$ | $-4.05977I$                           | $0. + 6.92820I$ |
| $a = -0.500000 + 0.866025I$ |                                       |                 |
| $b = -0.500000 - 0.866025I$ |                                       |                 |
| $u = -0.500000 - 0.866025I$ | $4.05977I$                            | $0. - 6.92820I$ |
| $a = -0.500000 - 0.866025I$ |                                       |                 |
| $b = -0.500000 + 0.866025I$ |                                       |                 |

$$\text{IV. } \Gamma_4^u = \langle b - u, a - 1, u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ u + 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u \\ u + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 1 \\ u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ 2u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -3



(iv) **u**-Polynomials at the component

| Crossings                        | <b>u</b> -Polynomials at each crossing |
|----------------------------------|--|
| $c_1, c_2, c_6$<br>$c_7, c_{10}$ | $u^2 - u + 1$                          |
| $c_3, c_4, c_8$                  | $u^2 + u + 1$                          |
| $c_5, c_9$                       | $u^2$                                  |

(v) Riley Polynomials at the component

| Crossings   | Riley Polynomials at each crossing |
|---|------------------------------------|
| $c_1, c_2, c_3$<br>$c_4, c_6, c_7$<br>$c_8, c_{10}$ | $y^2 + y + 1$                      |
| $c_5, c_9$  | $y^2$                              |

(vi) Complex Volumes and Cusp Shapes

| Solutions to $I_4^u$  | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---------------------------------------|------------|
| $u = -0.500000 + 0.866025I$<br>$a = 1.00000$<br>$b = -0.500000 + 0.866025I$ | 0                                     | -3.00000   |
| $u = -0.500000 - 0.866025I$<br>$a = 1.00000$<br>$b = -0.500000 - 0.866025I$ | 0                                     | -3.00000   |

### V. u-Polynomials

| Crossings          | u-Polynomials at each crossing   |
|--------------------|--|
| $c_1, c_6$         | $((u^2 - u + 1)^2)(u^{10} + u^9 + \dots - u + 1)$ $\cdot (u^{26} + 2u^{25} + \dots + u + 1)$ |
| $c_2, c_7, c_{10}$ | $((u^2 - u + 1)^2)(u^{10} + 3u^9 + \dots + 5u + 1)(u^{26} + 8u^{25} + \dots + 13u + 1)$      |
| $c_3, c_8$         | $((u^2 + u + 1)^2)(u^{10} + u^9 + \dots - u + 1)$ $\cdot (u^{26} + 2u^{25} + \dots + u + 1)$ |
| $c_4$              | $((u^2 + u + 1)^2)(u^{10} + 3u^9 + \dots + 5u + 1)(u^{26} + 8u^{25} + \dots + 13u + 1)$      |
| $c_5, c_9$         | $u^4(u^{10} - 5u^9 + \dots - 8u + 4)(u^{13} + 2u^{12} + \dots + 3u + 2)^2$                   |

## VI. Riley Polynomials

| Crossings                   | Riley Polynomials at each crossing   |
|-----------------------------|--|
| $c_1, c_3, c_6$<br>$c_8$    | $((y^2 + y + 1)^2)(y^{10} + 3y^9 + \dots + 5y + 1)(y^{26} + 8y^{25} + \dots + 13y + 1)$    |
| $c_2, c_4, c_7$<br>$c_{10}$ | $((y^2 + y + 1)^2)(y^{10} + 11y^9 + \dots + 13y + 1)(y^{26} + 20y^{25} + \dots - 11y + 1)$ |
| $c_5, c_9$                  | $y^4(y^{10} + 5y^9 + \dots + 32y + 16)(y^{13} + 10y^{12} + \dots - 7y - 4)^2$              |