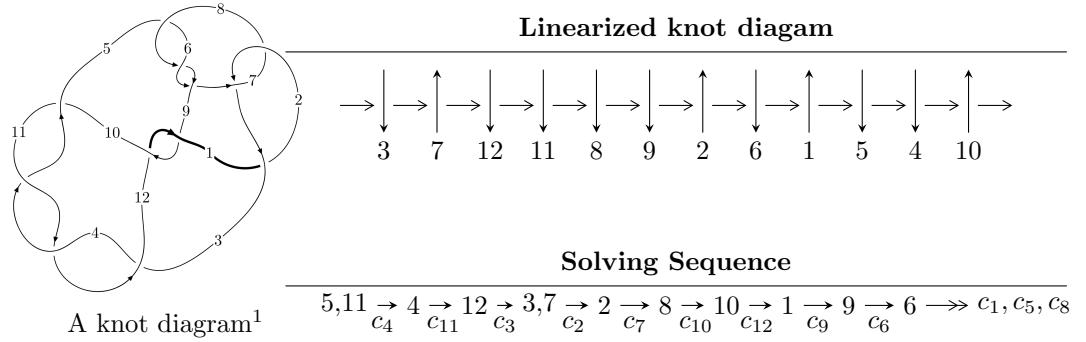


$12a_{0689}$  ( $K12a_{0689}$ )



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle u^{61} - u^{60} + \dots + b - 1, u^{64} - 2u^{63} + \dots + a + 2, u^{65} - 2u^{64} + \dots + u - 1 \rangle$$

$$I_2^u = \langle b - 1, u^3 + u^2 + a + 3u + 1, u^4 + u^3 + 3u^2 + 2u + 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 69 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle u^{61} - u^{60} + \cdots + b - 1, \ u^{64} - 2u^{63} + \cdots + a + 2, \ u^{65} - 2u^{64} + \cdots + u - 1 \rangle^{\text{I.}}$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^2 + 1 \\ -u^4 - 2u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^{64} + 2u^{63} + \cdots - 4u - 2 \\ -u^{61} + u^{60} + \cdots + 2u + 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u^{11} - 6u^9 - 12u^7 - 8u^5 - u^3 - 2u \\ u^{13} + 7u^{11} + 17u^9 + 16u^7 + 6u^5 + 5u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^{64} + 2u^{63} + \cdots - 3u - 1 \\ -u^{61} + u^{60} + \cdots + u + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^5 + 2u^3 - u \\ u^5 + 3u^3 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^9 + 4u^7 + 3u^5 - 2u^3 + u \\ u^9 + 5u^7 + 7u^5 + 2u^3 + u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^{64} + 2u^{63} + \cdots - 3u - 1 \\ -u^{61} + u^{60} + \cdots + 2u + 1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $-u^{64} + 2u^{63} + \cdots - 10u - 3$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^{65} + 27u^{64} + \cdots - 1984u - 256$
$c_2, c_7$	$u^{65} + u^{64} + \cdots + 24u + 16$
$c_3, c_4, c_{10}$ $c_{11}$	$u^{65} - 2u^{64} + \cdots + u - 1$
$c_5, c_6, c_8$	$u^{65} - 5u^{64} + \cdots + u + 1$
$c_9, c_{12}$	$u^{65} + 12u^{64} + \cdots + 1405u + 131$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{65} + 15y^{64} + \cdots - 1257472y - 65536$
$c_2, c_7$	$y^{65} + 27y^{64} + \cdots - 1984y - 256$
$c_3, c_4, c_{10}$ $c_{11}$	$y^{65} + 72y^{64} + \cdots + 5y - 1$
$c_5, c_6, c_8$	$y^{65} - 57y^{64} + \cdots - 33y - 1$
$c_9, c_{12}$	$y^{65} + 36y^{64} + \cdots + 294081y - 17161$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.176961 + 0.834517I$		
$a = 1.40512 + 0.84371I$	$-1.61201 + 5.84427I$	$-3.29017 - 6.41076I$
$b = 0.99803 + 1.05009I$		
$u = -0.176961 - 0.834517I$		
$a = 1.40512 - 0.84371I$	$-1.61201 - 5.84427I$	$-3.29017 + 6.41076I$
$b = 0.99803 - 1.05009I$		
$u = 0.587464 + 0.605812I$		
$a = 1.99776 + 1.35416I$	$-6.38469 - 11.44690I$	$-7.68442 + 9.11849I$
$b = 1.11944 - 1.68323I$		
$u = 0.587464 - 0.605812I$		
$a = 1.99776 - 1.35416I$	$-6.38469 + 11.44690I$	$-7.68442 - 9.11849I$
$b = 1.11944 + 1.68323I$		
$u = 0.562023 + 0.589176I$		
$a = -1.70682 - 1.57560I$	$-1.08419 - 7.25640I$	$-4.14361 + 8.68494I$
$b = -1.43980 + 1.35021I$		
$u = 0.562023 - 0.589176I$		
$a = -1.70682 + 1.57560I$	$-1.08419 + 7.25640I$	$-4.14361 - 8.68494I$
$b = -1.43980 - 1.35021I$		
$u = -0.434655 + 0.677483I$		
$a = -0.76935 - 1.40090I$	$-3.21792 - 0.04263I$	$-6.95293 - 1.05086I$
$b = 0.761684 - 0.558455I$		
$u = -0.434655 - 0.677483I$		
$a = -0.76935 + 1.40090I$	$-3.21792 + 0.04263I$	$-6.95293 + 1.05086I$
$b = 0.761684 + 0.558455I$		
$u = -0.559917 + 0.567694I$		
$a = 0.51071 - 1.73794I$	$-4.08533 + 5.19997I$	$-6.95238 - 6.01151I$
$b = 0.802552 + 0.895254I$		
$u = -0.559917 - 0.567694I$		
$a = 0.51071 + 1.73794I$	$-4.08533 - 5.19997I$	$-6.95238 + 6.01151I$
$b = 0.802552 - 0.895254I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.618882 + 0.492399I$		
$a = -0.690536 - 0.334295I$	$-11.32390 - 2.09934I$	$-11.90160 + 3.33110I$
$b = -0.479177 - 0.103237I$		
$u = 0.618882 - 0.492399I$		
$a = -0.690536 + 0.334295I$	$-11.32390 + 2.09934I$	$-11.90160 - 3.33110I$
$b = -0.479177 + 0.103237I$		
$u = 0.541988 + 0.545165I$		
$a = 1.05538 + 1.71117I$	$-3.16680 - 2.60582I$	$-8.31028 + 4.78950I$
$b = 1.65899 - 0.67000I$		
$u = 0.541988 - 0.545165I$		
$a = 1.05538 - 1.71117I$	$-3.16680 + 2.60582I$	$-8.31028 - 4.78950I$
$b = 1.65899 + 0.67000I$		
$u = -0.083204 + 0.762987I$		
$a = -1.47229 - 0.86393I$	$3.00423 + 2.37974I$	$3.15751 - 4.54472I$
$b = -1.103520 - 0.489689I$		
$u = -0.083204 - 0.762987I$		
$a = -1.47229 + 0.86393I$	$3.00423 - 2.37974I$	$3.15751 + 4.54472I$
$b = -1.103520 + 0.489689I$		
$u = -0.491902 + 0.580057I$		
$a = -0.057631 + 1.353840I$	$0.50199 + 2.34229I$	$-0.41176 - 4.10900I$
$b = -0.650246 - 0.287216I$		
$u = -0.491902 - 0.580057I$		
$a = -0.057631 - 1.353840I$	$0.50199 - 2.34229I$	$-0.41176 + 4.10900I$
$b = -0.650246 + 0.287216I$		
$u = 0.628694 + 0.357461I$		
$a = -0.979961 - 0.460853I$	$-7.11475 + 7.31767I$	$-9.69803 - 3.17801I$
$b = 0.98668 + 1.61826I$		
$u = 0.628694 - 0.357461I$		
$a = -0.979961 + 0.460853I$	$-7.11475 - 7.31767I$	$-9.69803 + 3.17801I$
$b = 0.98668 - 1.61826I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.573521 + 0.396534I$	$-4.58689 - 1.30630I$	$-8.69876 - 0.72424I$
$a = -1.36868 + 0.72006I$		
$b = 0.700052 - 0.740111I$		
$u = -0.573521 - 0.396534I$	$-4.58689 + 1.30630I$	$-8.69876 + 0.72424I$
$a = -1.36868 - 0.72006I$		
$b = 0.700052 + 0.740111I$		
$u = 0.547064 + 0.428484I$	$-3.51243 - 1.15687I$	$-10.08026 + 3.04392I$
$a = -0.648498 + 0.866630I$		
$b = 1.39053 + 0.84768I$		
$u = 0.547064 - 0.428484I$	$-3.51243 + 1.15687I$	$-10.08026 - 3.04392I$
$a = -0.648498 - 0.866630I$		
$b = 1.39053 - 0.84768I$		
$u = 0.584549 + 0.365784I$	$-1.73587 + 3.32691I$	$-6.23751 - 2.52670I$
$a = 1.077210 - 0.038246I$		
$b = -1.19511 - 1.33848I$		
$u = 0.584549 - 0.365784I$	$-1.73587 - 3.32691I$	$-6.23751 + 2.52670I$
$a = 1.077210 + 0.038246I$		
$b = -1.19511 + 1.33848I$		
$u = 0.061107 + 0.669375I$	$-0.052808 - 1.067730I$	$0.522323 + 0.688585I$
$a = 1.86977 + 0.92766I$		
$b = 1.256900 - 0.293507I$		
$u = 0.061107 - 0.669375I$	$-0.052808 + 1.067730I$	$0.522323 - 0.688585I$
$a = 1.86977 - 0.92766I$		
$b = 1.256900 + 0.293507I$		
$u = 0.12768 + 1.41720I$	$-1.52933 + 4.66645I$	0
$a = -0.219186 + 0.941667I$		
$b = 0.74968 + 1.50855I$		
$u = 0.12768 - 1.41720I$	$-1.52933 - 4.66645I$	0
$a = -0.219186 - 0.941667I$		
$b = 0.74968 - 1.50855I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.549260 + 0.140873I$		
$a = -0.771450 - 0.335665I$	$-4.80826 + 3.38006I$	$-10.74337 - 4.24525I$
$b = 0.703771 + 0.825353I$		
$u = -0.549260 - 0.140873I$		
$a = -0.771450 + 0.335665I$	$-4.80826 - 3.38006I$	$-10.74337 + 4.24525I$
$b = 0.703771 - 0.825353I$		
$u = -0.397665 + 0.374585I$		
$a = 0.879636 - 0.312320I$	$-0.161016 + 0.965316I$	$-1.89057 - 5.10054I$
$b = -0.225733 + 0.046292I$		
$u = -0.397665 - 0.374585I$		
$a = 0.879636 + 0.312320I$	$-0.161016 - 0.965316I$	$-1.89057 + 5.10054I$
$b = -0.225733 - 0.046292I$		
$u = 0.10989 + 1.46208I$		
$a = 0.189540 - 1.294750I$	$4.08220 + 1.01313I$	0
$b = -0.77445 - 1.43903I$		
$u = 0.10989 - 1.46208I$		
$a = 0.189540 + 1.294750I$	$4.08220 - 1.01313I$	0
$b = -0.77445 + 1.43903I$		
$u = -0.12743 + 1.47880I$		
$a = -0.875843 + 0.046170I$	$1.48337 + 1.06097I$	0
$b = 0.504534 - 0.571892I$		
$u = -0.12743 - 1.47880I$		
$a = -0.875843 - 0.046170I$	$1.48337 - 1.06097I$	0
$b = 0.504534 + 0.571892I$		
$u = 0.13453 + 1.49947I$		
$a = 0.60759 + 1.59619I$	$2.82349 - 3.48654I$	0
$b = 1.18048 + 1.16499I$		
$u = 0.13453 - 1.49947I$		
$a = 0.60759 - 1.59619I$	$2.82349 + 3.48654I$	0
$b = 1.18048 - 1.16499I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.18092 + 1.50611I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -0.976025 - 0.476688I$	$-4.77664 - 4.96073I$	0
$b = -0.478652 - 0.330158I$		
$u = 0.18092 - 1.50611I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -0.976025 + 0.476688I$	$-4.77664 + 4.96073I$	0
$b = -0.478652 + 0.330158I$		
$u = -0.10169 + 1.52949I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = 0.709783 - 0.051260I$	$6.41258 + 2.64230I$	0
$b = 0.019341 + 0.265567I$		
$u = -0.10169 - 1.52949I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = 0.709783 + 0.051260I$	$6.41258 - 2.64230I$	0
$b = 0.019341 - 0.265567I$		
$u = 0.15680 + 1.54612I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = 2.43920 + 0.89187I$	$3.81275 - 5.11770I$	0
$b = 1.88545 - 0.56925I$		
$u = 0.15680 - 1.54612I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = 2.43920 - 0.89187I$	$3.81275 + 5.11770I$	0
$b = 1.88545 + 0.56925I$		
$u = -0.352512 + 0.264358I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = 0.901458 - 0.344587I$	$-0.166994 + 0.943418I$	$-4.05068 - 5.99237I$
$b = -0.244395 - 0.205512I$		
$u = -0.352512 - 0.264358I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = 0.901458 + 0.344587I$	$-0.166994 - 0.943418I$	$-4.05068 + 5.99237I$
$b = -0.244395 + 0.205512I$		
$u = -0.16566 + 1.55185I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = 1.25278 - 0.66968I$	$2.98284 + 7.83055I$	0
$b = 0.885611 + 1.024030I$		
$u = -0.16566 - 1.55185I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = 1.25278 + 0.66968I$	$2.98284 - 7.83055I$	0
$b = 0.885611 - 1.024030I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.14397 + 1.55990I$		
$a = -0.870671 + 0.798870I$	$7.68999 + 4.65543I$	0
$b = -0.814944 - 0.452840I$		
$u = -0.14397 - 1.55990I$		
$a = -0.870671 - 0.798870I$	$7.68999 - 4.65543I$	0
$b = -0.814944 + 0.452840I$		
$u = 0.16814 + 1.55987I$		
$a = -2.69089 - 0.22163I$	$6.09318 - 9.92009I$	0
$b = -1.62903 + 1.34795I$		
$u = 0.16814 - 1.55987I$		
$a = -2.69089 + 0.22163I$	$6.09318 + 9.92009I$	0
$b = -1.62903 - 1.34795I$		
$u = 0.17903 + 1.56537I$		
$a = 2.62610 - 0.20365I$	$0.8595 - 14.2592I$	0
$b = 1.23470 - 1.72300I$		
$u = 0.17903 - 1.56537I$		
$a = 2.62610 + 0.20365I$	$0.8595 + 14.2592I$	0
$b = 1.23470 + 1.72300I$		
$u = 0.00918 + 1.57703I$		
$a = 2.49998 + 0.14603I$	$7.58996 - 1.27405I$	0
$b = 1.54857 - 0.46319I$		
$u = 0.00918 - 1.57703I$		
$a = 2.49998 - 0.14603I$	$7.58996 + 1.27405I$	0
$b = 1.54857 + 0.46319I$		
$u = -0.12188 + 1.58644I$		
$a = 0.279040 - 1.317600I$	$4.43394 + 1.99633I$	0
$b = 0.885103 - 0.421580I$		
$u = -0.12188 - 1.58644I$		
$a = 0.279040 + 1.317600I$	$4.43394 - 1.99633I$	0
$b = 0.885103 + 0.421580I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.01577 + 1.59335I$		
$a = -2.24569 - 0.79934I$	$10.99110 + 2.69855I$	0
$b = -1.46124 - 0.34322I$		
$u = -0.01577 - 1.59335I$		
$a = -2.24569 + 0.79934I$	$10.99110 - 2.69855I$	0
$b = -1.46124 + 0.34322I$		
$u = -0.03506 + 1.60818I$		
$a = 2.00100 + 1.33814I$	$6.66598 + 6.54137I$	0
$b = 1.26274 + 1.00587I$		
$u = -0.03506 - 1.60818I$		
$a = 2.00100 - 1.33814I$	$6.66598 - 6.54137I$	0
$b = 1.26274 - 1.00587I$		
$u = 0.266227$		
$a = -2.91703$	-2.12035	-4.69260
$b = 0.922923$		

$$\text{II. } I_2^u = \langle b - 1, u^3 + u^2 + a + 3u + 1, u^4 + u^3 + 3u^2 + 2u + 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^2 + 1 \\ u^3 + u^2 + 2u + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^3 - u^2 - 3u - 1 \\ 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^2 + 1 \\ u^3 + u^2 + 2u + 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^3 - u^2 - 3u - 1 \\ 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^2 + 1 \\ u^3 + u^2 + 2u + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^3 - u^2 - 3u \\ 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-3u^3 - 3u^2 - 10u - 8$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_7$	$u^4$
$c_3, c_4$	$u^4 + u^3 + 3u^2 + 2u + 1$
$c_5, c_6$	$(u - 1)^4$
$c_8$	$(u + 1)^4$
$c_9$	$u^4 - u^3 + u^2 + 1$
$c_{10}, c_{11}$	$u^4 - u^3 + 3u^2 - 2u + 1$
$c_{12}$	$u^4 + u^3 + u^2 + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_7$	$y^4$
$c_3, c_4, c_{10}$ $c_{11}$	$y^4 + 5y^3 + 7y^2 + 2y + 1$
$c_5, c_6, c_8$	$(y - 1)^4$
$c_9, c_{12}$	$y^4 + y^3 + 3y^2 + 2y + 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.395123 + 0.506844I$		
$a = 0.043315 - 1.227190I$	$-1.85594 + 1.41510I$	$-4.47493 - 4.18840I$
$b = 1.00000$		
$u = -0.395123 - 0.506844I$		
$a = 0.043315 + 1.227190I$	$-1.85594 - 1.41510I$	$-4.47493 + 4.18840I$
$b = 1.00000$		
$u = -0.10488 + 1.55249I$		
$a = 0.956685 - 0.641200I$	$5.14581 + 3.16396I$	$-2.02507 - 3.47609I$
$b = 1.00000$		
$u = -0.10488 - 1.55249I$		
$a = 0.956685 + 0.641200I$	$5.14581 - 3.16396I$	$-2.02507 + 3.47609I$
$b = 1.00000$		

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$u^4(u^{65} + 27u^{64} + \dots - 1984u - 256)$
$c_2, c_7$	$u^4(u^{65} + u^{64} + \dots + 24u + 16)$
$c_3, c_4$	$(u^4 + u^3 + 3u^2 + 2u + 1)(u^{65} - 2u^{64} + \dots + u - 1)$
$c_5, c_6$	$((u - 1)^4)(u^{65} - 5u^{64} + \dots + u + 1)$
$c_8$	$((u + 1)^4)(u^{65} - 5u^{64} + \dots + u + 1)$
$c_9$	$(u^4 - u^3 + u^2 + 1)(u^{65} + 12u^{64} + \dots + 1405u + 131)$
$c_{10}, c_{11}$	$(u^4 - u^3 + 3u^2 - 2u + 1)(u^{65} - 2u^{64} + \dots + u - 1)$
$c_{12}$	$(u^4 + u^3 + u^2 + 1)(u^{65} + 12u^{64} + \dots + 1405u + 131)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$y^4(y^{65} + 15y^{64} + \dots - 1257472y - 65536)$
$c_2, c_7$	$y^4(y^{65} + 27y^{64} + \dots - 1984y - 256)$
$c_3, c_4, c_{10}$ $c_{11}$	$(y^4 + 5y^3 + 7y^2 + 2y + 1)(y^{65} + 72y^{64} + \dots + 5y - 1)$
$c_5, c_6, c_8$	$((y - 1)^4)(y^{65} - 57y^{64} + \dots - 33y - 1)$
$c_9, c_{12}$	$(y^4 + y^3 + 3y^2 + 2y + 1)(y^{65} + 36y^{64} + \dots + 294081y - 17161)$