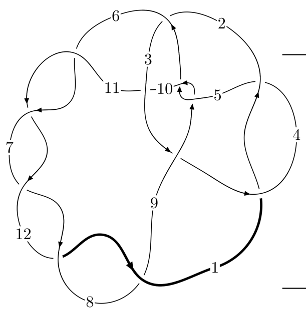
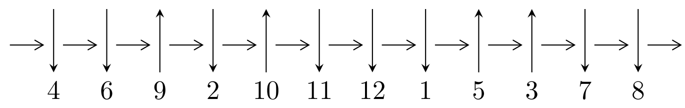
# $12a_{0909}$ $(K12a_{0909})$




A knot diagram[1]

**Linearized knot diagam**




**Solving Sequence**

$$8{,}12 \underset{c_{12}}{\rightarrow} 1{,}4 \underset{c_1}{\rightarrow} 2 \underset{c_4}{\rightarrow} 5 \underset{c_8}{\rightarrow} 9 \underset{c_3}{\rightarrow} 3 \underset{c_7}{\rightarrow} 7 \underset{c_{11}}{\rightarrow} 11 \underset{c_6}{\rightarrow} 6 \underset{c_{10}}{\rightarrow} 10 \twoheadrightarrow c_2, c_5, c_9$$

**Ideals for irreducible components[2] of $X_{\mathbf{par}}$**

$$I_1^u = \langle -2.13178 \times 10^{29} u^{50} + 5.14130 \times 10^{28} u^{49} + \cdots + 1.65837 \times 10^{29} b - 2.15608 \times 10^{27},$$
$$-1.15795 \times 10^{29} u^{50} + 4.35679 \times 10^{28} u^{49} + \cdots + 1.65837 \times 10^{29} a + 6.77432 \times 10^{28},\ u^{51} - 2u^{50} + \cdots - u +$$
$$I_2^u = \langle b - 1,\ a - 1,\ u + 1 \rangle$$

* 2 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 52 representations.

[1]The image of knot diagram is generated by the software "**Draw programme**" developed by Andrew Bartholomew(`http://www.layer8.co.uk/maths/draw/index.htm#Running-draw`), where we modified some parts for our purpose(`https://github.com/CATsTAILs/LinksPainter`).

[2]All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

**I.**

$\boldsymbol{I_1^u = \langle -2.13\times10^{29}u^{50}+5.14\times10^{28}u^{49}+\cdots+1.66\times10^{29}b-2.16\times10^{27},\ -1.16\times10^{29}u^{50}+4.36\times10^{28}u^{49}+\cdots+1.66\times10^{29}a+6.77\times10^{28},\ u^{51}-2u^{50}+\cdots-u+1\rangle}$

**(i) Arc colorings**

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.698249u^{50} - 0.262716u^{49} + \cdots - 11.7227u - 0.408493 \\ 1.28547u^{50} - 0.310022u^{49} + \cdots + 0.0538566u + 0.0130012 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0.674876u^{50} - 0.229531u^{49} + \cdots - 12.1488u + 0.603479 \\ 1.22229u^{50} - 0.317787u^{49} + \cdots + 0.264395u + 0.0354543 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0.137080u^{50} + 0.0189458u^{49} + \cdots + 0.718302u - 1.06415 \\ 0.412183u^{50} + 0.00870488u^{49} + \cdots - 0.371603u - 0.128201 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -0.568625u^{50} + 0.203922u^{49} + \cdots - 12.2253u + 0.626397 \\ -1.07781u^{50} + 0.232253u^{49} + \cdots - 0.243835u + 1.04522 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u^2 + 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^3 + 2u \\ -u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -0.275103u^{50} + 0.0102409u^{49} + \cdots + 1.08990u - 0.935948 \\ -0.412183u^{50} - 0.00870488u^{49} + \cdots + 0.371603u + 0.128201 \end{pmatrix}$$

**(ii) Obstruction class** $= -1$

**(iii) Cusp Shapes** $= -1.08773u^{50} + 4.19995u^{49} + \cdots - 16.8202u - 2.33236$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_4$ | $u^{51} - 3u^{50} + \cdots - 87u + 9$ |
| $c_2$ | $17(17u^{51} - 112u^{50} + \cdots + 151u - 17)$ |
| $c_3$ | $17(17u^{51} - 228u^{50} + \cdots - 4473u + 2377)$ |
| $c_5, c_9$ | $u^{51} - 15u^{49} + \cdots - 3u - 1$ |
| $c_6, c_7, c_8$ <br> $c_{11}, c_{12}$ | $u^{51} - 2u^{50} + \cdots - u + 1$ |
| $c_{10}$ | $u^{51} + 3u^{50} + \cdots - 291u - 51$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_4$ | $y^{51} - 39y^{50} + \cdots + 5175y - 81$ |
| $c_2$ | $289(289y^{51} - 39098y^{50} + \cdots + 18449y - 289)$ |
| $c_3$ | $289(289y^{51} - 30598y^{50} + \cdots + 2.10558 \times 10^8 y - 5650129)$ |
| $c_5, c_9$ | $y^{51} - 30y^{50} + \cdots + 9y - 1$ |
| $c_6, c_7, c_8$<br>$c_{11}, c_{12}$ | $y^{51} - 70y^{50} + \cdots + 9y - 1$ |
| $c_{10}$ | $y^{51} + 9y^{50} + \cdots + 43575y - 2601$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_1^u$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 1.047260 + 0.151588I$<br>$a = -0.338923 + 0.162440I$<br>$b = -0.065433 + 0.787075I$ | $-4.15821 - 2.51727I$ | 0 |
| $u = 1.047260 - 0.151588I$<br>$a = -0.338923 - 0.162440I$<br>$b = -0.065433 - 0.787075I$ | $-4.15821 + 2.51727I$ | 0 |
| $u = -1.077900 + 0.210349I$<br>$a = -0.414562 + 0.513194I$<br>$b = -0.211649 - 0.453660I$ | $-1.01874 + 6.10618I$ | 0 |
| $u = -1.077900 - 0.210349I$<br>$a = -0.414562 - 0.513194I$<br>$b = -0.211649 + 0.453660I$ | $-1.01874 - 6.10618I$ | 0 |
| $u = -0.890510$<br>$a = 0.780361$<br>$b = 0.512763$ | $-1.69978$ | $-4.88630$ |
| $u = -1.120720 + 0.024853I$<br>$a = -2.15005 - 0.20348I$<br>$b = -1.208940 - 0.347028I$ | $-6.84706 + 0.17814I$ | 0 |
| $u = -1.120720 - 0.024853I$<br>$a = -2.15005 + 0.20348I$<br>$b = -1.208940 + 0.347028I$ | $-6.84706 - 0.17814I$ | 0 |
| $u = 1.126260 + 0.086381I$<br>$a = -1.61198 + 0.21718I$<br>$b = -0.738481 + 0.732354I$ | $-5.85482 - 3.55367I$ | 0 |
| $u = 1.126260 - 0.086381I$<br>$a = -1.61198 - 0.21718I$<br>$b = -0.738481 - 0.732354I$ | $-5.85482 + 3.55367I$ | 0 |
| $u = -0.438374 + 0.708518I$<br>$a = 0.366935 + 0.769343I$<br>$b = -0.844551 - 0.361467I$ | $-4.09880 + 2.34673I$ | $-13.7642 - 6.0920I$ |

| Solutions to $I_1^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -0.438374 - 0.708518I$<br>$a = 0.366935 - 0.769343I$<br>$b = -0.844551 + 0.361467I$ | $-4.09880 - 2.34673I$ | $-13.7642 + 6.0920I$ |
| $u = -1.170700 + 0.336077I$<br>$a = 1.80050 + 0.84662I$<br>$b = 1.55082 + 0.18031I$ | $-5.46811 + 12.11320I$ | $0$ |
| $u = -1.170700 - 0.336077I$<br>$a = 1.80050 - 0.84662I$<br>$b = 1.55082 - 0.18031I$ | $-5.46811 - 12.11320I$ | $0$ |
| $u = -1.140670 + 0.479406I$<br>$a = 1.072900 + 0.758069I$<br>$b = 0.825477 - 0.456373I$ | $-4.33250 - 0.67320I$ | $0$ |
| $u = -1.140670 - 0.479406I$<br>$a = 1.072900 - 0.758069I$<br>$b = 0.825477 + 0.456373I$ | $-4.33250 + 0.67320I$ | $0$ |
| $u = 0.436309 + 0.619055I$<br>$a = 0.571431 - 1.171610I$<br>$b = -0.742635 + 0.081884I$ | $-0.40580 - 8.83491I$ | $-5.93498 + 8.65489I$ |
| $u = 0.436309 - 0.619055I$<br>$a = 0.571431 + 1.171610I$<br>$b = -0.742635 - 0.081884I$ | $-0.40580 + 8.83491I$ | $-5.93498 - 8.65489I$ |
| $u = 1.190350 + 0.364165I$<br>$a = 1.55085 - 0.82569I$<br>$b = 1.381760 + 0.003819I$ | $-9.25343 - 5.99551I$ | $0$ |
| $u = 1.190350 - 0.364165I$<br>$a = 1.55085 + 0.82569I$<br>$b = 1.381760 - 0.003819I$ | $-9.25343 + 5.99551I$ | $0$ |
| $u = 0.299142 + 0.682434I$<br>$a = -0.097433 - 0.386283I$<br>$b = -0.974446 + 0.425322I$ | $0.02412 + 4.64525I$ | $-6.01152 - 5.00660I$ |

| Solutions to $I_1^u$ | $\sqrt{-1}(\text{vol}+\sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 0.299142 - 0.682434I$<br>$a = -0.097433 + 0.386283I$<br>$b = -0.974446 - 0.425322I$ | $0.02412 - 4.64525I$ | $-6.01152 + 5.00660I$ |
| $u = -1.34834$<br>$a = 1.42591$<br>$b = 1.94067$ | $-2.22043$ | $0$ |
| $u = 0.380677 + 0.380330I$<br>$a = 1.68381 - 0.54757I$<br>$b = -0.0243995 - 0.1158830I$ | $3.01854 + 1.08074I$ | $-0.68742 + 2.46829I$ |
| $u = 0.380677 - 0.380330I$<br>$a = 1.68381 + 0.54757I$<br>$b = -0.0243995 + 0.1158830I$ | $3.01854 - 1.08074I$ | $-0.68742 - 2.46829I$ |
| $u = 0.295738 + 0.444672I$<br>$a = -0.187973 + 0.763692I$<br>$b = -0.269582 + 0.799798I$ | $3.28922 - 3.89695I$ | $-0.59685 + 7.26004I$ |
| $u = 0.295738 - 0.444672I$<br>$a = -0.187973 - 0.763692I$<br>$b = -0.269582 - 0.799798I$ | $3.28922 + 3.89695I$ | $-0.59685 - 7.26004I$ |
| $u = -0.390664 + 0.229819I$<br>$a = -0.02035 - 1.55893I$<br>$b = 0.805995 - 0.679439I$ | $-1.04347 + 2.52944I$ | $-8.36979 - 8.40912I$ |
| $u = -0.390664 - 0.229819I$<br>$a = -0.02035 + 1.55893I$<br>$b = 0.805995 + 0.679439I$ | $-1.04347 - 2.52944I$ | $-8.36979 + 8.40912I$ |
| $u = 0.400437$<br>$a = -0.797854$<br>$b = 1.13406$ | $-2.10246$ | $-9.62010$ |
| $u = -0.216099 + 0.323578I$<br>$a = 0.636723 - 0.775106I$<br>$b = 0.035450 - 0.334693I$ | $-0.210137 + 0.904024I$ | $-4.53571 - 7.39815I$ |

| Solutions to $I_1^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -0.216099 - 0.323578I$<br>$a = 0.636723 + 0.775106I$<br>$b = 0.035450 + 0.334693I$ | $-0.210137 - 0.904024I$ | $-4.53571 + 7.39815I$ |
| $u = 0.330272$<br>$a = -1.79695$<br>$b = 0.951690$ | $-2.13241$ | $-5.70430$ |
| $u = -0.111190 + 0.292412I$<br>$a = 4.56602 - 1.68963I$<br>$b = 0.566891 + 0.031398I$ | $-0.202413 - 0.749454I$ | $1.38825 - 9.49663I$ |
| $u = -0.111190 - 0.292412I$<br>$a = 4.56602 + 1.68963I$<br>$b = 0.566891 - 0.031398I$ | $-0.202413 + 0.749454I$ | $1.38825 + 9.49663I$ |
| $u = 1.72101 + 0.02434I$<br>$a = 0.523521 + 0.113299I$<br>$b = 1.54117 + 0.66770I$ | $-11.27570 - 0.29543I$ | 0 |
| $u = 1.72101 - 0.02434I$<br>$a = 0.523521 - 0.113299I$<br>$b = 1.54117 - 0.66770I$ | $-11.27570 + 0.29543I$ | 0 |
| $u = -1.73799$<br>$a = -7.71129$<br>$b = -22.2616$ | $-13.2279$ | 0 |
| $u = -1.74438 + 0.03663I$<br>$a = -0.436565 + 0.244633I$<br>$b = -0.910713 - 0.160686I$ | $-14.2469 + 3.2818I$ | 0 |
| $u = -1.74438 - 0.03663I$<br>$a = -0.436565 - 0.244633I$<br>$b = -0.910713 + 0.160686I$ | $-14.2469 - 3.2818I$ | 0 |
| $u = 1.74976 + 0.05001I$<br>$a = -0.269790 - 0.861011I$<br>$b = -0.63067 - 1.27223I$ | $-11.21760 - 7.16998I$ | 0 |

| Solutions to $I_1^u$ | $\sqrt{-1}(\mathrm{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = 1.74976 - 0.05001I$<br>$a = -0.269790 + 0.861011I$<br>$b = -0.63067 + 1.27223I$ | $-11.21760 + 7.16998I$ | 0 |
| $u = 1.76103 + 0.00664I$<br>$a = -2.28196 + 0.03399I$<br>$b = -5.41561 + 0.35170I$ | $-17.3264 - 0.3153I$ | 0 |
| $u = 1.76103 - 0.00664I$<br>$a = -2.28196 - 0.03399I$<br>$b = -5.41561 - 0.35170I$ | $-17.3264 + 0.3153I$ | 0 |
| $u = -1.76201 + 0.01966I$<br>$a = -1.82088 + 0.20804I$<br>$b = -4.16074 - 0.13010I$ | $-16.3513 + 3.9914I$ | 0 |
| $u = -1.76201 - 0.01966I$<br>$a = -1.82088 - 0.20804I$<br>$b = -4.16074 + 0.13010I$ | $-16.3513 - 3.9914I$ | 0 |
| $u = 1.77148 + 0.08896I$<br>$a = 2.05324 - 0.41872I$<br>$b = 5.09853 - 1.25029I$ | $-16.0463 - 13.9639I$ | 0 |
| $u = 1.77148 - 0.08896I$<br>$a = 2.05324 + 0.41872I$<br>$b = 5.09853 + 1.25029I$ | $-16.0463 + 13.9639I$ | 0 |
| $u = -1.77662 + 0.09436I$<br>$a = 1.91281 + 0.46826I$<br>$b = 4.74582 + 1.21071I$ | $19.5628 + 7.9919I$ | 0 |
| $u = -1.77662 - 0.09436I$<br>$a = 1.91281 - 0.46826I$<br>$b = 4.74582 - 1.21071I$ | $19.5628 - 7.9919I$ | 0 |
| $u = 1.79336 + 0.11215I$<br>$a = 1.64752 - 0.41305I$<br>$b = 4.21301 - 0.77958I$ | $-14.9728 - 1.9325I$ | 0 |

| Solutions to $I_1^u$ | | $\sqrt{-1}(\mathrm{vol}+\sqrt{-1}CS)$ | Cusp shape |
|---|---|---|---|
| $u=$ | $1.79336-0.11215I$ | | |
| $a=$ | $1.64752+0.41305I$ | $-14.9728+1.9325I$ | 0 |
| $b=$ | $4.21301+0.77958I$ | | |

## II. $I_2^u = \langle b-1,\ a-1,\ u+1 \rangle$

**(i) Arc colorings**

$a_8 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$a_{12} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$a_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$a_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$a_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$a_5 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$a_9 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$a_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$a_7 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$a_{11} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$a_6 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$a_{10} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

**(ii) Obstruction class** $= -1$

**(iii) Cusp Shapes** $= -6$

**(iv) u-Polynomials at the component**

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_4, c_{10}$ | $u$ |
| $c_2$ | $u - 1$ |
| $c_3, c_5, c_6$<br>$c_7, c_8, c_9$<br>$c_{11}, c_{12}$ | $u + 1$ |

**(v) Riley Polynomials at the component**

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_4, c_{10}$ | $y$ |
| $c_2, c_3, c_5$<br>$c_6, c_7, c_8$<br>$c_9, c_{11}, c_{12}$ | $y-1$ |

**(vi) Complex Volumes and Cusp Shapes**

| Solutions to $I_2^u$ | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---|---|
| $u = -1.00000$<br>$a = \ \ 1.00000$<br>$b = \ \ 1.00000$ | $-1.64493$ | $-6.00000$ |

## III. u-Polynomials

| Crossings | u-Polynomials at each crossing |
|---|---|
| $c_1, c_4$ | $u(u^{51} - 3u^{50} + \cdots - 87u + 9)$ |
| $c_2$ | $17(u-1)(17u^{51} - 112u^{50} + \cdots + 151u - 17)$ |
| $c_3$ | $17(u+1)(17u^{51} - 228u^{50} + \cdots - 4473u + 2377)$ |
| $c_5, c_9$ | $(u+1)(u^{51} - 15u^{49} + \cdots - 3u - 1)$ |
| $c_6, c_7, c_8$ $c_{11}, c_{12}$ | $(u+1)(u^{51} - 2u^{50} + \cdots - u + 1)$ |
| $c_{10}$ | $u(u^{51} + 3u^{50} + \cdots - 291u - 51)$ |

## IV. Riley Polynomials

| Crossings | Riley Polynomials at each crossing |
|---|---|
| $c_1, c_4$ | $y(y^{51} - 39y^{50} + \cdots + 5175y - 81)$ |
| $c_2$ | $289(y-1)(289y^{51} - 39098y^{50} + \cdots + 18449y - 289)$ |
| $c_3$ | $289(y-1)(289y^{51} - 30598y^{50} + \cdots + 2.10558 \times 10^8 y - 5650129)$ |
| $c_5, c_9$ | $(y-1)(y^{51} - 30y^{50} + \cdots + 9y - 1)$ |
| $c_6, c_7, c_8$<br>$c_{11}, c_{12}$ | $(y-1)(y^{51} - 70y^{50} + \cdots + 9y - 1)$ |
| $c_{10}$ | $y(y^{51} + 9y^{50} + \cdots + 43575y - 2601)$ |