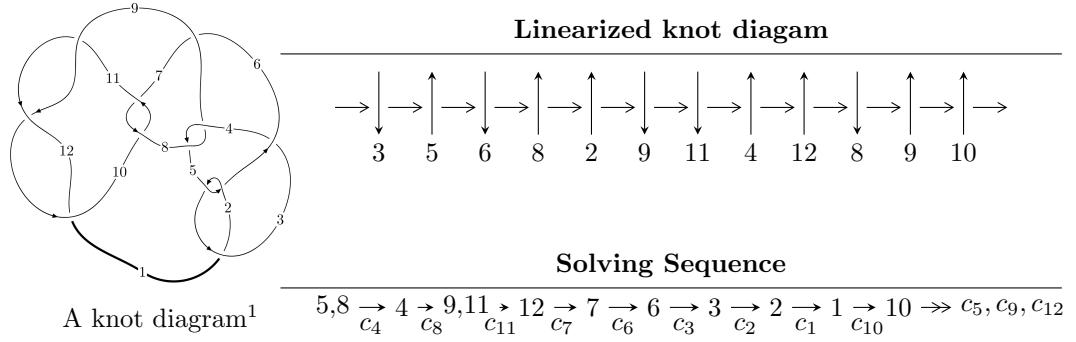


$12n_{0010}$  ( $K12n_{0010}$ )



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$\begin{aligned}
 I_1^u &= \langle 373570485293358u^{28} + 1033333880231479u^{27} + \dots + 1737205994835979b - 46103598198659, \\
 &\quad - 452869533551973u^{28} - 703336104853671u^{27} + \dots + 1737205994835979a - 136865573763131, \\
 &\quad u^{29} + 2u^{28} + \dots - u - 1 \rangle \\
 I_2^u &= \langle -u^3 + b - u - 1, a, u^4 + u^2 + u + 1 \rangle \\
 I_3^u &= \langle -u^5 + u^4 - 2u^3 + 2u^2 + b - 2u + 2, a, u^6 - u^5 + 2u^4 - 2u^3 + 2u^2 - 2u + 1 \rangle
 \end{aligned}$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 39 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

**I.**

$$I_1^u = \langle 3.74 \times 10^{14}u^{28} + 1.03 \times 10^{15}u^{27} + \dots + 1.74 \times 10^{15}b - 4.61 \times 10^{13}, -4.53 \times 10^{14}u^{28} - 7.03 \times 10^{14}u^{27} + \dots + 1.74 \times 10^{15}a - 1.37 \times 10^{14}, u^{29} + 2u^{28} + \dots - u - 1 \rangle$$

(i) **Arc colorings**

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0.260688u^{28} + 0.404866u^{27} + \dots + 1.65378u + 0.0787849 \\ -0.215041u^{28} - 0.594825u^{27} + \dots + 2.17523u + 0.0265389 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} u \\ -0.574992u^{28} - 1.22877u^{27} + \dots + 1.37727u + 0.0642647 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0.281422u^{28} + 0.665341u^{27} + \dots + 0.425718u - 0.328077 \\ -0.300941u^{28} - 0.245733u^{27} + \dots + 0.372268u - 0.413702 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0.0992626u^{28} + 0.229078u^{27} + \dots + 0.144178u - 0.116511 \\ -0.422114u^{28} - 0.580655u^{27} + \dots - 0.163376u - 0.274080 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.0599722u^{28} + 0.00259463u^{27} + \dots - 0.0401153u + 1.59959 \\ 0.138580u^{28} + 0.0401668u^{27} + \dots - 0.139073u + 1.22534 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.0786083u^{28} - 0.0375722u^{27} + \dots + 0.0989573u + 0.374257 \\ 0.138580u^{28} + 0.0401668u^{27} + \dots - 0.139073u + 1.22534 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0.493107u^{28} + 0.774647u^{27} + \dots + 0.437369u + 0.188122 \\ 0.393844u^{28} + 0.545569u^{27} + \dots + 0.293191u + 0.304633 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0.260688u^{28} + 0.404866u^{27} + \dots + 1.65378u + 0.0787849 \\ -0.314304u^{28} - 0.823903u^{27} + \dots + 2.03105u + 0.143050 \end{pmatrix}$$

(ii) **Obstruction class = -1**

(iii) **Cusp Shapes**

$$= -\frac{7883435024310839}{1737205994835979}u^{28} - \frac{14219216887312602}{1737205994835979}u^{27} + \dots + \frac{21942781332157203}{1737205994835979}u + \frac{8811379112410107}{1737205994835979}$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^{29} + 12u^{28} + \cdots + u - 1$
$c_2, c_5$	$u^{29} + 2u^{28} + \cdots + u - 1$
$c_3$	$u^{29} - 2u^{28} + \cdots + 120u - 36$
$c_4, c_8$	$u^{29} - 2u^{28} + \cdots - u + 1$
$c_6$	$u^{29} - 10u^{28} + \cdots - 469083u + 52489$
$c_7, c_{10}$	$u^{29} - 5u^{28} + \cdots - 3072u - 1024$
$c_9, c_{11}, c_{12}$	$u^{29} + 11u^{28} + \cdots - 30u^2 - 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{29} + 12y^{28} + \cdots + 85y - 1$
$c_2, c_5$	$y^{29} + 12y^{28} + \cdots + y - 1$
$c_3$	$y^{29} + 12y^{28} + \cdots - 12456y - 1296$
$c_4, c_8$	$y^{29} + 30y^{27} + \cdots + y - 1$
$c_6$	$y^{29} + 92y^{28} + \cdots - 44517246691y - 2755095121$
$c_7, c_{10}$	$y^{29} + 63y^{28} + \cdots + 3670016y - 1048576$
$c_9, c_{11}, c_{12}$	$y^{29} - 51y^{28} + \cdots - 60y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.269747 + 0.997117I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -0.080093 + 0.634791I$	$-3.76543 - 0.40137I$	$-7.74784 + 0.84155I$
$b = -0.459723 - 0.451799I$		
$u = 0.269747 - 0.997117I$		
$a = -0.080093 - 0.634791I$	$-3.76543 + 0.40137I$	$-7.74784 - 0.84155I$
$b = -0.459723 + 0.451799I$		
$u = -0.520591 + 0.795499I$		
$a = 0.040524 + 0.941049I$	$0.03144 - 1.92773I$	$1.38434 + 3.11728I$
$b = 1.201380 + 0.062565I$		
$u = -0.520591 - 0.795499I$		
$a = 0.040524 - 0.941049I$	$0.03144 + 1.92773I$	$1.38434 - 3.11728I$
$b = 1.201380 - 0.062565I$		
$u = -0.896581 + 0.315451I$		
$a = -1.90204 + 1.76626I$	$4.34061 - 5.06790I$	$7.76076 + 6.40955I$
$b = -0.36922 + 3.53096I$		
$u = -0.896581 - 0.315451I$		
$a = -1.90204 - 1.76626I$	$4.34061 + 5.06790I$	$7.76076 - 6.40955I$
$b = -0.36922 - 3.53096I$		
$u = 0.888576 + 0.223844I$		
$a = 2.22962 + 1.30457I$	$4.83420 - 0.14491I$	$9.33140 - 0.02437I$
$b = 1.34311 + 2.77938I$		
$u = 0.888576 - 0.223844I$		
$a = 2.22962 - 1.30457I$	$4.83420 + 0.14491I$	$9.33140 + 0.02437I$
$b = 1.34311 - 2.77938I$		
$u = 0.586489 + 0.957504I$		
$a = -0.302708 + 0.858093I$	$-1.89724 + 6.62062I$	$-1.77451 - 5.81131I$
$b = -1.34326 - 0.63754I$		
$u = 0.586489 - 0.957504I$		
$a = -0.302708 - 0.858093I$	$-1.89724 - 6.62062I$	$-1.77451 + 5.81131I$
$b = -1.34326 + 0.63754I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.538341 + 0.454787I$		
$a = -0.661454 + 0.909012I$	$0.49621 - 1.44403I$	$2.32169 + 4.35214I$
$b = 0.67192 + 1.39992I$		
$u = -0.538341 - 0.454787I$		
$a = -0.661454 - 0.909012I$	$0.49621 + 1.44403I$	$2.32169 - 4.35214I$
$b = 0.67192 - 1.39992I$		
$u = -0.454661 + 0.440048I$		
$a = -0.603679 + 0.541804I$	$0.61760 - 1.38123I$	$3.89803 + 4.67424I$
$b = -0.128589 + 1.154160I$		
$u = -0.454661 - 0.440048I$		
$a = -0.603679 - 0.541804I$	$0.61760 + 1.38123I$	$3.89803 - 4.67424I$
$b = -0.128589 - 1.154160I$		
$u = 0.571853 + 0.250420I$		
$a = 0.590811 + 0.162782I$	$-0.23394 - 2.60554I$	$1.51395 + 2.58658I$
$b = 0.832609 + 0.563271I$		
$u = 0.571853 - 0.250420I$		
$a = 0.590811 - 0.162782I$	$-0.23394 + 2.60554I$	$1.51395 - 2.58658I$
$b = 0.832609 - 0.563271I$		
$u = 0.600458$		
$a = 1.34216$	2.41354	4.07200
$b = -0.231076$		
$u = -0.092427 + 0.519848I$		
$a = -0.242741 + 1.091430I$	$2.03975 + 2.27000I$	$-9.41057 + 5.26980I$
$b = 0.12625 + 2.93959I$		
$u = -0.092427 - 0.519848I$		
$a = -0.242741 - 1.091430I$	$2.03975 - 2.27000I$	$-9.41057 - 5.26980I$
$b = 0.12625 - 2.93959I$		
$u = -1.10611 + 1.07911I$		
$a = 0.66217 - 1.55771I$	$15.6166 - 12.3453I$	$4.17843 + 6.31172I$
$b = -4.13956 - 3.94336I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.10611 - 1.07911I$		
$a = 0.66217 + 1.55771I$	$15.6166 + 12.3453I$	$4.17843 - 6.31172I$
$b = -4.13956 + 3.94336I$		
$u = 1.10589 + 1.08681I$		
$a = -0.81738 - 1.52507I$	$17.5116 + 6.4339I$	$6.30123 - 2.11610I$
$b = 3.91452 - 4.33465I$		
$u = 1.10589 - 1.08681I$		
$a = -0.81738 + 1.52507I$	$17.5116 - 6.4339I$	$6.30123 + 2.11610I$
$b = 3.91452 + 4.33465I$		
$u = 1.10403 + 1.10245I$		
$a = -1.17352 - 1.22596I$	$17.4749 + 1.6883I$	$6.38533 - 1.83848I$
$b = 2.76455 - 5.02114I$		
$u = 1.10403 - 1.10245I$		
$a = -1.17352 + 1.22596I$	$17.4749 - 1.6883I$	$6.38533 + 1.83848I$
$b = 2.76455 + 5.02114I$		
$u = -1.10116 + 1.10760I$		
$a = 1.23857 - 1.06154I$	$15.5469 + 4.2345I$	$4.31422 - 2.32649I$
$b = -2.25205 - 5.03203I$		
$u = -1.10116 - 1.10760I$		
$a = 1.23857 + 1.06154I$	$15.5469 - 4.2345I$	$4.31422 + 2.32649I$
$b = -2.25205 + 5.03203I$		
$u = -1.11694 + 1.09740I$		
$a = 0.85084 - 1.21443I$	$10.89410 - 4.09225I$	$1.50754 + 2.07565I$
$b = -3.04641 - 4.13857I$		
$u = -1.11694 - 1.09740I$		
$a = 0.85084 + 1.21443I$	$10.89410 + 4.09225I$	$1.50754 - 2.07565I$
$b = -3.04641 + 4.13857I$		

$$\text{II. } I_2^u = \langle -u^3 + b - u - 1, a, u^4 + u^2 + u + 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_5 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_8 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_4 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_9 &= \begin{pmatrix} u \\ u^3 + u \end{pmatrix} \\ a_{11} &= \begin{pmatrix} 0 \\ u^3 + u + 1 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} -u \\ 1 \end{pmatrix} \\ a_7 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_6 &= \begin{pmatrix} u^3 \\ -u^2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u^3 + u^2 + 1 \\ -u \end{pmatrix} \\ a_2 &= \begin{pmatrix} u^3 + u^2 + u + 1 \\ -u \end{pmatrix} \\ a_1 &= \begin{pmatrix} -u \\ -u^3 - u \end{pmatrix} \\ a_{10} &= \begin{pmatrix} 0 \\ u^3 + u + 1 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $3u^3 - 5u^2 - u + 3$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_6$	$u^4 - 2u^3 + 3u^2 - u + 1$
$c_2, c_4$	$u^4 + u^2 + u + 1$
$c_3$	$u^4 + 3u^3 + 4u^2 + 3u + 2$
$c_5, c_8$	$u^4 + u^2 - u + 1$
$c_7, c_{10}$	$u^4$
$c_9$	$(u + 1)^4$
$c_{11}, c_{12}$	$(u - 1)^4$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_6$	$y^4 + 2y^3 + 7y^2 + 5y + 1$
$c_2, c_4, c_5$ $c_8$	$y^4 + 2y^3 + 3y^2 + y + 1$
$c_3$	$y^4 - y^3 + 2y^2 + 7y + 4$
$c_7, c_{10}$	$y^4$
$c_9, c_{11}, c_{12}$	$(y - 1)^4$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.547424 + 0.585652I$		
$a = 0$	$2.62503 - 1.39709I$	$4.96170 + 3.59727I$
$b = 0.851808 + 0.911292I$		
$u = -0.547424 - 0.585652I$		
$a = 0$	$2.62503 + 1.39709I$	$4.96170 - 3.59727I$
$b = 0.851808 - 0.911292I$		
$u = 0.547424 + 1.120870I$		
$a = 0$	$-0.98010 + 7.64338I$	$1.53830 - 8.45840I$
$b = -0.351808 + 0.720342I$		
$u = 0.547424 - 1.120870I$		
$a = 0$	$-0.98010 - 7.64338I$	$1.53830 + 8.45840I$
$b = -0.351808 - 0.720342I$		

### III.

$$I_3^u = \langle -u^5 + u^4 - 2u^3 + 2u^2 + b - 2u + 2, a, u^6 - u^5 + 2u^4 - 2u^3 + 2u^2 - 2u + 1 \rangle$$

(i) **Arc colorings**

$$\begin{aligned} a_5 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_8 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_4 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_9 &= \begin{pmatrix} u \\ u^3 + u \end{pmatrix} \\ a_{11} &= \begin{pmatrix} 0 \\ u^5 - u^4 + 2u^3 - 2u^2 + 2u - 2 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} -u \\ u^5 - u^4 + u^3 - 2u^2 + u - 2 \end{pmatrix} \\ a_7 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_6 &= \begin{pmatrix} u^3 \\ u^5 + u^3 + u \end{pmatrix} \\ a_3 &= \begin{pmatrix} -u^5 + u^4 - 2u^3 + 2u^2 - 2u + 2 \\ -u^5 - 2u^3 + u^2 - u + 1 \end{pmatrix} \\ a_2 &= \begin{pmatrix} u^4 + u^2 - u + 1 \\ -u^5 - 2u^3 + u^2 - u + 1 \end{pmatrix} \\ a_1 &= \begin{pmatrix} -u \\ -u^3 - u \end{pmatrix} \\ a_{10} &= \begin{pmatrix} 0 \\ u^5 - u^4 + 2u^3 - 2u^2 + 2u - 2 \end{pmatrix} \end{aligned}$$

(ii) **Obstruction class** = 1

(iii) **Cusp Shapes** =  $-2u^5 + u^4 + u^2 + u + 4$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_6$	$u^6 - 3u^5 + 4u^4 - 2u^3 + 1$
$c_2, c_4$	$u^6 - u^5 + 2u^4 - 2u^3 + 2u^2 - 2u + 1$
$c_3$	$(u^3 - u^2 + 1)^2$
$c_5, c_8$	$u^6 + u^5 + 2u^4 + 2u^3 + 2u^2 + 2u + 1$
$c_7, c_{10}$	$u^6$
$c_9$	$(u + 1)^6$
$c_{11}, c_{12}$	$(u - 1)^6$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_6$	$y^6 - y^5 + 4y^4 - 2y^3 + 8y^2 + 1$
$c_2, c_4, c_5$ $c_8$	$y^6 + 3y^5 + 4y^4 + 2y^3 + 1$
$c_3$	$(y^3 - y^2 + 2y - 1)^2$
$c_7, c_{10}$	$y^6$
$c_9, c_{11}, c_{12}$	$(y - 1)^6$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.498832 + 1.001300I$		
$a = 0$	$1.37919 - 2.82812I$	$4.90478 + 3.87141I$
$b = 0.398606 + 0.800120I$		
$u = -0.498832 - 1.001300I$		
$a = 0$	$1.37919 + 2.82812I$	$4.90478 - 3.87141I$
$b = 0.398606 - 0.800120I$		
$u = 0.284920 + 1.115140I$		
$a = 0$	-2.75839	$0.235367 - 0.997558I$
$b = -0.215080 + 0.841795I$		
$u = 0.284920 - 1.115140I$		
$a = 0$	-2.75839	$0.235367 + 0.997558I$
$b = -0.215080 - 0.841795I$		
$u = 0.713912 + 0.305839I$		
$a = 0$	$1.37919 - 2.82812I$	$5.35985 + 0.59776I$
$b = -1.183530 + 0.507021I$		
$u = 0.713912 - 0.305839I$		
$a = 0$	$1.37919 + 2.82812I$	$5.35985 - 0.59776I$
$b = -1.183530 - 0.507021I$		

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u^4 - 2u^3 + 3u^2 - u + 1)(u^6 - 3u^5 + 4u^4 - 2u^3 + 1)$ $\cdot (u^{29} + 12u^{28} + \dots + u - 1)$
$c_2$	$(u^4 + u^2 + u + 1)(u^6 - u^5 + 2u^4 - 2u^3 + 2u^2 - 2u + 1)$ $\cdot (u^{29} + 2u^{28} + \dots + u - 1)$
$c_3$	$((u^3 - u^2 + 1)^2)(u^4 + 3u^3 + \dots + 3u + 2)(u^{29} - 2u^{28} + \dots + 120u - 36)$
$c_4$	$(u^4 + u^2 + u + 1)(u^6 - u^5 + 2u^4 - 2u^3 + 2u^2 - 2u + 1)$ $\cdot (u^{29} - 2u^{28} + \dots - u + 1)$
$c_5$	$(u^4 + u^2 - u + 1)(u^6 + u^5 + 2u^4 + 2u^3 + 2u^2 + 2u + 1)$ $\cdot (u^{29} + 2u^{28} + \dots + u - 1)$
$c_6$	$(u^4 - 2u^3 + 3u^2 - u + 1)(u^6 - 3u^5 + 4u^4 - 2u^3 + 1)$ $\cdot (u^{29} - 10u^{28} + \dots - 469083u + 52489)$
$c_7, c_{10}$	$u^{10}(u^{29} - 5u^{28} + \dots - 3072u - 1024)$
$c_8$	$(u^4 + u^2 - u + 1)(u^6 + u^5 + 2u^4 + 2u^3 + 2u^2 + 2u + 1)$ $\cdot (u^{29} - 2u^{28} + \dots - u + 1)$
$c_9$	$((u + 1)^{10})(u^{29} + 11u^{28} + \dots - 30u^2 - 1)$
$c_{11}, c_{12}$	$((u - 1)^{10})(u^{29} + 11u^{28} + \dots - 30u^2 - 1)$

## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$(y^4 + 2y^3 + 7y^2 + 5y + 1)(y^6 - y^5 + 4y^4 - 2y^3 + 8y^2 + 1)$ $\cdot (y^{29} + 12y^{28} + \dots + 85y - 1)$
$c_2, c_5$	$(y^4 + 2y^3 + 3y^2 + y + 1)(y^6 + 3y^5 + 4y^4 + 2y^3 + 1)$ $\cdot (y^{29} + 12y^{28} + \dots + y - 1)$
$c_3$	$(y^3 - y^2 + 2y - 1)^2(y^4 - y^3 + 2y^2 + 7y + 4)$ $\cdot (y^{29} + 12y^{28} + \dots - 12456y - 1296)$
$c_4, c_8$	$(y^4 + 2y^3 + 3y^2 + y + 1)(y^6 + 3y^5 + 4y^4 + 2y^3 + 1)$ $\cdot (y^{29} + 30y^{27} + \dots + y - 1)$
$c_6$	$(y^4 + 2y^3 + 7y^2 + 5y + 1)(y^6 - y^5 + 4y^4 - 2y^3 + 8y^2 + 1)$ $\cdot (y^{29} + 92y^{28} + \dots - 44517246691y - 2755095121)$
$c_7, c_{10}$	$y^{10}(y^{29} + 63y^{28} + \dots + 3670016y - 1048576)$
$c_9, c_{11}, c_{12}$	$((y - 1)^{10})(y^{29} - 51y^{28} + \dots - 60y - 1)$