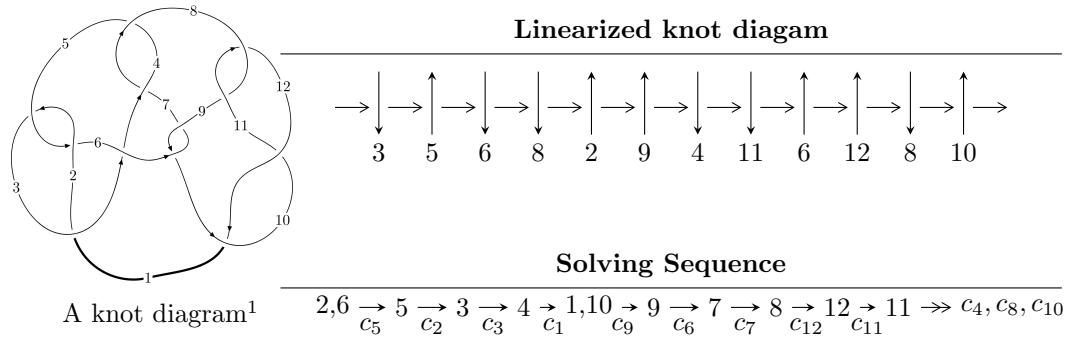


$12n_{0019}$  ( $K12n_{0019}$ )



### Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$I_1^u = \langle 222u^{17} + 1807u^{16} + \dots + 536b - 1177, -19u^{17} - 156u^{16} + \dots + 8a + 89, u^{18} + 8u^{17} + \dots - 8u + 1 \rangle$$

$$I_2^u = \langle b, -u^4a - 2u^3a + u^4 - 3u^2a - u^3 + a^2 - 2au - 2u^2 - a - 5u - 3, u^5 + u^4 + 2u^3 + u^2 + u + 1 \rangle$$

$$I_3^u = \langle -a^3u - a^3 - 3a^2 - au + 3b + 2a + u + 4, a^4 - a^3u + 3a^3 - a^2u + a^2 - 4a - u - 3, u^2 - u + 1 \rangle$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 36 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle 222u^{17} + 1807u^{16} + \cdots + 536b - 1177, -19u^{17} - 156u^{16} + \cdots + 8a + 89, u^{18} + 8u^{17} + \cdots - 8u + 1 \rangle$$

(i) **Arc colorings**

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^3 \\ u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^3 \\ u^5 + u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} \frac{19}{8}u^{17} + \frac{39}{2}u^{16} + \cdots + \frac{79}{2}u - \frac{89}{8} \\ -0.414179u^{17} - 3.37127u^{16} + \cdots - 6.08769u + 2.19590 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 2.78918u^{17} + 22.8713u^{16} + \cdots + 45.5877u - 13.3209 \\ -0.414179u^{17} - 3.37127u^{16} + \cdots - 6.08769u + 2.19590 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -1.65112u^{17} - 13.6642u^{16} + \cdots - 27.3918u + 7.50560 \\ 0.430970u^{17} + 3.58396u^{16} + \cdots + 6.21455u - 1.77985 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -1.69590u^{17} - 13.9813u^{16} + \cdots - 27.3134u + 7.47948 \\ 0.345149u^{17} + 2.83022u^{16} + \cdots + 4.67724u - 1.35075 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0.447761u^{17} + 3.79664u^{16} + \cdots + 7.84142u - 0.613806 \\ -0.345149u^{17} - 2.83022u^{16} + \cdots - 4.67724u + 1.35075 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1.43657u^{17} + 11.9049u^{16} + \cdots + 24.4235u - 6.30784 \\ -0.468284u^{17} - 3.88993u^{16} + \cdots - 6.77425u + 2.21642 \end{pmatrix}$$

(ii) **Obstruction class** = -1

(iii) **Cusp Shapes** =  $\frac{1867}{536}u^{17} + \frac{15631}{536}u^{16} + \cdots + \frac{28407}{536}u - \frac{3763}{268}$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^{18} + 2u^{17} + \cdots - 34u + 1$
$c_2, c_5$	$u^{18} + 8u^{17} + \cdots - 8u + 1$
$c_3$	$u^{18} - 8u^{17} + \cdots - 16496u + 1921$
$c_4, c_7$	$u^{18} + 2u^{17} + \cdots - 384u + 256$
$c_6, c_9$	$u^{18} + 2u^{17} + \cdots + 1024u^2 + 1024$
$c_8, c_{11}$	$u^{18} - 9u^{17} + \cdots + 5u + 1$
$c_{10}, c_{12}$	$u^{18} + u^{17} + \cdots + 7u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{18} + 34y^{17} + \cdots - 706y + 1$
$c_2, c_5$	$y^{18} + 2y^{17} + \cdots - 34y + 1$
$c_3$	$y^{18} + 42y^{17} + \cdots - 77040466y + 3690241$
$c_4, c_7$	$y^{18} + 30y^{17} + \cdots + 409600y + 65536$
$c_6, c_9$	$y^{18} + 50y^{17} + \cdots + 2097152y + 1048576$
$c_8, c_{11}$	$y^{18} - y^{17} + \cdots - 7y + 1$
$c_{10}, c_{12}$	$y^{18} + 47y^{17} + \cdots - 199y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.489678 + 0.809386I$		
$a = -3.46446 + 1.37116I$	$-0.02354 + 3.71255I$	$2.6622 - 33.9545I$
$b = -0.367948 - 0.217959I$		
$u = 0.489678 - 0.809386I$		
$a = -3.46446 - 1.37116I$	$-0.02354 - 3.71255I$	$2.6622 + 33.9545I$
$b = -0.367948 + 0.217959I$		
$u = -0.528473 + 1.113200I$		
$a = 0.884253 - 0.149684I$	$-6.98798 - 6.29888I$	$-7.63956 + 6.18005I$
$b = -0.293472 - 1.150100I$		
$u = -0.528473 - 1.113200I$		
$a = 0.884253 + 0.149684I$	$-6.98798 + 6.29888I$	$-7.63956 - 6.18005I$
$b = -0.293472 + 1.150100I$		
$u = 0.402685 + 0.640215I$		
$a = -0.600704 - 0.110262I$	$-0.176698 + 1.378410I$	$-2.62845 - 4.45652I$
$b = 0.079711 + 0.564353I$		
$u = 0.402685 - 0.640215I$		
$a = -0.600704 + 0.110262I$	$-0.176698 - 1.378410I$	$-2.62845 + 4.45652I$
$b = 0.079711 - 0.564353I$		
$u = 0.166779 + 0.714203I$		
$a = -0.576482 + 0.150196I$	$-0.194005 + 1.320020I$	$-1.40154 - 3.97468I$
$b = 0.406152 + 0.438776I$		
$u = 0.166779 - 0.714203I$		
$a = -0.576482 - 0.150196I$	$-0.194005 - 1.320020I$	$-1.40154 + 3.97468I$
$b = 0.406152 - 0.438776I$		
$u = -0.79804 + 1.31718I$		
$a = -1.30880 - 1.44991I$	$14.5520 - 13.1732I$	$-2.14093 + 5.47150I$
$b = -1.88686 + 2.04182I$		
$u = -0.79804 - 1.31718I$		
$a = -1.30880 + 1.44991I$	$14.5520 + 13.1732I$	$-2.14093 - 5.47150I$
$b = -1.88686 - 2.04182I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.48576 + 0.43889I$		
$a = -0.636632 - 0.933823I$	$17.4544 + 5.4859I$	$-0.93598 - 1.55559I$
$b = -2.70328 - 3.24263I$		
$u = -1.48576 - 0.43889I$		
$a = -0.636632 + 0.933823I$	$17.4544 - 5.4859I$	$-0.93598 + 1.55559I$
$b = -2.70328 + 3.24263I$		
$u = -1.39001 + 1.00947I$		
$a = -1.31713 + 0.69280I$	$-5.22275 + 0.41218I$	$-1.70669 + 0.I$
$b = -0.33733 + 5.04758I$		
$u = -1.39001 - 1.00947I$		
$a = -1.31713 - 0.69280I$	$-5.22275 - 0.41218I$	$-1.70669 + 0.I$
$b = -0.33733 - 5.04758I$		
$u = -1.06945 + 1.38280I$		
$a = 1.27493 + 1.39729I$	$11.82800 - 4.92111I$	$-2.64479 + 1.56009I$
$b = 3.58701 - 2.69224I$		
$u = -1.06945 - 1.38280I$		
$a = 1.27493 - 1.39729I$	$11.82800 + 4.92111I$	$-2.64479 - 1.56009I$
$b = 3.58701 + 2.69224I$		
$u = 0.212586 + 0.037327I$		
$a = -0.25498 + 2.94572I$	$0.024368 - 1.375910I$	$0.93572 + 4.18536I$
$b = 0.516021 - 0.465095I$		
$u = 0.212586 - 0.037327I$		
$a = -0.25498 - 2.94572I$	$0.024368 + 1.375910I$	$0.93572 - 4.18536I$
$b = 0.516021 + 0.465095I$		

$$\text{II. } I_2^u = \langle b, -u^4a + u^4 + \cdots - a - 3, u^5 + u^4 + 2u^3 + u^2 + u + 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_5 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u \\ u^3 + u \end{pmatrix} \\ a_4 &= \begin{pmatrix} -u^3 \\ u^3 + u \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^3 \\ -u^4 - u^3 - u^2 - 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} a \\ 0 \end{pmatrix} \\ a_9 &= \begin{pmatrix} a \\ 0 \end{pmatrix} \\ a_7 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -u^3 \\ u^4 + u^3 + u^2 + 1 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} -u^4 - u^3 - 3u^2 + a - 2u - 1 \\ -u^4 - u^3 - u^2 - 1 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} 2u^3a - u^4 - 2u^3 + au - 3u^2 + 2a - 2u - 1 \\ -2u^4a - 2u^3a - 2u^2a - au - 2a \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-u^4a - 3u^3a + u^4 - 4u^2a - 5u^3 - 6u^2 - 2a - 9u - 5$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$(u^5 - 3u^4 + 4u^3 - u^2 - u + 1)^2$
$c_2$	$(u^5 - u^4 + 2u^3 - u^2 + u - 1)^2$
$c_3, c_4$	$(u^5 + u^4 - 2u^3 - u^2 + u - 1)^2$
$c_5$	$(u^5 + u^4 + 2u^3 + u^2 + u + 1)^2$
$c_6, c_9$	$u^{10}$
$c_7$	$(u^5 - u^4 - 2u^3 + u^2 + u + 1)^2$
$c_8, c_{12}$	$(u^2 - u + 1)^5$
$c_{10}, c_{11}$	$(u^2 + u + 1)^5$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$(y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)^2$
$c_2, c_5$	$(y^5 + 3y^4 + 4y^3 + y^2 - y - 1)^2$
$c_3, c_4, c_7$	$(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2$
$c_6, c_9$	$y^{10}$
$c_8, c_{10}, c_{11}$ $c_{12}$	$(y^2 + y + 1)^5$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.339110 + 0.822375I$		
$a = 1.20942 + 2.19910I$	$-0.329100 - 0.499304I$	$2.94328 - 6.15174I$
$b = 0$		
$u = 0.339110 + 0.822375I$		
$a = -2.50919 - 0.05217I$	$-0.32910 + 3.56046I$	$-6.96704 - 8.14994I$
$b = 0$		
$u = 0.339110 - 0.822375I$		
$a = 1.20942 - 2.19910I$	$-0.329100 + 0.499304I$	$2.94328 + 6.15174I$
$b = 0$		
$u = 0.339110 - 0.822375I$		
$a = -2.50919 + 0.05217I$	$-0.32910 - 3.56046I$	$-6.96704 + 8.14994I$
$b = 0$		
$u = -0.766826$		
$a = 0.337181 + 0.584015I$	$-2.40108 + 2.02988I$	$-0.15429 - 1.95361I$
$b = 0$		
$u = -0.766826$		
$a = 0.337181 - 0.584015I$	$-2.40108 - 2.02988I$	$-0.15429 + 1.95361I$
$b = 0$		
$u = -0.455697 + 1.200150I$		
$a = 0.358089 + 0.327409I$	$-5.87256 - 2.37095I$	$-5.14480 + 4.03066I$
$b = 0$		
$u = -0.455697 + 1.200150I$		
$a = 0.104500 - 0.473819I$	$-5.87256 - 6.43072I$	$-0.67715 + 5.27500I$
$b = 0$		
$u = -0.455697 - 1.200150I$		
$a = 0.358089 - 0.327409I$	$-5.87256 + 2.37095I$	$-5.14480 - 4.03066I$
$b = 0$		
$u = -0.455697 - 1.200150I$		
$a = 0.104500 + 0.473819I$	$-5.87256 + 6.43072I$	$-0.67715 - 5.27500I$
$b = 0$		

$$\text{III. } I_3^u = \langle -a^3u - a^3 - 3a^2 - au + 3b + 2a + u + 4, a^4 - a^3u + 3a^3 - a^2u + a^2 - 4a - u - 3, u^2 - u + 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_5 &= \begin{pmatrix} 1 \\ u-1 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u \\ u-1 \end{pmatrix} \\ a_4 &= \begin{pmatrix} 1 \\ u-1 \end{pmatrix} \\ a_1 &= \begin{pmatrix} -1 \\ 0 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} a \\ \frac{1}{3}a^3u + \frac{1}{3}au + \dots - \frac{2}{3}a - \frac{4}{3} \end{pmatrix} \\ a_9 &= \begin{pmatrix} -\frac{1}{3}a^3u - \frac{1}{3}au + \dots + \frac{5}{3}a + \frac{4}{3} \\ \frac{1}{3}a^3u + \frac{1}{3}au + \dots - \frac{2}{3}a - \frac{4}{3} \end{pmatrix} \\ a_7 &= \begin{pmatrix} -\frac{1}{3}a^3u - \frac{4}{3}a^2u + \dots - a - \frac{4}{3} \\ \frac{2}{3}a^3u + \frac{2}{3}a^2u + \dots + a + \frac{5}{3} \end{pmatrix} \\ a_8 &= \begin{pmatrix} -\frac{1}{3}a^3u - \frac{4}{3}a^2u + \dots - a - \frac{4}{3} \\ \frac{2}{3}a^3u + \frac{2}{3}a^2u + \dots + a + \frac{5}{3} \end{pmatrix} \\ a_{12} &= \begin{pmatrix} \frac{1}{3}a^3u - \frac{2}{3}a^2u + \dots + \frac{4}{3}a^2 - \frac{5}{3} \\ \frac{2}{3}a^3u + \frac{2}{3}a^2u + \dots + a + \frac{5}{3} \end{pmatrix} \\ a_{11} &= \begin{pmatrix} \frac{4}{3}a^3u + \frac{4}{3}a^2u + \dots + \frac{1}{3}a^2 + \frac{1}{3} \\ -\frac{1}{3}a^3u - \frac{1}{3}a^2u + \dots + a + \frac{5}{3} \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = 1

$$(iii) \text{ Cusp Shapes} = -\frac{7}{3}a^3u + \frac{11}{3}a^3 - 5a^2u + 4a^2 + \frac{11}{3}au - \frac{25}{3}a + \frac{25}{3}u - \frac{44}{3}$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_3, c_5$	$(u^2 - u + 1)^4$
$c_2$	$(u^2 + u + 1)^4$
$c_4, c_7$	$u^8$
$c_6, c_{10}$	$(u^4 + u^3 + 3u^2 + 2u + 1)^2$
$c_8$	$(u^4 + u^3 + u^2 + 1)^2$
$c_9, c_{12}$	$(u^4 - u^3 + 3u^2 - 2u + 1)^2$
$c_{11}$	$(u^4 - u^3 + u^2 + 1)^2$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_5$	$(y^2 + y + 1)^4$
$c_4, c_7$	$y^8$
$c_6, c_9, c_{10}$ $c_{12}$	$(y^4 + 5y^3 + 7y^2 + 2y + 1)^2$
$c_8, c_{11}$	$(y^4 + y^3 + 3y^2 + 2y + 1)^2$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.500000 + 0.866025I$		
$a = -0.715307 - 0.631577I$	$0.211005 + 0.614778I$	$0.01166 + 7.13374I$
$b = -0.395123 + 0.506844I$		
$u = 0.500000 + 0.866025I$		
$a = 1.248740 + 0.225872I$	$-6.79074 - 1.13408I$	$-8.12668 + 3.09304I$
$b = -0.10488 + 1.55249I$		
$u = 0.500000 + 0.866025I$		
$a = -1.44025 - 0.04422I$	$-6.79074 + 5.19385I$	$-5.34148 - 0.51945I$
$b = -0.10488 - 1.55249I$		
$u = 0.500000 + 0.866025I$		
$a = -1.59319 + 1.31595I$	$0.21101 + 3.44499I$	$4.95650 - 5.37720I$
$b = -0.395123 - 0.506844I$		
$u = 0.500000 - 0.866025I$		
$a = -0.715307 + 0.631577I$	$0.211005 - 0.614778I$	$0.01166 - 7.13374I$
$b = -0.395123 - 0.506844I$		
$u = 0.500000 - 0.866025I$		
$a = 1.248740 - 0.225872I$	$-6.79074 + 1.13408I$	$-8.12668 - 3.09304I$
$b = -0.10488 - 1.55249I$		
$u = 0.500000 - 0.866025I$		
$a = -1.44025 + 0.04422I$	$-6.79074 - 5.19385I$	$-5.34148 + 0.51945I$
$b = -0.10488 + 1.55249I$		
$u = 0.500000 - 0.866025I$		
$a = -1.59319 - 1.31595I$	$0.21101 - 3.44499I$	$4.95650 + 5.37720I$
$b = -0.395123 + 0.506844I$		

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u^2 - u + 1)^4(u^5 - 3u^4 + 4u^3 - u^2 - u + 1)^2 \cdot (u^{18} + 2u^{17} + \dots - 34u + 1)$
$c_2$	$((u^2 + u + 1)^4)(u^5 - u^4 + \dots + u - 1)^2(u^{18} + 8u^{17} + \dots - 8u + 1)$
$c_3$	$(u^2 - u + 1)^4(u^5 + u^4 - 2u^3 - u^2 + u - 1)^2 \cdot (u^{18} - 8u^{17} + \dots - 16496u + 1921)$
$c_4$	$u^8(u^5 + u^4 + \dots + u - 1)^2(u^{18} + 2u^{17} + \dots - 384u + 256)$
$c_5$	$((u^2 - u + 1)^4)(u^5 + u^4 + \dots + u + 1)^2(u^{18} + 8u^{17} + \dots - 8u + 1)$
$c_6$	$u^{10}(u^4 + u^3 + 3u^2 + 2u + 1)^2(u^{18} + 2u^{17} + \dots + 1024u^2 + 1024)$
$c_7$	$u^8(u^5 - u^4 + \dots + u + 1)^2(u^{18} + 2u^{17} + \dots - 384u + 256)$
$c_8$	$((u^2 - u + 1)^5)(u^4 + u^3 + u^2 + 1)^2(u^{18} - 9u^{17} + \dots + 5u + 1)$
$c_9$	$u^{10}(u^4 - u^3 + 3u^2 - 2u + 1)^2(u^{18} + 2u^{17} + \dots + 1024u^2 + 1024)$
$c_{10}$	$((u^2 + u + 1)^5)(u^4 + u^3 + 3u^2 + 2u + 1)^2(u^{18} + u^{17} + \dots + 7u + 1)$
$c_{11}$	$((u^2 + u + 1)^5)(u^4 - u^3 + u^2 + 1)^2(u^{18} - 9u^{17} + \dots + 5u + 1)$
$c_{12}$	$((u^2 - u + 1)^5)(u^4 - u^3 + 3u^2 - 2u + 1)^2(u^{18} + u^{17} + \dots + 7u + 1)$

## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$(y^2 + y + 1)^4(y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)^2 \cdot (y^{18} + 34y^{17} + \dots - 706y + 1)$
$c_2, c_5$	$(y^2 + y + 1)^4(y^5 + 3y^4 + 4y^3 + y^2 - y - 1)^2 \cdot (y^{18} + 2y^{17} + \dots - 34y + 1)$
$c_3$	$(y^2 + y + 1)^4(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2 \cdot (y^{18} + 42y^{17} + \dots - 77040466y + 3690241)$
$c_4, c_7$	$y^8(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2 \cdot (y^{18} + 30y^{17} + \dots + 409600y + 65536)$
$c_6, c_9$	$y^{10}(y^4 + 5y^3 + 7y^2 + 2y + 1)^2 \cdot (y^{18} + 50y^{17} + \dots + 2097152y + 1048576)$
$c_8, c_{11}$	$((y^2 + y + 1)^5)(y^4 + y^3 + 3y^2 + 2y + 1)^2(y^{18} - y^{17} + \dots - 7y + 1)$
$c_{10}, c_{12}$	$(y^2 + y + 1)^5(y^4 + 5y^3 + 7y^2 + 2y + 1)^2 \cdot (y^{18} + 47y^{17} + \dots - 199y + 1)$