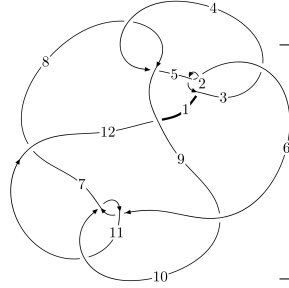
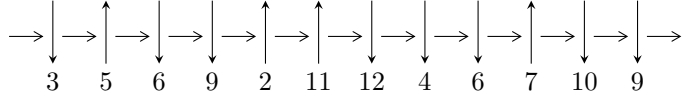


12n<sub>0037</sub> (K12n<sub>0037</sub>)



A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**

$$6, 11 \xrightarrow{c_6} 2, 7 \xrightarrow{c_5} 5 \xrightarrow{c_2} 3 \xrightarrow{c_1} 1 \xrightarrow{c_{10}} 10 \xrightarrow{c_{11}} 12 \xrightarrow{c_7} 8 \xrightarrow{c_9} 9 \xrightarrow{c_4} 4 \twoheadrightarrow c_3, c_8, c_{12}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle u^{28} - 2u^{27} + \dots + 2b - 3u, 2u^{28} - 4u^{27} + \dots + 2a - 2, u^{30} - 3u^{29} + \dots + 4u - 1 \rangle$$

$$I_2^u = \langle -2u^4a - 4u^3a - 2u^4 + 3u^2a - 4u^3 - 8au + 3u^2 + 19b - 7a - 8u - 7, \\ u^3a - u^2a - 2u^3 + a^2 + au + 2u^2 - u + 2, u^5 - u^4 + 2u^3 - u^2 + u - 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 40 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle u^{28} - 2u^{27} + \dots + 2b - 3u, 2u^{28} - 4u^{27} + \dots + 2a - 2, u^{30} - 3u^{29} + \dots + 4u - 1 \rangle \quad \text{I.}$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u^{28} + 2u^{27} + \dots + \frac{3}{2}u + 1 \\ -\frac{1}{2}u^{28} + u^{27} + \dots + \frac{5}{2}u^2 + \frac{3}{2}u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -\frac{1}{2}u^{28} + 2u^{27} + \dots + \frac{3}{2}u^3 + 4u \\ -\frac{1}{2}u^{28} + 2u^{27} + \dots + \frac{17}{2}u^2 - \frac{3}{2}u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -3u^{28} + 6u^{27} + \dots + \frac{17}{2}u^2 + \frac{5}{2}u \\ -\frac{3}{2}u^{28} + 3u^{27} + \dots + \frac{17}{2}u^2 - \frac{3}{2}u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^{11} - 2u^9 - 2u^7 - u^3 \\ -u^{11} - 3u^9 - 4u^7 - u^5 + u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^3 \\ u^5 + u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^6 - u^4 + 1 \\ u^8 + 2u^6 + 2u^4 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^3 \\ u^3 + u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -\frac{3}{2}u^{28} + 3u^{27} + \dots + \frac{3}{2}u^3 + 4u \\ -\frac{3}{2}u^{28} + 3u^{27} + \dots + \frac{17}{2}u^2 - \frac{3}{2}u \end{pmatrix}$$

(ii) Obstruction class = -1

$$(iii) \text{ Cusp Shapes} = 4u^{29} - \frac{25}{2}u^{28} + \dots - \frac{59}{2}u + \frac{7}{2}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{30} + 22u^{29} + \dots + 23u + 1$
$c_2, c_5$	$u^{30} + 6u^{29} + \dots + 5u + 1$
$c_3$	$u^{30} - 6u^{29} + \dots + 5u + 1$
$c_4, c_8$	$u^{30} - u^{29} + \dots - 2048u - 1024$
$c_6, c_{10}$	$u^{30} - 3u^{29} + \dots + 4u - 1$
$c_7, c_9$	$u^{30} + 3u^{29} + \dots + 2u - 1$
$c_{11}$	$u^{30} + 19u^{29} + \dots + 8u + 1$
$c_{12}$	$u^{30} - 13u^{29} + \dots - 29592u + 1669$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{30} - 22y^{29} + \dots - 241y + 1$
$c_2, c_5$	$y^{30} + 22y^{29} + \dots + 23y + 1$
$c_3$	$y^{30} - 66y^{29} + \dots + 199y + 1$
$c_4, c_8$	$y^{30} - 55y^{29} + \dots - 4194304y + 1048576$
$c_6, c_{10}$	$y^{30} + 19y^{29} + \dots + 8y + 1$
$c_7, c_9$	$y^{30} - 45y^{29} + \dots + 8y + 1$
$c_{11}$	$y^{30} - 13y^{29} + \dots + 36y + 1$
$c_{12}$	$y^{30} - 145y^{29} + \dots - 1268048336y + 2785561$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.976696 + 0.048976I$ $a = 0.12463 + 2.08766I$ $b = -0.56420 + 1.41173I$	$-16.9810 - 6.1121I$	$-7.59589 + 2.67108I$
$u = 0.976696 - 0.048976I$ $a = 0.12463 - 2.08766I$ $b = -0.56420 - 1.41173I$	$-16.9810 + 6.1121I$	$-7.59589 - 2.67108I$
$u = 0.465293 + 0.912238I$ $a = 1.93554 - 1.46111I$ $b = 0.008467 - 0.943478I$	$-1.62988 + 2.08354I$	$-8.16060 - 3.45084I$
$u = 0.465293 - 0.912238I$ $a = 1.93554 + 1.46111I$ $b = 0.008467 + 0.943478I$	$-1.62988 - 2.08354I$	$-8.16060 + 3.45084I$
$u = 0.960868$ $a = -0.707740$ $b = -1.15384$	$-12.5512$	$-5.29830$
$u = -0.166593 + 0.933326I$ $a = -0.92254 - 2.23571I$ $b = 0.617718 - 0.920078I$	$-1.07529 - 3.17191I$	$-11.28317 + 2.37568I$
$u = -0.166593 - 0.933326I$ $a = -0.92254 + 2.23571I$ $b = 0.617718 + 0.920078I$	$-1.07529 + 3.17191I$	$-11.28317 - 2.37568I$
$u = 0.231283 + 1.116780I$ $a = -0.01646 + 3.64757I$ $b = 0.357122 + 1.153660I$	$-3.77750 + 3.57782I$	$-10.98746 - 4.01828I$
$u = 0.231283 - 1.116780I$ $a = -0.01646 - 3.64757I$ $b = 0.357122 - 1.153660I$	$-3.77750 - 3.57782I$	$-10.98746 + 4.01828I$
$u = -0.828254 + 0.182083I$ $a = 0.61725 - 1.74485I$ $b = -0.176405 - 1.246890I$	$-5.31113 + 2.21238I$	$-8.26070 - 1.34538I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.828254 - 0.182083I$ $a = 0.61725 + 1.74485I$ $b = -0.176405 + 1.246890I$	$-5.31113 - 2.21238I$	$-8.26070 + 1.34538I$
$u = 0.247708 + 0.775830I$ $a = 0.759182 + 0.262675I$ $b = 0.064907 + 0.268421I$	$-0.445314 + 1.227680I$	$-5.13147 - 5.35598I$
$u = 0.247708 - 0.775830I$ $a = 0.759182 - 0.262675I$ $b = 0.064907 - 0.268421I$	$-0.445314 - 1.227680I$	$-5.13147 + 5.35598I$
$u = -0.431100 + 1.154270I$ $a = -0.376542 + 0.218469I$ $b = -0.568898 - 0.159015I$	$-4.80479 - 4.01525I$	$-8.38777 + 4.38030I$
$u = -0.431100 - 1.154270I$ $a = -0.376542 - 0.218469I$ $b = -0.568898 + 0.159015I$	$-4.80479 + 4.01525I$	$-8.38777 - 4.38030I$
$u = -0.538187 + 1.166640I$ $a = 1.43637 + 2.76799I$ $b = -0.271108 + 1.248230I$	$-8.20594 - 7.19126I$	$-10.97609 + 5.35204I$
$u = -0.538187 - 1.166640I$ $a = 1.43637 - 2.76799I$ $b = -0.271108 - 1.248230I$	$-8.20594 + 7.19126I$	$-10.97609 - 5.35204I$
$u = -0.033664 + 0.692526I$ $a = 1.161840 - 0.009465I$ $b = 0.453127 + 0.691055I$	$-0.304353 + 1.377760I$	$-5.31625 - 4.96434I$
$u = -0.033664 - 0.692526I$ $a = 1.161840 + 0.009465I$ $b = 0.453127 - 0.691055I$	$-0.304353 - 1.377760I$	$-5.31625 + 4.96434I$
$u = -0.343267 + 1.272440I$ $a = -0.45988 - 3.28730I$ $b = -0.116378 - 1.358180I$	$-9.80427 - 1.78849I$	$-12.19268 + 1.56602I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.343267 - 1.272440I$ $a = -0.45988 + 3.28730I$ $b = -0.116378 + 1.358180I$	$-9.80427 + 1.78849I$	$-12.19268 - 1.56602I$
$u = -0.662170$ $a = 0.443818$ $b = -0.426253$	$-1.60545$	$-5.56060$
$u = 0.486497 + 1.299810I$ $a = -1.68061 - 0.72887I$ $b = -1.177550 + 0.042833I$	$-16.5686 + 5.1451I$	$-8.39302 - 2.77801I$
$u = 0.486497 - 1.299810I$ $a = -1.68061 + 0.72887I$ $b = -1.177550 - 0.042833I$	$-16.5686 - 5.1451I$	$-8.39302 + 2.77801I$
$u = 0.517727 + 1.292950I$ $a = 0.64113 - 3.50293I$ $b = -0.59774 - 1.40210I$	$18.6656 + 11.4427I$	$-10.35634 - 5.55493I$
$u = 0.517727 - 1.292950I$ $a = 0.64113 + 3.50293I$ $b = -0.59774 + 1.40210I$	$18.6656 - 11.4427I$	$-10.35634 + 5.55493I$
$u = 0.459310 + 1.324670I$ $a = -1.26343 + 2.96678I$ $b = -0.55077 + 1.45009I$	$18.1990 - 1.0261I$	$-10.88309 + 0.I$
$u = 0.459310 - 1.324670I$ $a = -1.26343 - 2.96678I$ $b = -0.55077 - 1.45009I$	$18.1990 + 1.0261I$	$-10.88309 + 0.I$
$u = 0.307204 + 0.297461I$ $a = 1.17548 + 0.93277I$ $b = 0.311753 + 0.846408I$	$-0.35666 + 1.51654I$	$-2.64602 - 3.80074I$
$u = 0.307204 - 0.297461I$ $a = 1.17548 - 0.93277I$ $b = 0.311753 - 0.846408I$	$-0.35666 - 1.51654I$	$-2.64602 + 3.80074I$

$$\text{II. } I_2^u = \langle -2u^4a - 2u^4 + \dots - 7a - 7, u^3a - u^2a - 2u^3 + a^2 + au + 2u^2 - u + 2, u^5 - u^4 + 2u^3 - u^2 + u - 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} a \\ 0.105263au^4 + 0.105263u^4 + \dots + 0.368421a + 0.368421 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -0.105263au^4 - 0.105263u^4 + \dots + 0.631579a - 0.368421 \\ 0.105263au^4 + 0.105263u^4 + \dots + 0.368421a - 0.631579 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^3 - u^2 + a + u - 1 \\ 0.105263au^4 + 0.105263u^4 + \dots + 0.368421a - 0.631579 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^3 \\ u^4 - u^3 + u^2 + 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^3 \\ u^3 + u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^3 \\ u^3 + u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -0.105263au^4 - 0.105263u^4 + \dots + 0.631579a - 0.368421 \\ 0.105263au^4 + 0.105263u^4 + \dots + 0.368421a - 0.631579 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes

$$= \frac{3}{19}u^4a - \frac{13}{19}u^3a - \frac{16}{19}u^4 + \frac{5}{19}u^2a + \frac{82}{19}u^3 - \frac{7}{19}au - \frac{90}{19}u^2 - \frac{37}{19}a + \frac{69}{19}u - \frac{170}{19}$$



(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_3, c_5$	$(u^2 - u + 1)^5$
$c_2$	$(u^2 + u + 1)^5$
$c_4, c_8$	$u^{10}$
$c_6$	$(u^5 - u^4 + 2u^3 - u^2 + u - 1)^2$
$c_7$	$(u^5 + u^4 - 2u^3 - u^2 + u - 1)^2$
$c_9, c_{12}$	$(u^5 - u^4 - 2u^3 + u^2 + u + 1)^2$
$c_{10}$	$(u^5 + u^4 + 2u^3 + u^2 + u + 1)^2$
$c_{11}$	$(u^5 + 3u^4 + 4u^3 + u^2 - u - 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_5$	$(y^2 + y + 1)^5$
$c_4, c_8$	$y^{10}$
$c_6, c_{10}$	$(y^5 + 3y^4 + 4y^3 + y^2 - y - 1)^2$
$c_7, c_9, c_{12}$	$(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2$
$c_{11}$	$(y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.339110 + 0.822375I$ $a = 0.523653 + 0.423720I$ $b = 0.500000 + 0.866025I$	$-0.329100 + 0.499304I$	$-5.91654 + 2.81652I$
$u = -0.339110 + 0.822375I$ $a = -1.39487 - 1.53138I$ $b = 0.500000 - 0.866025I$	$-0.32910 - 3.56046I$	$-1.60756 + 7.85087I$
$u = -0.339110 - 0.822375I$ $a = 0.523653 - 0.423720I$ $b = 0.500000 - 0.866025I$	$-0.329100 - 0.499304I$	$-5.91654 - 2.81652I$
$u = -0.339110 - 0.822375I$ $a = -1.39487 + 1.53138I$ $b = 0.500000 + 0.866025I$	$-0.32910 + 3.56046I$	$-1.60756 - 7.85087I$
$u = 0.766826$ $a = -0.314857 + 1.186700I$ $b = 0.500000 + 0.866025I$	$-2.40108 + 2.02988I$	$-6.55976 - 2.76390I$
$u = 0.766826$ $a = -0.314857 - 1.186700I$ $b = 0.500000 - 0.866025I$	$-2.40108 - 2.02988I$	$-6.55976 + 2.76390I$
$u = 0.455697 + 1.200150I$ $a = 0.85051 - 1.45588I$ $b = 0.500000 - 0.866025I$	$-5.87256 + 2.37095I$	$-10.62344 - 1.09779I$
$u = 0.455697 + 1.200150I$ $a = -0.66443 + 2.33052I$ $b = 0.500000 + 0.866025I$	$-5.87256 + 6.43072I$	$-9.29269 - 5.42389I$
$u = 0.455697 - 1.200150I$ $a = 0.85051 + 1.45588I$ $b = 0.500000 + 0.866025I$	$-5.87256 - 2.37095I$	$-10.62344 + 1.09779I$
$u = 0.455697 - 1.200150I$ $a = -0.66443 - 2.33052I$ $b = 0.500000 - 0.866025I$	$-5.87256 - 6.43072I$	$-9.29269 + 5.42389I$

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u^2 - u + 1)^5)(u^{30} + 22u^{29} + \dots + 23u + 1)$
$c_2$	$((u^2 + u + 1)^5)(u^{30} + 6u^{29} + \dots + 5u + 1)$
$c_3$	$((u^2 - u + 1)^5)(u^{30} - 6u^{29} + \dots + 5u + 1)$
$c_4, c_8$	$u^{10}(u^{30} - u^{29} + \dots - 2048u - 1024)$
$c_5$	$((u^2 - u + 1)^5)(u^{30} + 6u^{29} + \dots + 5u + 1)$
$c_6$	$((u^5 - u^4 + 2u^3 - u^2 + u - 1)^2)(u^{30} - 3u^{29} + \dots + 4u - 1)$
$c_7$	$((u^5 + u^4 - 2u^3 - u^2 + u - 1)^2)(u^{30} + 3u^{29} + \dots + 2u - 1)$
$c_9$	$((u^5 - u^4 - 2u^3 + u^2 + u + 1)^2)(u^{30} + 3u^{29} + \dots + 2u - 1)$
$c_{10}$	$((u^5 + u^4 + 2u^3 + u^2 + u + 1)^2)(u^{30} - 3u^{29} + \dots + 4u - 1)$
$c_{11}$	$((u^5 + 3u^4 + 4u^3 + u^2 - u - 1)^2)(u^{30} + 19u^{29} + \dots + 8u + 1)$
$c_{12}$	$((u^5 - u^4 - 2u^3 + u^2 + u + 1)^2)(u^{30} - 13u^{29} + \dots - 29592u + 1669)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y^2 + y + 1)^5)(y^{30} - 22y^{29} + \dots - 241y + 1)$
$c_2, c_5$	$((y^2 + y + 1)^5)(y^{30} + 22y^{29} + \dots + 23y + 1)$
$c_3$	$((y^2 + y + 1)^5)(y^{30} - 66y^{29} + \dots + 199y + 1)$
$c_4, c_8$	$y^{10}(y^{30} - 55y^{29} + \dots - 4194304y + 1048576)$
$c_6, c_{10}$	$((y^5 + 3y^4 + 4y^3 + y^2 - y - 1)^2)(y^{30} + 19y^{29} + \dots + 8y + 1)$
$c_7, c_9$	$((y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2)(y^{30} - 45y^{29} + \dots + 8y + 1)$
$c_{11}$	$((y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)^2)(y^{30} - 13y^{29} + \dots + 36y + 1)$
$c_{12}$	$(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2$ $\cdot (y^{30} - 145y^{29} + \dots - 1268048336y + 2785561)$