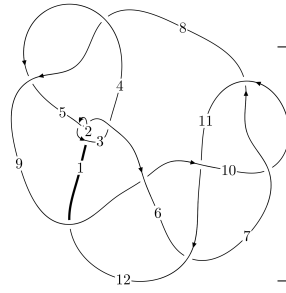
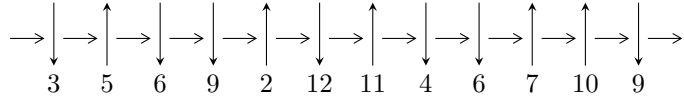


12n<sub>0040</sub> (K12n<sub>0040</sub>)



A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**

$$2,6 \xrightarrow{c_5} 5 \xrightarrow{c_2} 3 \xrightarrow{c_1} 1,10 \xrightarrow{c_9} 9 \xrightarrow{c_4} 4 \xrightarrow{c_{12}} 12 \xrightarrow{c_6} 7 \xrightarrow{c_{11}} 11 \xrightarrow{c_7} 8 \twoheadrightarrow c_3, c_8, c_{10}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle 34u^{42} + 241u^{41} + \dots + 32b + 95, 27u^{42} + 149u^{41} + \dots + 32a + 121, u^{43} + 7u^{42} + \dots + 8u + 1 \rangle$$

$$I_2^u = \langle -au + 3b + 2a, a^6 - a^5u - a^5 - 3a^4u + 12a^3u - 6a^3 - 9au + 18a - 27, u^2 - u + 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 55 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle 34u^{42} + 241u^{41} + \dots + 32b + 95, 27u^{42} + 149u^{41} + \dots + 32a + 121, u^{43} + 7u^{42} + \dots + 8u + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^3 \\ u^5 + u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -0.843750u^{42} - 4.65625u^{41} + \dots - 25.8125u - 3.78125 \\ -1.06250u^{42} - 7.53125u^{41} + \dots - 14.0313u - 2.96875 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1.90625u^{42} - 12.1875u^{41} + \dots - 39.8438u - 6.75000 \\ -1.06250u^{42} - 7.53125u^{41} + \dots - 14.0313u - 2.96875 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^3 \\ u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0.0312500u^{41} + 0.187500u^{40} + \dots + 0.218750u - 0.968750 \\ -\frac{1}{32}u^{42} - \frac{7}{32}u^{41} + \dots + \frac{7}{4}u - \frac{1}{32} \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -0.156250u^{42} - 0.687500u^{41} + \dots + 0.0312500u + 1.31250 \\ -0.281250u^{42} - 2.09375u^{41} + \dots - 3.18750u - 0.406250 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -0.125000u^{42} - 1.53125u^{41} + \dots - 3.09375u - 0.843750 \\ 0.281250u^{42} + 2.15625u^{41} + \dots + 4.62500u + 0.468750 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1.28125u^{42} + 9.50000u^{41} + \dots + 47.5313u + 7.81250 \\ 0.875000u^{42} + 5.59375u^{41} + \dots + 9.15625u + 2.03125 \end{pmatrix}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = -\frac{21}{4}u^{42} - \frac{619}{16}u^{41} + \dots - \frac{887}{8}u - \frac{183}{8}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{43} + 29u^{42} + \dots - 6u - 1$
$c_2, c_5$	$u^{43} + 7u^{42} + \dots + 8u + 1$
$c_3$	$u^{43} - 7u^{42} + \dots - 15u + 2$
$c_4, c_8$	$u^{43} - u^{42} + \dots + 4096u + 4096$
$c_6$	$u^{43} - 9u^{42} + \dots + 301u - 32$
$c_7, c_{10}$	$u^{43} - 3u^{42} + \dots - 4u + 1$
$c_9$	$u^{43} + 3u^{42} + \dots + u^2 + 1$
$c_{11}$	$u^{43} - 19u^{42} + \dots - 2u - 1$
$c_{12}$	$u^{43} - 13u^{42} + \dots - 396502u - 27289$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{43} - 23y^{42} + \dots - 70y - 1$
$c_2, c_5$	$y^{43} + 29y^{42} + \dots - 6y - 1$
$c_3$	$y^{43} - 75y^{42} + \dots + 109y - 4$
$c_4, c_8$	$y^{43} - 65y^{42} + \dots + 150994944y - 16777216$
$c_6$	$y^{43} - 7y^{42} + \dots + 2985y - 1024$
$c_7, c_{10}$	$y^{43} - 19y^{42} + \dots - 2y - 1$
$c_9$	$y^{43} - 59y^{42} + \dots - 2y - 1$
$c_{11}$	$y^{43} + 13y^{42} + \dots - 54y - 1$
$c_{12}$	$y^{43} - 119y^{42} + \dots - 1101094234046y - 744689521$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.108850 + 1.019930I$ $a = -2.54446 - 0.82403I$ $b = 0.887627 + 0.888202I$	$0.432070 - 0.070776I$	$-2.00000 + 0.I$
$u = -0.108850 - 1.019930I$ $a = -2.54446 + 0.82403I$ $b = 0.887627 - 0.888202I$	$0.432070 + 0.070776I$	$-2.00000 + 0.I$
$u = -1.03310$ $a = 1.81625$ $b = -1.75986$	$-5.16147$	$0.303180$
$u = 0.631242 + 0.697176I$ $a = -0.026925 + 1.068100I$ $b = 0.114579 - 0.360320I$	$1.44756 + 4.15054I$	$-1.66670 - 7.98021I$
$u = 0.631242 - 0.697176I$ $a = -0.026925 - 1.068100I$ $b = 0.114579 + 0.360320I$	$1.44756 - 4.15054I$	$-1.66670 + 7.98021I$
$u = 0.630221 + 0.873340I$ $a = 0.381127 - 0.768103I$ $b = -0.219981 + 0.236881I$	$0.968100 + 0.793721I$	$0$
$u = 0.630221 - 0.873340I$ $a = 0.381127 + 0.768103I$ $b = -0.219981 - 0.236881I$	$0.968100 - 0.793721I$	$0$
$u = -1.093460 + 0.077172I$ $a = 1.96976 - 0.17734I$ $b = -1.81042 + 0.05602I$	$-9.35130 + 7.40339I$	$0. - 4.50100I$
$u = -1.093460 - 0.077172I$ $a = 1.96976 + 0.17734I$ $b = -1.81042 - 0.05602I$	$-9.35130 - 7.40339I$	$0. + 4.50100I$
$u = 0.356528 + 0.829227I$ $a = -0.809224 - 0.565549I$ $b = 0.200199 + 0.315601I$	$-0.32126 + 1.54787I$	$-2.12659 - 4.62507I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.356528 - 0.829227I$ $a = -0.809224 + 0.565549I$ $b = 0.200199 - 0.315601I$	$-0.32126 - 1.54787I$	$-2.12659 + 4.62507I$
$u = -1.097840 + 0.043281I$ $a = -1.97365 + 0.09899I$ $b = 1.81083 - 0.03124I$	$-11.14880 + 1.85775I$	0
$u = -1.097840 - 0.043281I$ $a = -1.97365 - 0.09899I$ $b = 1.81083 + 0.03124I$	$-11.14880 - 1.85775I$	0
$u = -0.231681 + 1.107840I$ $a = -3.17472 - 0.52343I$ $b = 1.31188 + 0.91141I$	$-1.74448 - 7.62411I$	0
$u = -0.231681 - 1.107840I$ $a = -3.17472 + 0.52343I$ $b = 1.31188 - 0.91141I$	$-1.74448 + 7.62411I$	0
$u = 0.441020 + 1.047300I$ $a = -1.087770 + 0.250841I$ $b = 0.447924 + 0.048017I$	$-1.10611 + 1.42382I$	0
$u = 0.441020 - 1.047300I$ $a = -1.087770 - 0.250841I$ $b = 0.447924 - 0.048017I$	$-1.10611 - 1.42382I$	0
$u = -0.173823 + 1.139560I$ $a = 2.91930 + 0.38726I$ $b = -1.21169 - 0.76310I$	$-4.07750 - 2.66673I$	0
$u = -0.173823 - 1.139560I$ $a = 2.91930 - 0.38726I$ $b = -1.21169 + 0.76310I$	$-4.07750 + 2.66673I$	0
$u = 0.555429 + 1.053250I$ $a = 0.937490 - 0.560097I$ $b = -0.423819 + 0.102545I$	$-0.20886 + 5.67924I$	0

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.555429 - 1.053250I$ $a = 0.937490 + 0.560097I$ $b = -0.423819 - 0.102545I$	$-0.20886 - 5.67924I$	0
$u = -0.013036 + 1.234440I$ $a = 2.43278 + 0.00691I$ $b = -1.046990 - 0.429320I$	$-5.51855 - 0.23082I$	0
$u = -0.013036 - 1.234440I$ $a = 2.43278 - 0.00691I$ $b = -1.046990 + 0.429320I$	$-5.51855 + 0.23082I$	0
$u = 0.067968 + 1.257240I$ $a = -2.24159 + 0.11300I$ $b = 0.972223 + 0.322115I$	$-4.39423 + 4.95924I$	0
$u = 0.067968 - 1.257240I$ $a = -2.24159 - 0.11300I$ $b = 0.972223 - 0.322115I$	$-4.39423 - 4.95924I$	0
$u = -0.51561 + 1.34254I$ $a = 3.53483 - 0.69986I$ $b = -1.90537 - 0.30670I$	$-9.34262 - 5.50773I$	0
$u = -0.51561 - 1.34254I$ $a = 3.53483 + 0.69986I$ $b = -1.90537 + 0.30670I$	$-9.34262 + 5.50773I$	0
$u = -0.57404 + 1.34689I$ $a = 3.56336 - 0.81250I$ $b = -1.96829 - 0.24490I$	$-13.3037 - 13.3388I$	0
$u = -0.57404 - 1.34689I$ $a = 3.56336 + 0.81250I$ $b = -1.96829 + 0.24490I$	$-13.3037 + 13.3388I$	0
$u = -0.029590 + 0.530131I$ $a = 1.15627 + 1.68061I$ $b = 0.135978 - 0.751914I$	$1.74801 - 1.44634I$	$0.320350 + 0.404730I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.029590 - 0.530131I$ $a = 1.15627 - 1.68061I$ $b = 0.135978 + 0.751914I$	$1.74801 + 1.44634I$	$0.320350 - 0.404730I$
$u = -0.55869 + 1.36236I$ $a = -3.52852 + 0.79270I$ $b = 1.93730 + 0.24369I$	$-15.2690 - 7.7521I$	0
$u = -0.55869 - 1.36236I$ $a = -3.52852 - 0.79270I$ $b = 1.93730 - 0.24369I$	$-15.2690 + 7.7521I$	0
$u = -0.50713 + 1.39889I$ $a = -3.43056 + 0.73543I$ $b = 1.84983 + 0.24468I$	$-15.7130 - 3.8589I$	0
$u = -0.50713 - 1.39889I$ $a = -3.43056 - 0.73543I$ $b = 1.84983 - 0.24468I$	$-15.7130 + 3.8589I$	0
$u = -0.48108 + 1.40912I$ $a = 3.38956 - 0.70682I$ $b = -1.81208 - 0.24952I$	$-14.0896 + 1.7953I$	0
$u = -0.48108 - 1.40912I$ $a = 3.38956 + 0.70682I$ $b = -1.81208 + 0.24952I$	$-14.0896 - 1.7953I$	0
$u = 0.395813 + 0.305739I$ $a = 0.39533 + 1.71062I$ $b = 0.047086 - 0.461449I$	$1.64146 - 1.44258I$	$1.62393 + 1.37217I$
$u = 0.395813 - 0.305739I$ $a = 0.39533 - 1.71062I$ $b = 0.047086 + 0.461449I$	$1.64146 + 1.44258I$	$1.62393 - 1.37217I$
$u = -0.355224 + 0.327355I$ $a = 0.46521 + 1.45996I$ $b = 0.831377 - 0.614700I$	$0.49380 + 5.07125I$	$-1.64078 - 7.14289I$



Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.355224 - 0.327355I$		
$a = 0.46521 - 1.45996I$	$0.49380 - 5.07125I$	$-1.64078 + 7.14289I$
$b = 0.831377 + 0.614700I$		
$u = -0.321614 + 0.162353I$		
$a = -0.735715 - 0.820169I$	$-1.36971 + 0.61078I$	$-6.09478 - 1.69888I$
$b = -0.768263 + 0.294664I$		
$u = -0.321614 - 0.162353I$		
$a = -0.735715 + 0.820169I$	$-1.36971 - 0.61078I$	$-6.09478 + 1.69888I$
$b = -0.768263 - 0.294664I$		

$$\text{II. } I_2^u = \langle -au + 3b + 2a, -a^5u - 3a^4u + \cdots + 18a - 27, u^2 - u + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ u - 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u \\ u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} a \\ \frac{1}{3}au - \frac{2}{3}a \end{pmatrix}$$

$$a_9 = \begin{pmatrix} \frac{1}{3}au + \frac{1}{3}a \\ \frac{1}{3}au - \frac{2}{3}a \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ u - 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -\frac{1}{3}a^2 - 1 \\ -\frac{1}{3}a^2u + \frac{1}{3}a^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} \frac{1}{9}a^4u + \frac{1}{3}a^2u + \cdots - \frac{1}{3}a^2 + 1 \\ -\frac{1}{9}a^4u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} \frac{2}{9}a^4u - \frac{1}{3}a^2u + \cdots + \frac{1}{3}a^2 - 1 \\ -\frac{1}{9}a^4u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} \frac{1}{3}au + \frac{1}{3}a \\ \frac{1}{3}au - \frac{2}{3}a \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes

$$= -\frac{2}{27}a^5u + \frac{1}{27}a^5 - \frac{4}{9}a^4u + \frac{2}{9}a^3u - \frac{4}{9}a^3 + \frac{5}{3}a^2u - a^2 - \frac{2}{3}au + \frac{10}{3}a - 5u + 2$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_3, c_5$	$(u^2 - u + 1)^6$
$c_2$	$(u^2 + u + 1)^6$
$c_4, c_8$	$u^{12}$
$c_6, c_{11}$	$(u^6 - 3u^5 + 5u^4 - 4u^3 + 2u^2 - u + 1)^2$
$c_7, c_9, c_{12}$	$(u^6 - u^5 - u^4 + 2u^3 - u + 1)^2$
$c_{10}$	$(u^6 + u^5 - u^4 - 2u^3 + u + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_5$	$(y^2 + y + 1)^6$
$c_4, c_8$	$y^{12}$
$c_6, c_{11}$	$(y^6 + y^5 + 5y^4 + 6y^2 + 3y + 1)^2$
$c_7, c_9, c_{10}$ $c_{12}$	$(y^6 - 3y^5 + 5y^4 - 4y^3 + 2y^2 - y + 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.500000 + 0.866025I$ $a = 0.066864 + 1.367670I$ $b = -0.428243 - 0.664531I$	$1.89061 + 1.10558I$	$3.93112 - 2.76498I$
$u = 0.500000 + 0.866025I$ $a = 1.217870 - 0.625927I$ $b = -0.428243 + 0.664531I$	$1.89061 + 2.95419I$	$0.42156 - 3.46552I$
$u = 0.500000 + 0.866025I$ $a = -1.24734 - 1.31124I$ $b = 1.002190 + 0.295542I$	$-1.89061 + 1.10558I$	$-7.50338 - 2.58970I$
$u = 0.500000 + 0.866025I$ $a = -1.75924 - 0.42461I$ $b = 1.002190 - 0.295542I$	$-1.89061 + 2.95419I$	$-5.61650 - 4.08278I$
$u = 0.500000 + 0.866025I$ $a = 2.09482 + 0.09194I$ $b = -1.073950 + 0.558752I$	$-3.66314I$	$-4.13964 + 2.11509I$
$u = 0.500000 + 0.866025I$ $a = 1.12703 + 1.76820I$ $b = -1.073950 - 0.558752I$	$7.72290I$	$-1.09315 - 8.26466I$
$u = 0.500000 - 0.866025I$ $a = 0.066864 - 1.367670I$ $b = -0.428243 + 0.664531I$	$1.89061 - 1.10558I$	$3.93112 + 2.76498I$
$u = 0.500000 - 0.866025I$ $a = 1.217870 + 0.625927I$ $b = -0.428243 - 0.664531I$	$1.89061 - 2.95419I$	$0.42156 + 3.46552I$
$u = 0.500000 - 0.866025I$ $a = -1.24734 + 1.31124I$ $b = 1.002190 - 0.295542I$	$-1.89061 - 1.10558I$	$-7.50338 + 2.58970I$
$u = 0.500000 - 0.866025I$ $a = -1.75924 + 0.42461I$ $b = 1.002190 + 0.295542I$	$-1.89061 - 2.95419I$	$-5.61650 + 4.08278I$

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.500000 - 0.866025I$ $a = 2.09482 - 0.09194I$ $b = -1.073950 - 0.558752I$	3.66314I	-4.13964 - 2.11509I
$u = 0.500000 - 0.866025I$ $a = 1.12703 - 1.76820I$ $b = -1.073950 + 0.558752I$	- 7.72290I	-1.09315 + 8.26466I

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u^2 - u + 1)^6)(u^{43} + 29u^{42} + \dots - 6u - 1)$
$c_2$	$((u^2 + u + 1)^6)(u^{43} + 7u^{42} + \dots + 8u + 1)$
$c_3$	$((u^2 - u + 1)^6)(u^{43} - 7u^{42} + \dots - 15u + 2)$
$c_4, c_8$	$u^{12}(u^{43} - u^{42} + \dots + 4096u + 4096)$
$c_5$	$((u^2 - u + 1)^6)(u^{43} + 7u^{42} + \dots + 8u + 1)$
$c_6$	$((u^6 - 3u^5 + 5u^4 - 4u^3 + 2u^2 - u + 1)^2)(u^{43} - 9u^{42} + \dots + 301u - 32)$
$c_7$	$((u^6 - u^5 - u^4 + 2u^3 - u + 1)^2)(u^{43} - 3u^{42} + \dots - 4u + 1)$
$c_9$	$((u^6 - u^5 - u^4 + 2u^3 - u + 1)^2)(u^{43} + 3u^{42} + \dots + u^2 + 1)$
$c_{10}$	$((u^6 + u^5 - u^4 - 2u^3 + u + 1)^2)(u^{43} - 3u^{42} + \dots - 4u + 1)$
$c_{11}$	$((u^6 - 3u^5 + 5u^4 - 4u^3 + 2u^2 - u + 1)^2)(u^{43} - 19u^{42} + \dots - 2u - 1)$
$c_{12}$	$((u^6 - u^5 - u^4 + 2u^3 - u + 1)^2)(u^{43} - 13u^{42} + \dots - 396502u - 27289)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y^2 + y + 1)^6)(y^{43} - 23y^{42} + \dots - 70y - 1)$
$c_2, c_5$	$((y^2 + y + 1)^6)(y^{43} + 29y^{42} + \dots - 6y - 1)$
$c_3$	$((y^2 + y + 1)^6)(y^{43} - 75y^{42} + \dots + 109y - 4)$
$c_4, c_8$	$y^{12}(y^{43} - 65y^{42} + \dots + 1.50995 \times 10^8 y - 1.67772 \times 10^7)$
$c_6$	$((y^6 + y^5 + 5y^4 + 6y^2 + 3y + 1)^2)(y^{43} - 7y^{42} + \dots + 2985y - 1024)$
$c_7, c_{10}$	$((y^6 - 3y^5 + 5y^4 - 4y^3 + 2y^2 - y + 1)^2)(y^{43} - 19y^{42} + \dots - 2y - 1)$
$c_9$	$((y^6 - 3y^5 + 5y^4 - 4y^3 + 2y^2 - y + 1)^2)(y^{43} - 59y^{42} + \dots - 2y - 1)$
$c_{11}$	$((y^6 + y^5 + 5y^4 + 6y^2 + 3y + 1)^2)(y^{43} + 13y^{42} + \dots - 54y - 1)$
$c_{12}$	$(y^6 - 3y^5 + 5y^4 - 4y^3 + 2y^2 - y + 1)^2$ $\cdot (y^{43} - 119y^{42} + \dots - 1101094234046y - 744689521)$