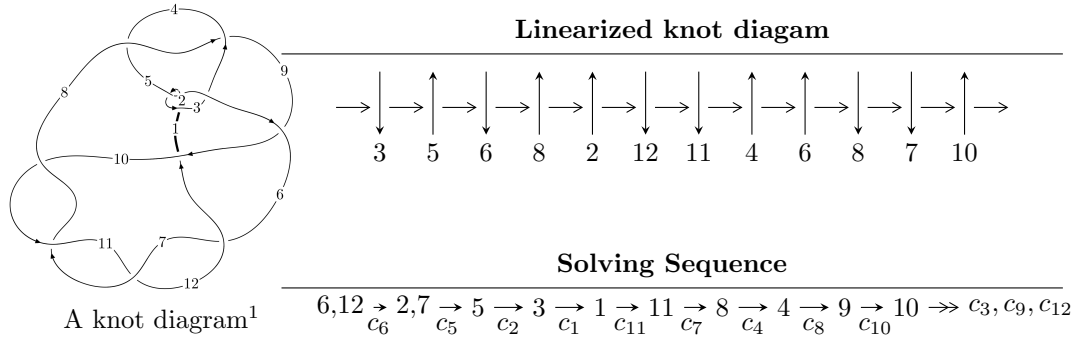


12n<sub>0048</sub> (K12n<sub>0048</sub>)



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle u^{31} - 2u^{30} + \dots + 2b + 2u, u^{30} - 2u^{29} + \dots + 2a + 4, u^{32} - 3u^{31} + \dots - 4u + 1 \rangle$$

$$I_2^u = \langle -au + b - u, u^3a - u^2a - u^3 + a^2 + 3au - 2u, u^4 - u^3 + 3u^2 - 2u + 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 40 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle u^{31} - 2u^{30} + \dots + 2b + 2u, u^{30} - 2u^{29} + \dots + 2a + 4, u^{32} - 3u^{31} + \dots - 4u + 1 \rangle$$

I.

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -\frac{1}{2}u^{30} + u^{29} + \dots - 2u - 2 \\ -\frac{1}{2}u^{31} + u^{30} + \dots - 2u^2 - u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} \frac{5}{2}u^{31} - \frac{13}{2}u^{30} + \dots + 15u - 4 \\ \frac{1}{2}u^{31} - 2u^{30} + \dots + 3u - 2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^{31} - \frac{7}{2}u^{30} + \dots + 12u - 4 \\ -\frac{1}{2}u^{31} + u^{30} + \dots - 4u^2 - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^7 + 4u^5 + 4u^3 \\ u^9 + 5u^7 + 7u^5 + 2u^3 + u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} \frac{3}{2}u^{31} - \frac{9}{2}u^{30} + \dots + 12u - 3 \\ -\frac{1}{2}u^{31} + u^{30} + \dots - 4u^2 - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^5 + 2u^3 - u \\ -u^5 - 3u^3 - u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^3 + 2u \\ u^5 + 3u^3 + u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $\frac{5}{2}u^{31} - 6u^{30} + \dots - 4u + \frac{11}{2}$

(iv) u-Polynomials at the component

| Crossings                      | u-Polynomials at each crossing             |
|--------------------------------|--|
| $c_1$                          | $u^{32} + 21u^{31} + \dots + 16u + 1$      |
| $c_2, c_5$                     | $u^{32} + 5u^{31} + \dots + 8u + 1$        |
| $c_3$                          | $u^{32} - 5u^{31} + \dots + 8u^2 + 1$      |
| $c_4, c_8$                     | $u^{32} - u^{31} + \dots + 128u + 256$     |
| $c_6, c_7, c_{10}$<br>$c_{11}$ | $u^{32} - 3u^{31} + \dots - 4u + 1$        |
| $c_9$                          | $u^{32} - 3u^{31} + \dots - 10410u + 8329$ |
| $c_{12}$                       | $u^{32} + 3u^{31} + \dots + 8u^2 + 1$      |

(v) Riley Polynomials at the component

| Crossings                      | Riley Polynomials at each crossing                   |
|--------------------------------|--|
| $c_1$                          | $y^{32} - 15y^{31} + \dots + 432y + 1$               |
| $c_2, c_5$                     | $y^{32} + 21y^{31} + \dots + 16y + 1$                |
| $c_3$                          | $y^{32} - 51y^{31} + \dots + 16y + 1$                |
| $c_4, c_8$                     | $y^{32} + 45y^{31} + \dots + 475136y + 65536$        |
| $c_6, c_7, c_{10}$<br>$c_{11}$ | $y^{32} + 35y^{31} + \dots + 16y + 1$                |
| $c_9$                          | $y^{32} + 63y^{31} + \dots + 4526986928y + 69372241$ |
| $c_{12}$                       | $y^{32} + 43y^{31} + \dots + 16y + 1$                |

(vi) Complex Volumes and Cusp Shapes

| Solutions to $I_1^u$   | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape            |
|--|---------------------------------------|-----------------------|
| $u = 0.712603 + 0.575097I$<br>$a = 1.17357 + 2.13596I$<br>$b = -0.53596 + 1.34356I$      | $-12.5938 - 8.0140I$                  | $-3.11734 + 5.63402I$ |
| $u = 0.712603 - 0.575097I$<br>$a = 1.17357 - 2.13596I$<br>$b = -0.53596 - 1.34356I$      | $-12.5938 + 8.0140I$                  | $-3.11734 - 5.63402I$ |
| $u = 0.744220 + 0.468541I$<br>$a = -0.60836 - 1.69690I$<br>$b = -0.49608 - 1.36523I$     | $-12.91530 + 3.15697I$                | $-3.83308 - 0.28463I$ |
| $u = 0.744220 - 0.468541I$<br>$a = -0.60836 + 1.69690I$<br>$b = -0.49608 + 1.36523I$     | $-12.91530 - 3.15697I$                | $-3.83308 + 0.28463I$ |
| $u = 0.701586 + 0.517087I$<br>$a = -0.496748 + 0.685785I$<br>$b = -1.054030 - 0.033443I$ | $-8.50888 - 2.34942I$                 | $-0.92927 + 2.77248I$ |
| $u = 0.701586 - 0.517087I$<br>$a = -0.496748 - 0.685785I$<br>$b = -1.054030 + 0.033443I$ | $-8.50888 + 2.34942I$                 | $-0.92927 - 2.77248I$ |
| $u = -0.490239 + 0.668149I$<br>$a = 1.72439 - 1.21578I$<br>$b = -0.125255 - 1.087690I$   | $-2.20368 + 2.67014I$                 | $-2.97925 - 3.94706I$ |
| $u = -0.490239 - 0.668149I$<br>$a = 1.72439 + 1.21578I$<br>$b = -0.125255 + 1.087690I$   | $-2.20368 - 2.67014I$                 | $-2.97925 + 3.94706I$ |
| $u = -0.599531 + 0.288987I$<br>$a = 0.01061 + 2.20427I$<br>$b = 0.026786 + 1.136550I$    | $-3.37323 + 1.08981I$                 | $-5.51512 - 2.69237I$ |
| $u = -0.599531 - 0.288987I$<br>$a = 0.01061 - 2.20427I$<br>$b = 0.026786 - 1.136550I$    | $-3.37323 - 1.08981I$                 | $-5.51512 + 2.69237I$ |

| Solutions to $I_1^u$   | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape           |
|--|---------------------------------------|----------------------|
| $u = -0.14693 + 1.41081I$<br>$a = -0.351474 + 1.272230I$<br>$b = 0.195534 + 1.213220I$   | $2.00066 + 3.66255I$                  | $0. - 3.26134I$      |
| $u = -0.14693 - 1.41081I$<br>$a = -0.351474 - 1.272230I$<br>$b = 0.195534 - 1.213220I$   | $2.00066 - 3.66255I$                  | $0. + 3.26134I$      |
| $u = -0.294946 + 0.485626I$<br>$a = 0.628330 - 0.123530I$<br>$b = -0.164482 + 0.198738I$ | $0.031344 + 1.111830I$                | $0.46087 - 6.46007I$ |
| $u = -0.294946 - 0.485626I$<br>$a = 0.628330 + 0.123530I$<br>$b = -0.164482 - 0.198738I$ | $0.031344 - 1.111830I$                | $0.46087 + 6.46007I$ |
| $u = 0.05664 + 1.45455I$<br>$a = -1.51625 - 0.70222I$<br>$b = 0.648137 - 1.003850I$      | $5.53715 - 3.59224I$                  | $0. + 2.25541I$      |
| $u = 0.05664 - 1.45455I$<br>$a = -1.51625 + 0.70222I$<br>$b = 0.648137 + 1.003850I$      | $5.53715 + 3.59224I$                  | $0. - 2.25541I$      |
| $u = -0.01210 + 1.48445I$<br>$a = -0.270899 - 0.532093I$<br>$b = 0.656453 + 0.503553I$   | $6.89090 + 1.46785I$                  | $4.83746 - 2.83876I$ |
| $u = -0.01210 - 1.48445I$<br>$a = -0.270899 + 0.532093I$<br>$b = 0.656453 - 0.503553I$   | $6.89090 - 1.46785I$                  | $4.83746 + 2.83876I$ |
| $u = 0.26257 + 1.48581I$<br>$a = -0.261839 - 0.635491I$<br>$b = -0.44075 - 1.37939I$     | $-6.59902 - 0.50025I$                 | $0$                  |
| $u = 0.26257 - 1.48581I$<br>$a = -0.261839 + 0.635491I$<br>$b = -0.44075 + 1.37939I$     | $-6.59902 + 0.50025I$                 | $0$                  |

| Solutions to $I_1^u$   | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape           |
|--|---------------------------------------|----------------------|
| $u = 0.23235 + 1.52047I$<br>$a = 0.332026 + 0.658818I$<br>$b = -1.044910 - 0.104621I$  | $-1.84666 - 5.75102I$                 | 0                    |
| $u = 0.23235 - 1.52047I$<br>$a = 0.332026 - 0.658818I$<br>$b = -1.044910 + 0.104621I$  | $-1.84666 + 5.75102I$                 | 0                    |
| $u = -0.07381 + 1.55331I$<br>$a = 0.782474 - 0.225010I$<br>$b = -0.293198 + 0.473940I$ | $7.02752 + 2.34797I$                  | 0                    |
| $u = -0.07381 - 1.55331I$<br>$a = 0.782474 + 0.225010I$<br>$b = -0.293198 - 0.473940I$ | $7.02752 - 2.34797I$                  | 0                    |
| $u = 0.038702 + 0.442217I$<br>$a = 0.809540 - 1.072710I$<br>$b = 0.420247 + 0.724423I$ | $0.54316 + 1.39338I$                  | $5.51393 - 4.82316I$ |
| $u = 0.038702 - 0.442217I$<br>$a = 0.809540 + 1.072710I$<br>$b = 0.420247 - 0.724423I$ | $0.54316 - 1.39338I$                  | $5.51393 + 4.82316I$ |
| $u = 0.23771 + 1.55222I$<br>$a = 1.49649 + 1.06121I$<br>$b = -0.56783 + 1.31462I$      | $-5.59324 - 11.51330I$                | 0                    |
| $u = 0.23771 - 1.55222I$<br>$a = 1.49649 - 1.06121I$<br>$b = -0.56783 - 1.31462I$      | $-5.59324 + 11.51330I$                | 0                    |
| $u = -0.12413 + 1.59090I$<br>$a = 1.46814 - 0.31922I$<br>$b = -0.237070 - 1.031430I$   | $5.45590 + 4.87027I$                  | 0                    |
| $u = -0.12413 - 1.59090I$<br>$a = 1.46814 + 0.31922I$<br>$b = -0.237070 + 1.031430I$   | $5.45590 - 4.87027I$                  | 0                    |

| Solutions to $I_1^u$       | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape           |
|----------------------------|---------------------------------------|----------------------|
| $u = 0.255308 + 0.267111I$ | $-0.17176 - 2.59226I$                 | $1.49287 + 0.71136I$ |
| $a = -1.91999 - 2.90858I$  |                                       |                      |
| $b = 0.512407 - 0.945593I$ |                                       |                      |
| $u = 0.255308 - 0.267111I$ | $-0.17176 + 2.59226I$                 | $1.49287 - 0.71136I$ |
| $a = -1.91999 + 2.90858I$  |                                       |                      |
| $b = 0.512407 + 0.945593I$ |                                       |                      |



**II.**

$$I_2^u = \langle -au + b - u, u^3a - u^2a - u^3 + a^2 + 3au - 2u, u^4 - u^3 + 3u^2 - 2u + 1 \rangle$$

**(i) Arc colorings**

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} a \\ au + u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^3 - au - u^2 + a + 2u \\ au + u - 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^3 - u^2 + a + 3u - 1 \\ au + u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ u^3 - u^2 + 2u - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^3 - au - u^2 + a + 2u \\ au + u - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ u^3 - u^2 + 2u - 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^3 + 2u \\ u^3 - u^2 + 2u - 1 \end{pmatrix}$$

**(ii) Obstruction class = 1**

$$\text{(iii) Cusp Shapes} = u^2a + 4u^3 - 4au - 3u^2 - a + 7u - 3$$

(iv) u-Polynomials at the component

| Crossings        | u-Polynomials at each crossing  |
|------------------|---------------------------------|
| $c_1, c_3, c_5$  | $(u^2 - u + 1)^4$               |
| $c_2$            | $(u^2 + u + 1)^4$               |
| $c_4, c_8$       | $u^8$                           |
| $c_6, c_7$       | $(u^4 - u^3 + 3u^2 - 2u + 1)^2$ |
| $c_9, c_{12}$    | $(u^4 + u^3 + u^2 + 1)^2$       |
| $c_{10}, c_{11}$ | $(u^4 + u^3 + 3u^2 + 2u + 1)^2$ |

(v) Riley Polynomials at the component

| Crossings                      | Riley Polynomials at each crossing |
|--------------------------------|------------------------------------|
| $c_1, c_2, c_3$<br>$c_5$       | $(y^2 + y + 1)^4$                  |
| $c_4, c_8$                     | $y^8$                              |
| $c_6, c_7, c_{10}$<br>$c_{11}$ | $(y^4 + 5y^3 + 7y^2 + 2y + 1)^2$   |
| $c_9, c_{12}$                  | $(y^4 + y^3 + 3y^2 + 2y + 1)^2$    |

(vi) Complex Volumes and Cusp Shapes

| Solutions to $I_2^u$        | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape            |
|-----------------------------|---------------------------------------|-----------------------|
| $u = 0.395123 + 0.506844I$  |                                       |                       |
| $a = 0.541116 + 0.214920I$  | $-0.211005 + 0.614778I$               | $-2.00436 + 1.31849I$ |
| $b = 0.500000 + 0.866025I$  |                                       |                       |
| $u = 0.395123 + 0.506844I$  |                                       |                       |
| $a = -1.58443 - 1.44211I$   | $-0.21101 - 3.44499I$                 | $0.99907 + 9.21934I$  |
| $b = 0.500000 - 0.866025I$  |                                       |                       |
| $u = 0.395123 - 0.506844I$  |                                       |                       |
| $a = 0.541116 - 0.214920I$  | $-0.211005 - 0.614778I$               | $-2.00436 - 1.31849I$ |
| $b = 0.500000 - 0.866025I$  |                                       |                       |
| $u = 0.395123 - 0.506844I$  |                                       |                       |
| $a = -1.58443 + 1.44211I$   | $-0.21101 + 3.44499I$                 | $0.99907 - 9.21934I$  |
| $b = 0.500000 + 0.866025I$  |                                       |                       |
| $u = 0.10488 + 1.55249I$    |                                       |                       |
| $a = -0.423047 - 0.283088I$ | $6.79074 - 1.13408I$                  | $1.85285 - 1.30164I$  |
| $b = 0.500000 + 0.866025I$  |                                       |                       |
| $u = 0.10488 + 1.55249I$    |                                       |                       |
| $a = -1.53364 - 0.35811I$   | $6.79074 - 5.19385I$                  | $5.65243 + 5.51994I$  |
| $b = 0.500000 - 0.866025I$  |                                       |                       |
| $u = 0.10488 - 1.55249I$    |                                       |                       |
| $a = -0.423047 + 0.283088I$ | $6.79074 + 1.13408I$                  | $1.85285 + 1.30164I$  |
| $b = 0.500000 - 0.866025I$  |                                       |                       |
| $u = 0.10488 - 1.55249I$    |                                       |                       |
| $a = -1.53364 + 0.35811I$   | $6.79074 + 5.19385I$                  | $5.65243 - 5.51994I$  |
| $b = 0.500000 + 0.866025I$  |                                       |                       |

### III. u-Polynomials

| Crossings        | u-Polynomials at each crossing  |
|------------------|---|
| $c_1$            | $((u^2 - u + 1)^4)(u^{32} + 21u^{31} + \dots + 16u + 1)$              |
| $c_2$            | $((u^2 + u + 1)^4)(u^{32} + 5u^{31} + \dots + 8u + 1)$                |
| $c_3$            | $((u^2 - u + 1)^4)(u^{32} - 5u^{31} + \dots + 8u^2 + 1)$              |
| $c_4, c_8$       | $u^8(u^{32} - u^{31} + \dots + 128u + 256)$                           |
| $c_5$            | $((u^2 - u + 1)^4)(u^{32} + 5u^{31} + \dots + 8u + 1)$                |
| $c_6, c_7$       | $((u^4 - u^3 + 3u^2 - 2u + 1)^2)(u^{32} - 3u^{31} + \dots - 4u + 1)$  |
| $c_9$            | $((u^4 + u^3 + u^2 + 1)^2)(u^{32} - 3u^{31} + \dots - 10410u + 8329)$ |
| $c_{10}, c_{11}$ | $((u^4 + u^3 + 3u^2 + 2u + 1)^2)(u^{32} - 3u^{31} + \dots - 4u + 1)$  |
| $c_{12}$         | $((u^4 + u^3 + u^2 + 1)^2)(u^{32} + 3u^{31} + \dots + 8u^2 + 1)$      |

#### IV. Riley Polynomials

| Crossings                      | Riley Polynomials at each crossing  |
|--------------------------------|---|
| $c_1$                          | $((y^2 + y + 1)^4)(y^{32} - 15y^{31} + \dots + 432y + 1)$                                       |
| $c_2, c_5$                     | $((y^2 + y + 1)^4)(y^{32} + 21y^{31} + \dots + 16y + 1)$  |
| $c_3$                          | $((y^2 + y + 1)^4)(y^{32} - 51y^{31} + \dots + 16y + 1)$  |
| $c_4, c_8$                     | $y^8(y^{32} + 45y^{31} + \dots + 475136y + 65536)$  |
| $c_6, c_7, c_{10}$<br>$c_{11}$ | $((y^4 + 5y^3 + 7y^2 + 2y + 1)^2)(y^{32} + 35y^{31} + \dots + 16y + 1)$                         |
| $c_9$                          | $(y^4 + y^3 + 3y^2 + 2y + 1)^2$<br>$\cdot (y^{32} + 63y^{31} + \dots + 4526986928y + 69372241)$ |
| $c_{12}$                       | $((y^4 + y^3 + 3y^2 + 2y + 1)^2)(y^{32} + 43y^{31} + \dots + 16y + 1)$                          |