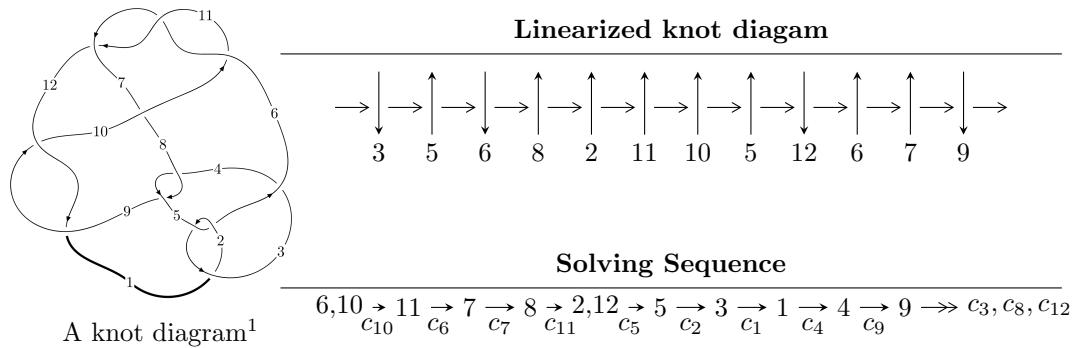


$12n_{0054}$  ( $K12n_{0054}$ )



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -2u^{23} + 2u^{22} + \dots + 2b - 2, u^{20} - u^{19} + \dots + 2a - 1, u^{24} - 3u^{23} + \dots - u - 1 \rangle$$

$$I_2^u = \langle u^4a + au + b - a + u, u^4a + u^3a + u^4 - 2u^2a + 2u^3 + a^2 - au - u^2 + a - 3u, u^5 + u^4 - 2u^3 - u^2 + u - 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 34 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle -2u^{23} + 2u^{22} + \dots + 2b - 2, u^{20} - u^{19} + \dots + 2a - 1, u^{24} - 3u^{23} + \dots - u - 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_6 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_{10} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_7 &= \begin{pmatrix} u \\ -u^3 + u \end{pmatrix} \\ a_8 &= \begin{pmatrix} -u^3 + 2u \\ -u^3 + u \end{pmatrix} \\ a_2 &= \begin{pmatrix} -\frac{1}{2}u^{20} + \frac{1}{2}u^{19} + \dots - \frac{7}{2}u + \frac{1}{2} \\ u^{23} - u^{22} + \dots + \frac{1}{2}u + 1 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} -u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix} \\ a_5 &= \begin{pmatrix} -u^{22} + 2u^{21} + \dots - \frac{5}{2}u - \frac{3}{2} \\ -u^{23} + \frac{3}{2}u^{22} + \dots - \frac{3}{2}u - 1 \end{pmatrix} \\ a_3 &= \begin{pmatrix} -u^{23} + 13u^{21} + \dots - \frac{7}{2}u - \frac{5}{2} \\ -\frac{1}{2}u^{20} + 5u^{18} + \dots + 3u^2 - \frac{1}{2}u \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^{10} - 5u^8 + 8u^6 - 3u^4 - u^2 - 1 \\ -u^{12} + 6u^{10} - 12u^8 + 8u^6 - u^4 + 2u^2 \end{pmatrix} \\ a_4 &= \begin{pmatrix} -u^{23} + 13u^{21} + \dots - \frac{7}{2}u - \frac{5}{2} \\ -4u^{23} + \frac{11}{2}u^{22} + \dots - \frac{9}{2}u - 3 \end{pmatrix} \\ a_9 &= \begin{pmatrix} u^6 - 3u^4 + 2u^2 + 1 \\ -u^8 + 4u^6 - 4u^4 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$\begin{aligned} &= -\frac{5}{2}u^{23} + \frac{7}{2}u^{22} + 25u^{21} - 29u^{20} - \frac{215}{2}u^{19} + 83u^{18} + 261u^{17} - 59u^{16} - \frac{775}{2}u^{15} - \frac{305}{2}u^{14} + \\ &325u^{13} + \frac{601}{2}u^{12} - 66u^{11} - \frac{251}{2}u^{10} - \frac{195}{2}u^9 - 33u^8 + 2u^7 - 2u^6 + 27u^5 - 8u^4 + 50u^3 + 7u^2 - 2u + \frac{5}{2} \end{aligned}$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^{24} + 2u^{23} + \cdots - 22u^2 + 1$
$c_2, c_5$	$u^{24} + 6u^{23} + \cdots + 4u + 1$
$c_3$	$u^{24} - 6u^{23} + \cdots + 22568u + 2857$
$c_4, c_8$	$u^{24} - u^{23} + \cdots + 2048u - 1024$
$c_6, c_{10}, c_{11}$	$u^{24} - 3u^{23} + \cdots - u - 1$
$c_7$	$u^{24} + 9u^{23} + \cdots + 193u + 37$
$c_9, c_{12}$	$u^{24} - u^{23} + \cdots + 3u - 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{24} + 46y^{23} + \cdots - 44y + 1$
$c_2, c_5$	$y^{24} + 2y^{23} + \cdots - 22y^2 + 1$
$c_3$	$y^{24} + 90y^{23} + \cdots + 73696224y + 8162449$
$c_4, c_8$	$y^{24} - 55y^{23} + \cdots + 3145728y + 1048576$
$c_6, c_{10}, c_{11}$	$y^{24} - 25y^{23} + \cdots - 7y + 1$
$c_7$	$y^{24} - 21y^{23} + \cdots - 29479y + 1369$
$c_9, c_{12}$	$y^{24} + 43y^{23} + \cdots - 7y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.576252 + 0.762796I$		
$a = 0.17468 - 1.56325I$	$15.5271 + 1.3395I$	$7.10412 + 0.50033I$
$b = 1.184550 - 0.408250I$		
$u = -0.576252 - 0.762796I$		
$a = 0.17468 + 1.56325I$	$15.5271 - 1.3395I$	$7.10412 - 0.50033I$
$b = 1.184550 + 0.408250I$		
$u = -0.514997 + 0.789630I$		
$a = -1.57008 - 0.30044I$	$15.3349 - 6.5027I$	$6.70839 + 4.44626I$
$b = -1.76900 - 0.98677I$		
$u = -0.514997 - 0.789630I$		
$a = -1.57008 + 0.30044I$	$15.3349 + 6.5027I$	$6.70839 - 4.44626I$
$b = -1.76900 + 0.98677I$		
$u = 1.225150 + 0.076811I$		
$a = 0.083295 + 0.422369I$	$2.02860 + 0.55793I$	$3.98398 + 0.47568I$
$b = 1.39234 - 0.93644I$		
$u = 1.225150 - 0.076811I$		
$a = 0.083295 - 0.422369I$	$2.02860 - 0.55793I$	$3.98398 - 0.47568I$
$b = 1.39234 + 0.93644I$		
$u = 0.386232 + 0.611238I$		
$a = 0.587872 - 0.504692I$	$0.95423 + 1.88035I$	$5.59254 - 3.14019I$
$b = 0.568781 - 0.120868I$		
$u = 0.386232 - 0.611238I$		
$a = 0.587872 + 0.504692I$	$0.95423 - 1.88035I$	$5.59254 + 3.14019I$
$b = 0.568781 + 0.120868I$		
$u = -1.368080 + 0.114668I$		
$a = 0.221757 - 0.722378I$	$3.28861 - 3.53789I$	$6.68532 + 4.97474I$
$b = 0.31647 + 2.25463I$		
$u = -1.368080 - 0.114668I$		
$a = 0.221757 + 0.722378I$	$3.28861 + 3.53789I$	$6.68532 - 4.97474I$
$b = 0.31647 - 2.25463I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.45120 + 0.24568I$		
$a = -0.508403 + 0.116308I$	$6.85595 - 5.06667I$	$9.81977 + 2.58134I$
$b = -1.21041 - 0.93814I$		
$u = -1.45120 - 0.24568I$		
$a = -0.508403 - 0.116308I$	$6.85595 + 5.06667I$	$9.81977 - 2.58134I$
$b = -1.21041 + 0.93814I$		
$u = 1.47737 + 0.05442I$		
$a = -0.580000 - 0.650952I$	$6.55976 + 3.21841I$	$9.78805 - 2.36901I$
$b = -1.53636 + 0.50809I$		
$u = 1.47737 - 0.05442I$		
$a = -0.580000 + 0.650952I$	$6.55976 - 3.21841I$	$9.78805 + 2.36901I$
$b = -1.53636 - 0.50809I$		
$u = 0.519837$		
$a = -1.00613$	1.05585	10.2690
$b = 0.367248$		
$u = -1.50378$		
$a = 0.695944$	7.71062	11.8740
$b = 0.281103$		
$u = 0.073930 + 0.488820I$		
$a = -1.54481 + 0.21964I$	$-1.23888 + 1.54124I$	$-1.90845 - 5.21623I$
$b = -1.121570 + 0.795516I$		
$u = 0.073930 - 0.488820I$		
$a = -1.54481 - 0.21964I$	$-1.23888 - 1.54124I$	$-1.90845 + 5.21623I$
$b = -1.121570 - 0.795516I$		
$u = 1.53161 + 0.28371I$		
$a = 0.782365 + 0.603573I$	$-17.4846 + 10.4436I$	$9.58510 - 4.63962I$
$b = 1.95290 - 1.78945I$		
$u = 1.53161 - 0.28371I$		
$a = 0.782365 - 0.603573I$	$-17.4846 - 10.4436I$	$9.58510 + 4.63962I$
$b = 1.95290 + 1.78945I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.55820 + 0.25583I$		
$a = 0.626457 - 0.749741I$	$-16.9383 + 2.4136I$	$10.12159 - 0.67000I$
$b = -0.376301 - 0.227538I$		
$u = 1.55820 - 0.25583I$		
$a = 0.626457 + 0.749741I$	$-16.9383 - 2.4136I$	$10.12159 + 0.67000I$
$b = -0.376301 + 0.227538I$		
$u = -0.349993 + 0.170334I$		
$a = 2.38196 - 1.38000I$	$0.46862 - 2.38365I$	$3.44790 + 2.07617I$
$b = 0.774418 + 0.002185I$		
$u = -0.349993 - 0.170334I$		
$a = 2.38196 + 1.38000I$	$0.46862 + 2.38365I$	$3.44790 - 2.07617I$
$b = 0.774418 - 0.002185I$		

$$I_2^u = \langle u^4a + au + b - a + u, \ u^4a + u^4 + \dots + a^2 + a, \ u^5 + u^4 - 2u^3 - u^2 + u - 1 \rangle$$

(i) **Arc colorings**

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^3 + 2u \\ -u^3 + u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} a \\ -u^4a - au + a - u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^4 + u^3 - 2u^2 + a - u + 1 \\ -u^4a + u^4 + u^2a - au - 2u^2 + a \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^4 + u^3 - 2u^2 + a - u + 1 \\ -u^4a + u^4 - au - 2u^2 + a - u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^4 + u^3 - 2u^2 + a - u + 1 \\ -u^4a + u^4 + u^2a - au - 2u^2 + a \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^3 + 2u \\ -u^3 + u \end{pmatrix}$$

(ii) **Obstruction class = 1**

(iii) **Cusp Shapes** =  $u^4a - u^3a - 2u^2a + 5u^3 + 5au - 9u + 10$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_3, c_5$	$(u^2 - u + 1)^5$
$c_2$	$(u^2 + u + 1)^5$
$c_4, c_8$	$u^{10}$
$c_6$	$(u^5 - u^4 - 2u^3 + u^2 + u + 1)^2$
$c_7$	$(u^5 + 3u^4 + 4u^3 + u^2 - u - 1)^2$
$c_9$	$(u^5 + u^4 + 2u^3 + u^2 + u + 1)^2$
$c_{10}, c_{11}$	$(u^5 + u^4 - 2u^3 - u^2 + u - 1)^2$
$c_{12}$	$(u^5 - u^4 + 2u^3 - u^2 + u - 1)^2$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_5$	$(y^2 + y + 1)^5$
$c_4, c_8$	$y^{10}$
$c_6, c_{10}, c_{11}$	$(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2$
$c_7$	$(y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)^2$
$c_9, c_{12}$	$(y^5 + 3y^4 + 4y^3 + y^2 - y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.21774$		
$a = -0.410598 + 0.711177I$	$2.40108 - 2.02988I$	$6.62546 + 2.50057I$
$b = -0.22546 - 1.71868I$		
$u = 1.21774$		
$a = -0.410598 - 0.711177I$	$2.40108 + 2.02988I$	$6.62546 - 2.50057I$
$b = -0.22546 + 1.71868I$		
$u = 0.309916 + 0.549911I$		
$a = -1.58413 + 0.01647I$	$0.329100 - 0.499304I$	$5.04069 - 0.50981I$
$b = -1.51295 + 0.11095I$		
$u = 0.309916 + 0.549911I$		
$a = 0.80632 + 1.36366I$	$0.32910 + 3.56046I$	$2.53179 - 8.01848I$
$b = 0.863922 + 0.161516I$		
$u = 0.309916 - 0.549911I$		
$a = -1.58413 - 0.01647I$	$0.329100 + 0.499304I$	$5.04069 + 0.50981I$
$b = -1.51295 - 0.11095I$		
$u = 0.309916 - 0.549911I$		
$a = 0.80632 - 1.36366I$	$0.32910 - 3.56046I$	$2.53179 + 8.01848I$
$b = 0.863922 - 0.161516I$		
$u = -1.41878 + 0.21917I$		
$a = 0.252108 + 0.649344I$	$5.87256 - 2.37095I$	$9.19707 + 1.05452I$
$b = -0.291925 - 0.343564I$		
$u = -1.41878 + 0.21917I$		
$a = 0.436295 - 0.543004I$	$5.87256 - 6.43072I$	$6.60498 + 6.63374I$
$b = 2.16641 + 1.32455I$		
$u = -1.41878 - 0.21917I$		
$a = 0.252108 - 0.649344I$	$5.87256 + 2.37095I$	$9.19707 - 1.05452I$
$b = -0.291925 + 0.343564I$		
$u = -1.41878 - 0.21917I$		
$a = 0.436295 + 0.543004I$	$5.87256 + 6.43072I$	$6.60498 - 6.63374I$
$b = 2.16641 - 1.32455I$		

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u^2 - u + 1)^5)(u^{24} + 2u^{23} + \dots - 22u^2 + 1)$
$c_2$	$((u^2 + u + 1)^5)(u^{24} + 6u^{23} + \dots + 4u + 1)$
$c_3$	$((u^2 - u + 1)^5)(u^{24} - 6u^{23} + \dots + 22568u + 2857)$
$c_4, c_8$	$u^{10}(u^{24} - u^{23} + \dots + 2048u - 1024)$
$c_5$	$((u^2 - u + 1)^5)(u^{24} + 6u^{23} + \dots + 4u + 1)$
$c_6$	$((u^5 - u^4 - 2u^3 + u^2 + u + 1)^2)(u^{24} - 3u^{23} + \dots - u - 1)$
$c_7$	$((u^5 + 3u^4 + 4u^3 + u^2 - u - 1)^2)(u^{24} + 9u^{23} + \dots + 193u + 37)$
$c_9$	$((u^5 + u^4 + 2u^3 + u^2 + u + 1)^2)(u^{24} - u^{23} + \dots + 3u - 1)$
$c_{10}, c_{11}$	$((u^5 + u^4 - 2u^3 - u^2 + u - 1)^2)(u^{24} - 3u^{23} + \dots - u - 1)$
$c_{12}$	$((u^5 - u^4 + 2u^3 - u^2 + u - 1)^2)(u^{24} - u^{23} + \dots + 3u - 1)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y^2 + y + 1)^5)(y^{24} + 46y^{23} + \dots - 44y + 1)$
$c_2, c_5$	$((y^2 + y + 1)^5)(y^{24} + 2y^{23} + \dots - 22y^2 + 1)$
$c_3$	$((y^2 + y + 1)^5)(y^{24} + 90y^{23} + \dots + 7.36962 \times 10^7 y + 8162449)$
$c_4, c_8$	$y^{10}(y^{24} - 55y^{23} + \dots + 3145728y + 1048576)$
$c_6, c_{10}, c_{11}$	$((y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2)(y^{24} - 25y^{23} + \dots - 7y + 1)$
$c_7$	$((y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)^2)(y^{24} - 21y^{23} + \dots - 29479y + 1369)$
$c_9, c_{12}$	$((y^5 + 3y^4 + 4y^3 + y^2 - y - 1)^2)(y^{24} + 43y^{23} + \dots - 7y + 1)$