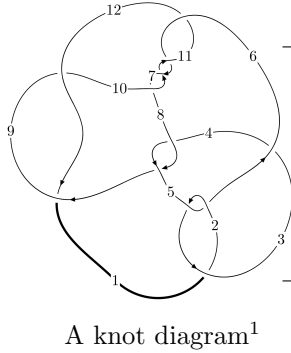
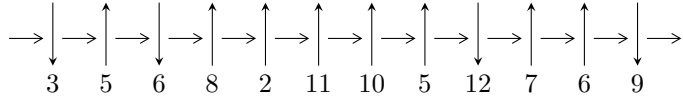


12n<sub>0065</sub> (K12n<sub>0065</sub>)



**Linearized knot diagram**



**Solving Sequence**

$$6,11 \xrightarrow{c_6} 7 \xrightarrow{c_{11}} 2,12 \xrightarrow{c_5} 5 \xrightarrow{c_2} 3 \xrightarrow{c_1} 1 \xrightarrow{c_{10}} 10 \xrightarrow{c_7} 8 \xrightarrow{c_4} 4 \xrightarrow{c_9} 9 \rightsquigarrow c_3, c_8, c_{12}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -u^{22} + 2u^{21} + \dots + 2b - 2u, -2u^{22} + 5u^{21} + \dots + 2a + 4, u^{23} - 3u^{22} + \dots - 4u + 1 \rangle$$

$$I_2^u = \langle -4u^3a - 2u^2a - 4u^3 - 11au - 2u^2 + 11b - 8a - 11u - 8, \\ u^3a + u^2a - u^3 + a^2 + 3au - 2u^2 + 2a - 4u - 2, u^4 + u^3 + 3u^2 + 2u + 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 31 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\langle -u^{22} + 2u^{21} + \dots + 2b - 2u, -2u^{22} + 5u^{21} + \dots + 2a + 4, u^{23} - 3u^{22} + \dots - 4u + 1 \rangle$$

I.  $I_1^u =$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^{22} - \frac{5}{2}u^{21} + \dots - \frac{11}{2}u^2 - 2 \\ \frac{1}{2}u^{22} - u^{21} + \dots + 2u^2 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -\frac{1}{2}u^{22} + \frac{3}{2}u^{21} + \dots - 7u + 2 \\ \frac{1}{2}u^{22} - u^{21} + \dots + u - 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -\frac{1}{2}u^{21} - 4u^{19} + \dots - 5u + 1 \\ \frac{3}{2}u^{22} - 4u^{21} + \dots + 4u - 2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^9 - 4u^7 - 3u^5 + 2u^3 - u \\ -u^9 - 5u^7 - 7u^5 - 2u^3 - u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ -u^4 - 2u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -\frac{3}{2}u^{22} + \frac{7}{2}u^{21} + \dots - 9u + 3 \\ \frac{3}{2}u^{22} - 4u^{21} + \dots + 4u - 2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^5 + 2u^3 - u \\ u^5 + 3u^3 + u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -\frac{3}{2}u^{22} + 3u^{21} - 21u^{20} + \frac{73}{2}u^{19} - 124u^{18} + 186u^{17} - 400u^{16} + 506u^{15} - \frac{1519}{2}u^{14} + \frac{1537}{2}u^{13} - 845u^{12} + 605u^{11} - \frac{1015}{2}u^{10} + 178u^9 - \frac{265}{2}u^8 - \frac{33}{2}u^7 - \frac{29}{2}u^6 - \frac{67}{2}u^5 - \frac{5}{2}u^4 - 49u^3 + \frac{19}{2}u^2 + \frac{7}{2}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{23} + 3u^{22} + \dots + 4u - 1$
$c_2, c_5$	$u^{23} + 5u^{22} + \dots - 4u - 1$
$c_3$	$u^{23} - 5u^{22} + \dots - 2678u - 593$
$c_4, c_8$	$u^{23} - u^{22} + \dots + 128u - 256$
$c_6, c_7, c_{10}$ $c_{11}$	$u^{23} + 3u^{22} + \dots - 4u - 1$
$c_9, c_{12}$	$u^{23} - u^{22} + \dots + 2u^2 - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{23} + 39y^{22} + \dots + 4y - 1$
$c_2, c_5$	$y^{23} + 3y^{22} + \dots + 4y - 1$
$c_3$	$y^{23} + 75y^{22} + \dots - 15091908y - 351649$
$c_4, c_8$	$y^{23} - 45y^{22} + \dots - 212992y - 65536$
$c_6, c_7, c_{10}$ $c_{11}$	$y^{23} + 25y^{22} + \dots + 4y - 1$
$c_9, c_{12}$	$y^{23} + 41y^{22} + \dots + 4y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.768656 + 0.574390I$ $a = 0.96411 - 1.30127I$ $b = -1.02013 - 1.06010I$	$14.5386 + 6.4049I$	$5.87992 - 4.56999I$
$u = 0.768656 - 0.574390I$ $a = 0.96411 + 1.30127I$ $b = -1.02013 + 1.06010I$	$14.5386 - 6.4049I$	$5.87992 + 4.56999I$
$u = 0.792664 + 0.509239I$ $a = -0.193691 - 0.034587I$ $b = -1.06482 + 1.00488I$	$14.7384 - 1.2262I$	$6.30437 - 0.37163I$
$u = 0.792664 - 0.509239I$ $a = -0.193691 + 0.034587I$ $b = -1.06482 - 1.00488I$	$14.7384 + 1.2262I$	$6.30437 + 0.37163I$
$u = -0.502322 + 0.520642I$ $a = 0.811150 + 0.610068I$ $b = -0.461371 + 0.176461I$	$0.70832 - 1.75933I$	$4.65925 + 3.45911I$
$u = -0.502322 - 0.520642I$ $a = 0.811150 - 0.610068I$ $b = -0.461371 - 0.176461I$	$0.70832 + 1.75933I$	$4.65925 - 3.45911I$
$u = -0.038925 + 1.309910I$ $a = 0.467966 - 0.365892I$ $b = 0.881690 - 0.526981I$	$-2.62006 - 1.64777I$	$3.52749 + 2.17174I$
$u = -0.038925 - 1.309910I$ $a = 0.467966 + 0.365892I$ $b = 0.881690 + 0.526981I$	$-2.62006 + 1.64777I$	$3.52749 - 2.17174I$
$u = 0.091640 + 1.402330I$ $a = -0.60254 + 2.18385I$ $b = 0.586427 + 1.117770I$	$-4.58949 + 3.87928I$	$1.77141 - 2.75540I$
$u = 0.091640 - 1.402330I$ $a = -0.60254 - 2.18385I$ $b = 0.586427 - 1.117770I$	$-4.58949 - 3.87928I$	$1.77141 + 2.75540I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.091228 + 0.543350I$ $a = 1.15824 - 1.74645I$ $b = 0.243917 - 0.711016I$	$-1.17938 - 1.49722I$	$-2.53958 + 5.27560I$
$u = -0.091228 - 0.543350I$ $a = 1.15824 + 1.74645I$ $b = 0.243917 + 0.711016I$	$-1.17938 + 1.49722I$	$-2.53958 - 5.27560I$
$u = -0.464696$ $a = 0.105740$ $b = 0.580286$	1.11404	10.0250
$u = -0.05085 + 1.53575I$ $a = 0.55421 - 2.04140I$ $b = 0.003120 - 0.853741I$	$-8.14932 - 2.13339I$	$-3.53476 + 3.27759I$
$u = -0.05085 - 1.53575I$ $a = 0.55421 + 2.04140I$ $b = 0.003120 + 0.853741I$	$-8.14932 + 2.13339I$	$-3.53476 - 3.27759I$
$u = 0.29102 + 1.52358I$ $a = -1.009020 + 0.577805I$ $b = -1.08811 + 0.92523I$	$8.14473 + 2.74909I$	$3.44638 - 0.72919I$
$u = 0.29102 - 1.52358I$ $a = -1.009020 - 0.577805I$ $b = -1.08811 - 0.92523I$	$8.14473 - 2.74909I$	$3.44638 + 0.72919I$
$u = -0.13755 + 1.55696I$ $a = 0.363925 + 1.080750I$ $b = -0.509748 + 0.419465I$	$-6.30985 - 4.03193I$	$1.37603 + 1.02672I$
$u = -0.13755 - 1.55696I$ $a = 0.363925 - 1.080750I$ $b = -0.509748 - 0.419465I$	$-6.30985 + 4.03193I$	$1.37603 - 1.02672I$
$u = 0.26608 + 1.55802I$ $a = 0.27719 - 2.09273I$ $b = -0.96060 - 1.09404I$	$7.54756 + 10.22760I$	$2.79101 - 4.77678I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.26608 - 1.55802I$ $a = 0.27719 + 2.09273I$ $b = -0.96060 + 1.09404I$	$7.54756 - 10.22760I$	$2.79101 + 4.77678I$
$u = 0.343175 + 0.152187I$ $a = -2.34441 + 0.12809I$ $b = 0.599485 + 0.898495I$	$0.46499 + 2.40467I$	$3.80616 - 1.75250I$
$u = 0.343175 - 0.152187I$ $a = -2.34441 - 0.12809I$ $b = 0.599485 - 0.898495I$	$0.46499 - 2.40467I$	$3.80616 + 1.75250I$

**II.**

$$I_2^u = \langle -4u^3a - 4u^3 + \cdots - 8a - 8, u^3a - u^3 + \cdots + 2a - 2, u^4 + u^3 + 3u^2 + 2u + 1 \rangle$$

**(i) Arc colorings**

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} a \\ 0.363636au^3 + 0.363636u^3 + \cdots + 0.727273a + 0.727273 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -0.363636au^3 + 0.636364u^3 + \cdots + 0.272727a + 1.27273 \\ 0.363636au^3 + 0.363636u^3 + \cdots + 0.727273a - 0.272727 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^3 + u^2 + a + 3u + 1 \\ 0.363636au^3 + 0.363636u^3 + \cdots + 0.727273a - 0.272727 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ u^3 + u^2 + 2u + 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -0.363636au^3 + 0.636364u^3 + \cdots + 0.272727a + 1.27273 \\ 0.363636au^3 + 0.363636u^3 + \cdots + 0.727273a - 0.272727 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ u^3 + u^2 + 2u + 1 \end{pmatrix}$$

**(ii) Obstruction class = 1**

**(iii) Cusp Shapes =  $-2u^3a - u^2a + 2u^3 - 6au + 3u^2 - 3a + 7u + 7$**



(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_3, c_5$	$(u^2 - u + 1)^4$
$c_2$	$(u^2 + u + 1)^4$
$c_4, c_8$	$u^8$
$c_6, c_7$	$(u^4 + u^3 + 3u^2 + 2u + 1)^2$
$c_9$	$(u^4 + u^3 + u^2 + 1)^2$
$c_{10}, c_{11}$	$(u^4 - u^3 + 3u^2 - 2u + 1)^2$
$c_{12}$	$(u^4 - u^3 + u^2 + 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_5$	$(y^2 + y + 1)^4$
$c_4, c_8$	$y^8$
$c_6, c_7, c_{10}$ $c_{11}$	$(y^4 + 5y^3 + 7y^2 + 2y + 1)^2$
$c_9, c_{12}$	$(y^4 + y^3 + 3y^2 + 2y + 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.395123 + 0.506844I$		
$a = 0.584432 + 0.289945I$	$0.211005 + 0.614778I$	$4.65255 + 0.59814I$
$b = 0.500000 + 0.866025I$		
$u = -0.395123 + 0.506844I$		
$a = -1.54112 - 1.51713I$	$0.21101 - 3.44499I$	$1.64912 + 8.49900I$
$b = 0.500000 - 0.866025I$		
$u = -0.395123 - 0.506844I$		
$a = 0.584432 - 0.289945I$	$0.211005 - 0.614778I$	$4.65255 - 0.59814I$
$b = 0.500000 - 0.866025I$		
$u = -0.395123 - 0.506844I$		
$a = -1.54112 + 1.51713I$	$0.21101 + 3.44499I$	$1.64912 - 8.49900I$
$b = 0.500000 + 0.866025I$		
$u = -0.10488 + 1.55249I$		
$a = 0.53364 + 1.37394I$	$-6.79074 - 1.13408I$	$1.99896 - 0.39034I$
$b = 0.500000 + 0.866025I$		
$u = -0.10488 + 1.55249I$		
$a = -0.57695 - 2.01514I$	$-6.79074 - 5.19385I$	$-1.80063 + 6.43123I$
$b = 0.500000 - 0.866025I$		
$u = -0.10488 - 1.55249I$		
$a = 0.53364 - 1.37394I$	$-6.79074 + 1.13408I$	$1.99896 + 0.39034I$
$b = 0.500000 - 0.866025I$		
$u = -0.10488 - 1.55249I$		
$a = -0.57695 + 2.01514I$	$-6.79074 + 5.19385I$	$-1.80063 - 6.43123I$
$b = 0.500000 + 0.866025I$		

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u^2 - u + 1)^4)(u^{23} + 3u^{22} + \dots + 4u - 1)$
$c_2$	$((u^2 + u + 1)^4)(u^{23} + 5u^{22} + \dots - 4u - 1)$
$c_3$	$((u^2 - u + 1)^4)(u^{23} - 5u^{22} + \dots - 2678u - 593)$
$c_4, c_8$	$u^8(u^{23} - u^{22} + \dots + 128u - 256)$
$c_5$	$((u^2 - u + 1)^4)(u^{23} + 5u^{22} + \dots - 4u - 1)$
$c_6, c_7$	$((u^4 + u^3 + 3u^2 + 2u + 1)^2)(u^{23} + 3u^{22} + \dots - 4u - 1)$
$c_9$	$((u^4 + u^3 + u^2 + 1)^2)(u^{23} - u^{22} + \dots + 2u^2 - 1)$
$c_{10}, c_{11}$	$((u^4 - u^3 + 3u^2 - 2u + 1)^2)(u^{23} + 3u^{22} + \dots - 4u - 1)$
$c_{12}$	$((u^4 - u^3 + u^2 + 1)^2)(u^{23} - u^{22} + \dots + 2u^2 - 1)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y^2 + y + 1)^4)(y^{23} + 39y^{22} + \dots + 4y - 1)$
$c_2, c_5$	$((y^2 + y + 1)^4)(y^{23} + 3y^{22} + \dots + 4y - 1)$
$c_3$	$((y^2 + y + 1)^4)(y^{23} + 75y^{22} + \dots - 1.50919 \times 10^7 y - 351649)$
$c_4, c_8$	$y^8(y^{23} - 45y^{22} + \dots - 212992y - 65536)$
$c_6, c_7, c_{10}$ $c_{11}$	$((y^4 + 5y^3 + 7y^2 + 2y + 1)^2)(y^{23} + 25y^{22} + \dots + 4y - 1)$
$c_9, c_{12}$	$((y^4 + y^3 + 3y^2 + 2y + 1)^2)(y^{23} + 41y^{22} + \dots + 4y - 1)$