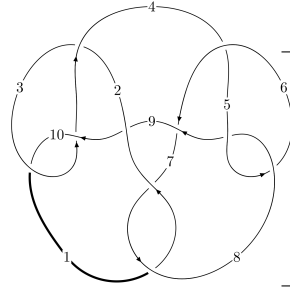
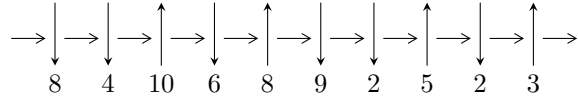


10₁₃₈ ($K10n_1$)



A knot diagram¹

Linearized knot diagram



Solving Sequence

$$5,8 \xrightarrow{c_5} 6 \xrightarrow{c_8} 2,9 \xrightarrow{c_9} 10 \xrightarrow{c_1} 1 \xrightarrow{c_4} 4 \xrightarrow{c_3} 3 \xrightarrow{c_7} 7 \longrightarrow c_2, c_6, c_{10}$$

Ideals for irreducible components² of X_{par}

$$I_1^u = \langle -u^4 + u^3 - u^2 + b + u, -u^4 + u^3 - 2u^2 + a + u - 1, u^7 - u^6 + 3u^5 - 2u^4 + 3u^3 - 2u^2 - 1 \rangle$$

$$I_2^u = \langle u^{13} - 2u^{12} + 6u^{11} - 7u^{10} + 11u^9 - 13u^8 + 14u^7 - 17u^6 + 10u^5 - 9u^4 + 7u^3 - 3u^2 + b + 3u, \\ -u^{12} + 2u^{11} - 5u^{10} + 6u^9 - 9u^8 + 11u^7 - 12u^6 + 13u^5 - 8u^4 + 8u^3 - 5u^2 + a + 3u - 2, \\ u^{14} - 2u^{13} + 6u^{12} - 8u^{11} + 13u^{10} - 16u^9 + 18u^8 - 21u^7 + 16u^6 - 15u^5 + 10u^4 - 6u^3 + 5u^2 - u + 1 \rangle$$

$$I_3^u = \langle b + u, a, u^2 + u + 1 \rangle$$

$$I_4^u = \langle b + 1, a, u^2 + u + 1 \rangle$$

* 4 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 25 representations.

¹The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

²All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\mathbf{I. } I_1^u = \langle -u^4 + u^3 - u^2 + b + u, -u^4 + u^3 - 2u^2 + a + u - 1, u^7 - u^6 + 3u^5 - 2u^4 + 3u^3 - 2u^2 - 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^4 - u^3 + 2u^2 - u + 1 \\ u^4 - u^3 + u^2 - u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^2 - 1 \\ u^5 + 2u^3 - u^2 + u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^4 - u^3 + 2u^2 - u + 1 \\ -u^6 + u^5 - u^4 - u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^2 + 1 \\ -u^4 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^3 + u^2 - u + 1 \\ u^6 + u^4 - u^3 + u^2 - u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u^4 + u^2 + 1 \\ u^4 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $-4u^6 + 6u^5 - 10u^4 + 10u^3 - 8u^2 + 8u - 4$

(iv) u-Polynomials at the component

| Crossings | u-Polynomials at each crossing |
|-----------------------------|--|
| c_1, c_7 | $u^7 + 5u^6 + 10u^5 + 13u^4 + 18u^3 + 20u^2 + 12u + 4$ |
| c_2, c_4 | $u^7 + 5u^6 + 11u^5 + 10u^4 - u^3 - 8u^2 - 4u - 1$ |
| c_3, c_5, c_8 c_{10} | $u^7 + u^6 + 3u^5 + 2u^4 + 3u^3 + 2u^2 + 1$ |
| c_6, c_9 | $u^7 - u^6 - 5u^5 + 2u^4 + 7u^3 + 4u^2 + 2u + 1$ |

(v) Riley Polynomials at the component

| Crossings | Riley Polynomials at each crossing |
|-----------------------------|---|
| c_1, c_7 | $y^7 - 5y^6 + 6y^5 + 15y^4 + 4y^3 - 72y^2 - 16y - 16$ |
| c_2, c_4 | $y^7 - 3y^6 + 19y^5 - 50y^4 + 83y^3 - 36y^2 - 1$ |
| c_3, c_5, c_8 c_{10} | $y^7 + 5y^6 + 11y^5 + 10y^4 - y^3 - 8y^2 - 4y - 1$ |
| c_6, c_9 | $y^7 - 11y^6 + 43y^5 - 62y^4 + 15y^3 + 8y^2 - 4y - 1$ |

(vi) Complex Volumes and Cusp Shapes

| Solutions to I_1^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|---|---------------------------------------|-----------------------|
| $u = 0.903382$ $a = 1.65758$ $b = -0.158515$ | -3.61413 | -1.15360 |
| $u = -0.237163 + 1.166790I$ $a = -0.931299 + 0.562572I$ $b = -0.626141 + 1.116010I$ | $-4.21141 - 3.35522I$ | $-7.88053 + 3.75965I$ |
| $u = -0.237163 - 1.166790I$ $a = -0.931299 - 0.562572I$ $b = -0.626141 - 1.116010I$ | $-4.21141 + 3.35522I$ | $-7.88053 - 3.75965I$ |
| $u = -0.266839 + 0.572668I$ $a = 0.482335 - 0.961495I$ $b = -0.260920 - 0.655876I$ | $-0.184850 - 1.357360I$ | $-2.08591 + 4.58406I$ |
| $u = -0.266839 - 0.572668I$ $a = 0.482335 + 0.961495I$ $b = -0.260920 + 0.655876I$ | $-0.184850 + 1.357360I$ | $-2.08591 - 4.58406I$ |
| $u = 0.552311 + 1.284990I$ $a = 0.120172 - 1.321830I$ $b = 0.46632 - 2.74126I$ | $-11.0685 + 10.4672I$ | $-6.45679 - 5.97165I$ |
| $u = 0.552311 - 1.284990I$ $a = 0.120172 + 1.321830I$ $b = 0.46632 + 2.74126I$ | $-11.0685 - 10.4672I$ | $-6.45679 + 5.97165I$ |

II.

$$I_2^u = \langle u^{13} - 2u^{12} + \dots + b + 3u, -u^{12} + 2u^{11} + \dots + a - 2, u^{14} - 2u^{13} + \dots - u + 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^{12} - 2u^{11} + \dots - 3u + 2 \\ -u^{13} + 2u^{12} + \dots + 3u^2 - 3u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^2 - 1 \\ u^{13} - u^{12} + \dots + u^2 + 3u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^{12} - 2u^{11} + \dots - 3u + 2 \\ -u^{13} + 3u^{12} + \dots - 4u + 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^2 + 1 \\ -u^4 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^{12} - u^{11} + \dots - 2u + 2 \\ -3u^{13} + 6u^{12} + \dots - 5u + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u^4 + u^2 + 1 \\ u^4 \end{pmatrix}$$

(ii) Obstruction class = -1**(iii) Cusp Shapes**

$$= 5u^{13} - 8u^{12} + 25u^{11} - 27u^{10} + 45u^9 - 53u^8 + 56u^7 - 68u^6 + 41u^5 - 40u^4 + 30u^3 - 14u^2 + 15u - 5$$

(iv) u-Polynomials at the component

| Crossings | u-Polynomials at each crossing |
|-----------------------------|--|
| c_1, c_7 | $(u^7 - 2u^6 - 3u^5 + 8u^4 - 2u^3 - 2u^2 - u + 2)^2$ |
| c_2, c_4 | $u^{14} + 8u^{13} + \dots + 9u + 1$ |
| c_3, c_5, c_8 c_{10} | $u^{14} + 2u^{13} + \dots + u + 1$ |
| c_6, c_9 | $u^{14} - 2u^{13} + \dots - 5u + 1$ |

(v) Riley Polynomials at the component

| Crossings | Riley Polynomials at each crossing |
|-----------------------------|--|
| c_1, c_7 | $(y^7 - 10y^6 + 37y^5 - 62y^4 + 50y^3 - 32y^2 + 9y - 4)^2$ |
| c_2, c_4 | $y^{14} - 4y^{13} + \dots - 15y + 1$ |
| c_3, c_5, c_8 c_{10} | $y^{14} + 8y^{13} + \dots + 9y + 1$ |
| c_6, c_9 | $y^{14} - 16y^{13} + \dots + 9y + 1$ |

(vi) Complex Volumes and Cusp Shapes

| Solutions to I_2^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|--|---------------------------------------|-----------------------|
| $u = 0.991355 + 0.114136I$ $a = -1.71230 - 0.09769I$ $b = 0.026394 - 0.197164I$ | $-7.46645 - 4.93043I$ | $-4.23989 + 2.98386I$ |
| $u = 0.991355 - 0.114136I$ $a = -1.71230 + 0.09769I$ $b = 0.026394 + 0.197164I$ | $-7.46645 + 4.93043I$ | $-4.23989 - 2.98386I$ |
| $u = 0.185175 + 0.946853I$ $a = -0.600533 + 0.684269I$ $b = 0.46039 + 1.77594I$ | $-1.11654 + 3.28492I$ | $-6.60141 - 2.44171I$ |
| $u = 0.185175 - 0.946853I$ $a = -0.600533 - 0.684269I$ $b = 0.46039 - 1.77594I$ | $-1.11654 - 3.28492I$ | $-6.60141 + 2.44171I$ |
| $u = -0.625804 + 0.953838I$ $a = 0.688899 + 0.343864I$ $b = 0.684697 + 0.025265I$ | $-1.11654 - 3.28492I$ | $-6.60141 + 2.44171I$ |
| $u = -0.625804 - 0.953838I$ $a = 0.688899 - 0.343864I$ $b = 0.684697 - 0.025265I$ | $-1.11654 + 3.28492I$ | $-6.60141 - 2.44171I$ |
| $u = -0.457566 + 0.656399I$ $a = 0.143355 - 0.834966I$ $b = -0.251357 - 0.560891I$ | $-0.165382 - 1.372840I$ | $-2.77344 + 4.48022I$ |
| $u = -0.457566 - 0.656399I$ $a = 0.143355 + 0.834966I$ $b = -0.251357 + 0.560891I$ | $-0.165382 + 1.372840I$ | $-2.77344 - 4.48022I$ |
| $u = 0.480471 + 1.270420I$ $a = -0.237920 + 1.237410I$ $b = -0.43046 + 2.68133I$ | $-7.46645 + 4.93043I$ | $-4.23989 - 2.98386I$ |
| $u = 0.480471 - 1.270420I$ $a = -0.237920 - 1.237410I$ $b = -0.43046 - 2.68133I$ | $-7.46645 - 4.93043I$ | $-4.23989 + 2.98386I$ |

| Solutions to I_2^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|-----------------------------|---------------------------------------|-----------------------|
| $u = 0.010735 + 0.596013I$ | | |
| $a = 0.813209 - 0.794860I$ | $-0.165382 - 1.372840I$ | $-2.77344 + 4.48022I$ |
| $b = -0.506054 - 0.754738I$ | | |
| $u = 0.010735 - 0.596013I$ | | |
| $a = 0.813209 + 0.794860I$ | $-0.165382 + 1.372840I$ | $-2.77344 - 4.48022I$ |
| $b = -0.506054 + 0.754738I$ | | |
| $u = 0.415634 + 1.342520I$ | | |
| $a = 0.405289 - 1.309100I$ | -12.1121 | $-7.77053 + 0.I$ |
| $b = 0.51639 - 2.58562I$ | | |
| $u = 0.415634 - 1.342520I$ | | |
| $a = 0.405289 + 1.309100I$ | -12.1121 | $-7.77053 + 0.I$ |
| $b = 0.51639 + 2.58562I$ | | |

$$\text{III. } I_3^u = \langle b + u, a, u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u \\ -u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ -u + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = $8u + 4$

(iv) u-Polynomials at the component

| Crossings | u-Polynomials at each crossing |
|------------------------------------|--------------------------------|
| c_1, c_7 | u^2 |
| c_2, c_3, c_4 c_6, c_8, c_9 | $u^2 - u + 1$ |
| c_5, c_{10} | $u^2 + u + 1$ |

(v) Riley Polynomials at the component

| Crossings | Riley Polynomials at each crossing |
|---|------------------------------------|
| c_1, c_7 | y^2 |
| c_2, c_3, c_4 c_5, c_6, c_8 c_9, c_{10} | $y^2 + y + 1$ |

(vi) Complex Volumes and Cusp Shapes

| Solutions to I_3^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|-----------------------------|---------------------------------------|-----------------|
| $u = -0.500000 + 0.866025I$ | | |
| $a = 0$ | $-4.05977I$ | $0. + 6.92820I$ |
| $b = 0.500000 - 0.866025I$ | | |
| $u = -0.500000 - 0.866025I$ | | |
| $a = 0$ | $4.05977I$ | $0. - 6.92820I$ |
| $b = 0.500000 + 0.866025I$ | | |

$$\text{IV. } I_4^u = \langle b + 1, a, u^2 + u + 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ u + 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ 2u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u \\ -u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u - 1 \\ -u - 2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -3

(iv) u-Polynomials at the component

| Crossings | u-Polynomials at each crossing |
|------------------------------------|--------------------------------|
| c_1, c_7 | u^2 |
| c_2, c_3, c_4 c_6, c_8, c_9 | $u^2 - u + 1$ |
| c_5, c_{10} | $u^2 + u + 1$ |

(v) Riley Polynomials at the component

| Crossings | Riley Polynomials at each crossing |
|---|------------------------------------|
| c_1, c_7 | y^2 |
| c_2, c_3, c_4 c_5, c_6, c_8 c_9, c_{10} | $y^2 + y + 1$ |

(vi) Complex Volumes and Cusp Shapes

| Solutions to I_4^u | $\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$ | Cusp shape |
|--|---------------------------------------|------------|
| $u = -0.500000 + 0.866025I$ $a = 0$ $b = -1.00000$ | 0 | -3.00000 |
| $u = -0.500000 - 0.866025I$ $a = 0$ $b = -1.00000$ | 0 | -3.00000 |

V. u-Polynomials

| Crossings | u-Polynomials at each crossing |
|---------------|--|
| c_1, c_7 | $u^4(u^7 - 2u^6 - 3u^5 + 8u^4 - 2u^3 - 2u^2 - u + 2)^2$ $\cdot (u^7 + 5u^6 + 10u^5 + 13u^4 + 18u^3 + 20u^2 + 12u + 4)$ |
| c_2, c_4 | $(u^2 - u + 1)^2(u^7 + 5u^6 + 11u^5 + 10u^4 - u^3 - 8u^2 - 4u - 1)$ $\cdot (u^{14} + 8u^{13} + \dots + 9u + 1)$ |
| c_3, c_8 | $(u^2 - u + 1)^2(u^7 + u^6 + 3u^5 + 2u^4 + 3u^3 + 2u^2 + 1)$ $\cdot (u^{14} + 2u^{13} + \dots + u + 1)$ |
| c_5, c_{10} | $(u^2 + u + 1)^2(u^7 + u^6 + 3u^5 + 2u^4 + 3u^3 + 2u^2 + 1)$ $\cdot (u^{14} + 2u^{13} + \dots + u + 1)$ |
| c_6, c_9 | $(u^2 - u + 1)^2(u^7 - u^6 - 5u^5 + 2u^4 + 7u^3 + 4u^2 + 2u + 1)$ $\cdot (u^{14} - 2u^{13} + \dots - 5u + 1)$ |

VI. Riley Polynomials

| Crossings | Riley Polynomials at each crossing |
|-----------------------------|---|
| c_1, c_7 | $y^4(y^7 - 10y^6 + 37y^5 - 62y^4 + 50y^3 - 32y^2 + 9y - 4)^2$ $\cdot (y^7 - 5y^6 + 6y^5 + 15y^4 + 4y^3 - 72y^2 - 16y - 16)$ |
| c_2, c_4 | $(y^2 + y + 1)^2(y^7 - 3y^6 + 19y^5 - 50y^4 + 83y^3 - 36y^2 - 1)$ $\cdot (y^{14} - 4y^{13} + \dots - 15y + 1)$ |
| c_3, c_5, c_8 c_{10} | $(y^2 + y + 1)^2(y^7 + 5y^6 + 11y^5 + 10y^4 - y^3 - 8y^2 - 4y - 1)$ $\cdot (y^{14} + 8y^{13} + \dots + 9y + 1)$ |
| c_6, c_9 | $(y^2 + y + 1)^2(y^7 - 11y^6 + 43y^5 - 62y^4 + 15y^3 + 8y^2 - 4y - 1)$ $\cdot (y^{14} - 16y^{13} + \dots + 9y + 1)$ |