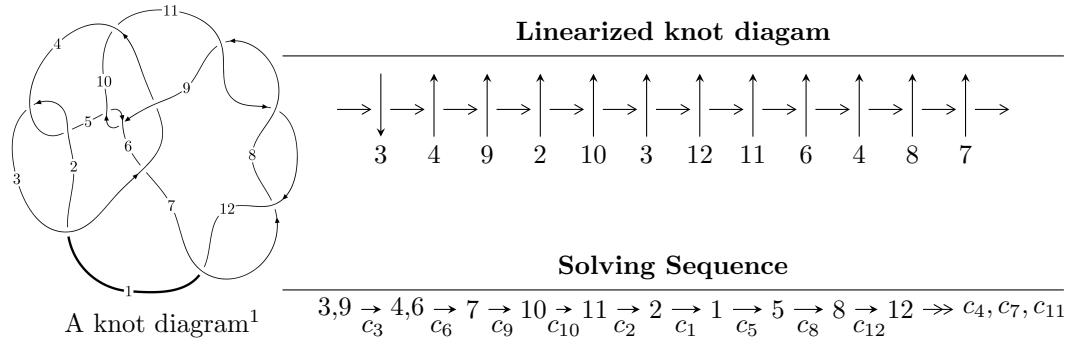


$12n_{0149}$  ( $K12n_{0149}$ )



### Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$\begin{aligned}
 I_1^u &= \langle -u^7 - 2u^6 + 6u^5 + 12u^4 - 8u^3 - 12u^2 + 2b + 13u + 8, \\
 &\quad 7u^7 + 9u^6 - 38u^5 - 59u^4 + 48u^3 + 59u^2 + 20a - 59u - 48, \\
 &\quad u^8 + 2u^7 - 4u^6 - 12u^5 - u^4 + 12u^3 - 2u^2 - 14u - 5 \rangle \\
 I_2^u &= \langle -u^3b - 2u^2b + b^2 + bu + 2u^2 - u - 2, u^2 + a, u^4 - u^2 + 1 \rangle \\
 I_3^u &= \langle b + u, a + u - 1, u^2 - u + 1 \rangle
 \end{aligned}$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 18 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle -u^7 - 2u^6 + \dots + 2b + 8, \text{ I. } 7u^7 + 9u^6 + \dots + 20a - 48, u^8 + 2u^7 + \dots - 14u - 5 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_3 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_9 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_4 &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_6 &= \begin{pmatrix} -0.350000u^7 - 0.450000u^6 + \dots + 2.95000u + 2.40000 \\ \frac{1}{2}u^7 + u^6 - 3u^5 - 6u^4 + 4u^3 + 6u^2 - \frac{13}{2}u - 4 \end{pmatrix} \\ a_7 &= \begin{pmatrix} \frac{3}{20}u^7 + \frac{11}{20}u^6 + \dots - \frac{71}{20}u - \frac{8}{5} \\ \frac{1}{2}u^7 + u^6 - 3u^5 - 6u^4 + 4u^3 + 6u^2 - \frac{13}{2}u - 4 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} -\frac{9}{20}u^7 - \frac{3}{20}u^6 + \dots + \frac{53}{20}u + \frac{4}{5} \\ 2u^7 + \frac{5}{4}u^6 + \dots - \frac{57}{4}u - \frac{27}{4} \end{pmatrix} \\ a_{11} &= \begin{pmatrix} \frac{11}{20}u^7 + \frac{7}{20}u^6 + \dots - \frac{67}{20}u - \frac{11}{5} \\ -\frac{1}{4}u^6 + \frac{1}{4}u^5 + \dots + \frac{7}{4}u + \frac{3}{4} \end{pmatrix} \\ a_2 &= \begin{pmatrix} -u^2 + 1 \\ u^4 \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^4 - u^2 + 1 \\ u^4 \end{pmatrix} \\ a_5 &= \begin{pmatrix} u^4 - u^2 + 1 \\ -u^6 - u^2 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -0.450000u^7 - 0.650000u^6 + \dots + 4.65000u + 2.80000 \\ -\frac{1}{2}u^7 - \frac{1}{2}u^6 + \dots + 5u + 2 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} \frac{13}{20}u^7 + \frac{1}{20}u^6 + \dots - \frac{81}{20}u - \frac{3}{5} \\ \frac{1}{2}u^7 - \frac{1}{4}u^6 + \dots - \frac{9}{4}u - \frac{1}{4} \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $-\frac{3}{2}u^7 - 2u^6 + \frac{15}{2}u^5 + \frac{27}{2}u^4 - \frac{15}{2}u^3 - \frac{33}{2}u^2 + 11u + \frac{49}{2}$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^8 - 20u^7 + \dots - 13476u + 625$
$c_2, c_4$	$u^8 - 12u^7 + 62u^6 - 188u^5 + 351u^4 - 436u^3 + 350u^2 - 176u + 25$
$c_3$	$u^8 - 2u^7 - 4u^6 + 12u^5 - u^4 - 12u^3 - 2u^2 + 14u - 5$
$c_5, c_9$	$u^8 + 2u^7 - 3u^6 + 8u^5 + 19u^4 + 26u^3 + 11u^2 + 4u - 4$
$c_6$	$u^8 - 12u^7 + 21u^6 + 102u^5 - 554u^4 - 2292u^3 + 749u^2 - 502u + 179$
$c_7, c_8, c_{11}$ $c_{12}$	$u^8 + 2u^7 + 6u^6 + 8u^5 + 5u^4 + 8u^3 - 12u^2 + 6u - 1$
$c_{10}$	$u^8 - 2u^7 - 3u^6 - 92u^5 - 70u^4 - 350u^3 - 705u^2 - 144u + 193$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^8 - 332y^7 + \dots - 198380076y + 390625$
$c_2, c_4$	$y^8 - 20y^7 + \dots - 13476y + 625$
$c_3$	$y^8 - 12y^7 + 62y^6 - 188y^5 + 351y^4 - 436y^3 + 350y^2 - 176y + 25$
$c_5, c_9$	$y^8 - 10y^7 + 15y^6 - 260y^5 - 145y^4 - 298y^3 - 239y^2 - 104y + 16$
$c_6$	$y^8 - 102y^7 + \dots + 16138y + 32041$
$c_7, c_8, c_{11}$ $c_{12}$	$y^8 + 8y^7 + 14y^6 - 60y^5 - 273y^4 - 292y^3 + 38y^2 - 12y + 1$
$c_{10}$	$y^8 - 10y^7 + \dots - 292866y + 37249$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.872971 + 0.618128I$ $a = -0.215446 - 0.731489I$ $b = -0.005364 + 0.460904I$	$-1.46286 + 2.16790I$	$9.57172 - 4.32976I$
$u = 0.872971 - 0.618128I$ $a = -0.215446 + 0.731489I$ $b = -0.005364 - 0.460904I$	$-1.46286 - 2.16790I$	$9.57172 + 4.32976I$
$u = -1.162380 + 0.411109I$ $a = -0.420013 - 0.734093I$ $b = 2.69985 + 1.42341I$	$-8.79021 - 1.33537I$	$7.32369 + 0.78408I$
$u = -1.162380 - 0.411109I$ $a = -0.420013 + 0.734093I$ $b = 2.69985 - 1.42341I$	$-8.79021 + 1.33537I$	$7.32369 - 0.78408I$
$u = -0.458955$ $a = 0.746207$ $b = -0.337573$	0.592549	17.1350
$u = -1.56303 + 0.67202I$ $a = 1.198220 - 0.500803I$ $b = -3.72972 + 3.74449I$	4.57005 - 8.46981I	$6.36910 + 3.46503I$
$u = -1.56303 - 0.67202I$ $a = 1.198220 + 0.500803I$ $b = -3.72972 - 3.74449I$	4.57005 + 8.46981I	$6.36910 - 3.46503I$
$u = 2.16384$ $a = 1.52826$ $b = -9.59194$	10.7735	8.33600

$$\text{II. } I_2^u = \langle -u^3b - 2u^2b + b^2 + bu + 2u^2 - u - 2, u^2 + a, u^4 - u^2 + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^2 \\ b \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^2 + b \\ b \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^3 - u \\ -u^3b + u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u^3b + u^3 - u \\ -bu + u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u^2 + 1 \\ u^2 - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ u^2 - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ -u^2 + 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^3 + u^2 - b + u \\ u^3b + u^2b - 2u^3 - b + 2u + 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} u^3b - u^3 - u^2 + u \\ u^3b - u^3 - u^2 + b + u - 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-4u^2 + 4$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_4$	$(u^2 - u + 1)^4$
$c_2$	$(u^2 + u + 1)^4$
$c_3$	$(u^4 - u^2 + 1)^2$
$c_5, c_9$	$(u^2 + 1)^4$
$c_6$	$u^8 + 4u^7 + 7u^6 + 16u^5 + 36u^4 + 50u^3 + 55u^2 + 50u + 25$
$c_7, c_8, c_{11}$ $c_{12}$	$(u^4 + 3u^2 + 1)^2$
$c_{10}$	$u^8 + 2u^7 + 3u^6 + 2u^5 - 4u^4 - 20u^3 - 5u^2 + 25$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y^2 + y + 1)^4$
$c_3$	$(y^2 - y + 1)^4$
$c_5, c_9$	$(y + 1)^8$
$c_6$	$y^8 - 2y^7 - 7y^6 - 42y^5 + 116y^4 + 210y^3 - 175y^2 + 250y + 625$
$c_7, c_8, c_{11}$ $c_{12}$	$(y^2 + 3y + 1)^4$
$c_{10}$	$y^8 + 2y^7 - 7y^6 + 42y^5 + 116y^4 - 210y^3 - 175y^2 - 250y + 625$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.866025 + 0.500000I$		
$a = -0.500000 - 0.866025I$	$-2.63189 + 2.02988I$	$2.00000 - 3.46410I$
$b = 1.035230 + 0.557008I$		
$u = 0.866025 + 0.500000I$		
$a = -0.500000 - 0.866025I$	$-10.52760 + 2.02988I$	$2.00000 - 3.46410I$
$b = -0.90126 + 1.67504I$		
$u = 0.866025 - 0.500000I$		
$a = -0.500000 + 0.866025I$	$-2.63189 - 2.02988I$	$2.00000 + 3.46410I$
$b = 1.035230 - 0.557008I$		
$u = 0.866025 - 0.500000I$		
$a = -0.500000 + 0.866025I$	$-10.52760 - 2.02988I$	$2.00000 + 3.46410I$
$b = -0.90126 - 1.67504I$		
$u = -0.866025 + 0.500000I$		
$a = -0.500000 + 0.866025I$	$-2.63189 - 2.02988I$	$2.00000 + 3.46410I$
$b = -0.035233 - 1.175040I$		
$u = -0.866025 + 0.500000I$		
$a = -0.500000 + 0.866025I$	$-10.52760 - 2.02988I$	$2.00000 + 3.46410I$
$b = 1.90126 - 0.05701I$		
$u = -0.866025 - 0.500000I$		
$a = -0.500000 - 0.866025I$	$-2.63189 + 2.02988I$	$2.00000 - 3.46410I$
$b = -0.035233 + 1.175040I$		
$u = -0.866025 - 0.500000I$		
$a = -0.500000 - 0.866025I$	$-10.52760 + 2.02988I$	$2.00000 - 3.46410I$
$b = 1.90126 + 0.05701I$		

$$\text{III. } I_3^u = \langle b + u, a + u - 1, u^2 - u + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u + 1 \\ -u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -2u + 1 \\ -u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u + 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u + 2 \\ -u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -2u + 2 \\ -u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -2u + 2 \\ -u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u \\ -1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $-4u + 8$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_2, c_3$ $c_4, c_{10}$	$u^2 + u + 1$
$c_5, c_9$	$(u + 1)^2$
$c_6, c_7, c_8$ $c_{11}, c_{12}$	$u^2 - u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$	
$c_4, c_6, c_7$	
$c_8, c_{10}, c_{11}$	$y^2 + y + 1$
$c_{12}$	
$c_5, c_9$	$(y - 1)^2$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.500000 + 0.866025I$		
$a = 0.500000 - 0.866025I$	$-1.64493 + 2.02988I$	$6.00000 - 3.46410I$
$b = -0.500000 - 0.866025I$		
$u = 0.500000 - 0.866025I$		
$a = 0.500000 + 0.866025I$	$-1.64493 - 2.02988I$	$6.00000 + 3.46410I$
$b = -0.500000 + 0.866025I$		

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u^2 - u + 1)^4)(u^2 + u + 1)(u^8 - 20u^7 + \dots - 13476u + 625)$
$c_2$	$(u^2 + u + 1)^5$ $\cdot (u^8 - 12u^7 + 62u^6 - 188u^5 + 351u^4 - 436u^3 + 350u^2 - 176u + 25)$
$c_3$	$(u^2 + u + 1)(u^4 - u^2 + 1)^2$ $\cdot (u^8 - 2u^7 - 4u^6 + 12u^5 - u^4 - 12u^3 - 2u^2 + 14u - 5)$
$c_4$	$(u^2 - u + 1)^4(u^2 + u + 1)$ $\cdot (u^8 - 12u^7 + 62u^6 - 188u^5 + 351u^4 - 436u^3 + 350u^2 - 176u + 25)$
$c_5, c_9$	$(u + 1)^2(u^2 + 1)^4$ $\cdot (u^8 + 2u^7 - 3u^6 + 8u^5 + 19u^4 + 26u^3 + 11u^2 + 4u - 4)$
$c_6$	$(u^2 - u + 1)$ $\cdot (u^8 - 12u^7 + 21u^6 + 102u^5 - 554u^4 - 2292u^3 + 749u^2 - 502u + 179)$ $\cdot (u^8 + 4u^7 + 7u^6 + 16u^5 + 36u^4 + 50u^3 + 55u^2 + 50u + 25)$
$c_7, c_8, c_{11}$ $c_{12}$	$(u^2 - u + 1)(u^4 + 3u^2 + 1)^2$ $\cdot (u^8 + 2u^7 + 6u^6 + 8u^5 + 5u^4 + 8u^3 - 12u^2 + 6u - 1)$
$c_{10}$	$(u^2 + u + 1)$ $\cdot (u^8 - 2u^7 - 3u^6 - 92u^5 - 70u^4 - 350u^3 - 705u^2 - 144u + 193)$ $\cdot (u^8 + 2u^7 + 3u^6 + 2u^5 - 4u^4 - 20u^3 - 5u^2 + 25)$

## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y^2 + y + 1)^5)(y^8 - 332y^7 + \dots - 1.98380 \times 10^8 y + 390625)$
$c_2, c_4$	$((y^2 + y + 1)^5)(y^8 - 20y^7 + \dots - 13476y + 625)$
$c_3$	$(y^2 - y + 1)^4(y^2 + y + 1)$ $\cdot (y^8 - 12y^7 + 62y^6 - 188y^5 + 351y^4 - 436y^3 + 350y^2 - 176y + 25)$
$c_5, c_9$	$(y - 1)^2(y + 1)^8$ $\cdot (y^8 - 10y^7 + 15y^6 - 260y^5 - 145y^4 - 298y^3 - 239y^2 - 104y + 16)$
$c_6$	$(y^2 + y + 1)(y^8 - 102y^7 + \dots + 16138y + 32041)$ $\cdot (y^8 - 2y^7 - 7y^6 - 42y^5 + 116y^4 + 210y^3 - 175y^2 + 250y + 625)$
$c_7, c_8, c_{11}$ $c_{12}$	$(y^2 + y + 1)(y^2 + 3y + 1)^4$ $\cdot (y^8 + 8y^7 + 14y^6 - 60y^5 - 273y^4 - 292y^3 + 38y^2 - 12y + 1)$
$c_{10}$	$(y^2 + y + 1)(y^8 - 10y^7 + \dots - 292866y + 37249)$ $\cdot (y^8 + 2y^7 - 7y^6 + 42y^5 + 116y^4 - 210y^3 - 175y^2 - 250y + 625)$