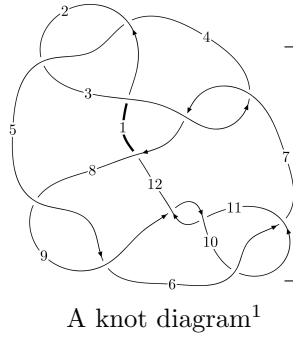
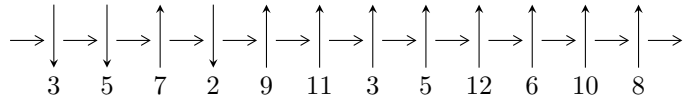


12n<sub>0159</sub> (K12n<sub>0159</sub>)



**Linearized knot diagram**



**Solving Sequence**

$$6,10 \xrightarrow{c_{10}} 11 \xrightarrow{c_6} 7 \xrightarrow{c_{11}} 12 \xrightarrow{c_9} 9 \xrightarrow{c_5} 3,5 \xrightarrow{c_2} 2 \xrightarrow{c_1} 1 \xrightarrow{c_4} 4 \xrightarrow{c_8} 8 \rightsquigarrow c_3, c_7, c_{12}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle u^{34} + 2u^{33} + \dots + b - 1, -u^{34} + 5u^{32} + \dots + a + 3u, u^{35} + 2u^{34} + \dots - 2u - 1 \rangle$$

$$I_2^u = \langle -u^5 + u^3 - u^2 + b - u, -u^7 + u^5 - u^4 - u^3 + a - 1, u^8 - u^7 - u^6 + 2u^5 + u^4 - 2u^3 + 2u - 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 43 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

I.

$$I_1^u = \langle u^{34} + 2u^{33} + \dots + b - 1, -u^{34} + 5u^{32} + \dots + a + 3u, u^{35} + 2u^{34} + \dots - 2u - 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^2 + 1 \\ -u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^4 - u^2 + 1 \\ u^4 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^{34} - 5u^{32} + \dots + 3u^2 - 3u \\ -u^{34} - 2u^{33} + \dots + 2u + 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^9 + 2u^7 - 3u^5 + 2u^3 - u \\ -u^9 + u^7 - u^5 + u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^{34} + u^{33} + \dots - 3u - 1 \\ -u^{29} + 5u^{27} + \dots - u^3 + 3u^2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^{26} - 5u^{24} + \dots + 3u^2 - 1 \\ u^{26} - 4u^{24} + \dots + 2u^4 + u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^{34} + u^{33} + \dots - 4u - 1 \\ u^{34} + u^{33} + \dots - u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^{14} + 3u^{12} - 6u^{10} + 7u^8 - 6u^6 + 4u^4 - 2u^2 + 1 \\ -u^{14} + 2u^{12} - 3u^{10} + 2u^8 + u^2 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$\begin{aligned} &= -7u^{34} - 6u^{33} + 39u^{32} + 43u^{31} - 141u^{30} - 173u^{29} + 355u^{28} + 506u^{27} - 686u^{26} - \\ &1134u^{25} + 1034u^{24} + 2071u^{23} - 1204u^{22} - 3115u^{21} + 1010u^{20} + 3917u^{19} - 424u^{18} - \\ &4134u^{17} - 365u^{16} + 3614u^{15} + 1028u^{14} - 2600u^{13} - 1314u^{12} + 1451u^{11} + 1204u^{10} - \\ &562u^9 - 821u^8 + 77u^7 + 436u^6 + 85u^5 - 158u^4 - 75u^3 + 20u^2 + 26u + 9 \end{aligned}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{35} + 49u^{34} + \dots + 102u + 1$
$c_2, c_4$	$u^{35} - 9u^{34} + \dots - 14u + 1$
$c_3, c_7$	$u^{35} - u^{34} + \dots - 640u + 256$
$c_5, c_8$	$u^{35} - 2u^{34} + \dots + 108u - 36$
$c_6, c_{10}$	$u^{35} + 2u^{34} + \dots - 2u - 1$
$c_9, c_{11}$	$u^{35} - 12u^{34} + \dots + 2u - 1$
$c_{12}$	$u^{35} + 36u^{33} + \dots - 4u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{35} - 117y^{34} + \dots + 6150y - 1$
$c_2, c_4$	$y^{35} - 49y^{34} + \dots + 102y - 1$
$c_3, c_7$	$y^{35} + 51y^{34} + \dots + 835584y - 65536$
$c_5, c_8$	$y^{35} - 12y^{34} + \dots + 6840y - 1296$
$c_6, c_{10}$	$y^{35} - 12y^{34} + \dots + 2y - 1$
$c_9, c_{11}$	$y^{35} + 24y^{34} + \dots + 2y - 1$
$c_{12}$	$y^{35} + 72y^{34} + \dots + 2y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.003180 + 0.076770I$ $a = -0.427602 + 0.732202I$ $b = 0.040765 - 1.055450I$	$1.87836 + 2.29361I$	$9.35864 - 3.99437I$
$u = 1.003180 - 0.076770I$ $a = -0.427602 - 0.732202I$ $b = 0.040765 + 1.055450I$	$1.87836 - 2.29361I$	$9.35864 + 3.99437I$
$u = -0.792898 + 0.645336I$ $a = -0.232938 - 0.928895I$ $b = 0.112888 - 0.720959I$	$-1.72435 - 2.14542I$	$5.01876 + 4.63119I$
$u = -0.792898 - 0.645336I$ $a = -0.232938 + 0.928895I$ $b = 0.112888 + 0.720959I$	$-1.72435 + 2.14542I$	$5.01876 - 4.63119I$
$u = -0.698567 + 0.764106I$ $a = 0.18575 + 2.20654I$ $b = -1.02864 + 2.02071I$	$-3.83985 + 2.10941I$	$1.06203 - 1.84479I$
$u = -0.698567 - 0.764106I$ $a = 0.18575 - 2.20654I$ $b = -1.02864 - 2.02071I$	$-3.83985 - 2.10941I$	$1.06203 + 1.84479I$
$u = 0.753011 + 0.738009I$ $a = 1.37904 - 1.86948I$ $b = -0.93662 - 2.24841I$	$-4.73261 + 0.86629I$	$0.579778 - 0.147183I$
$u = 0.753011 - 0.738009I$ $a = 1.37904 + 1.86948I$ $b = -0.93662 + 2.24841I$	$-4.73261 - 0.86629I$	$0.579778 + 0.147183I$
$u = -0.650584 + 0.839948I$ $a = -0.26514 - 2.94245I$ $b = 1.70386 - 2.51843I$	$-13.3967 + 6.2863I$	$1.00799 - 1.99078I$
$u = -0.650584 - 0.839948I$ $a = -0.26514 + 2.94245I$ $b = 1.70386 + 2.51843I$	$-13.3967 - 6.2863I$	$1.00799 + 1.99078I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.597910 + 0.716545I$ $a = -0.474108 + 0.553918I$ $b = 0.273509 + 0.526339I$	$0.466250 - 0.951396I$	$10.00295 + 0.38249I$
$u = 0.597910 - 0.716545I$ $a = -0.474108 - 0.553918I$ $b = 0.273509 - 0.526339I$	$0.466250 + 0.951396I$	$10.00295 - 0.38249I$
$u = -0.922847$ $a = -1.31041$ $b = -1.21035$	$0.182011$	$10.8590$
$u = -1.08201$ $a = 0.437613$ $b = 0.472974$	$5.82108$	$17.0620$
$u = 1.111560 + 0.128216I$ $a = 1.138050 - 0.133066I$ $b = 0.300334 + 1.303030I$	$-6.72567 + 5.75996I$	$7.32314 - 3.54445I$
$u = 1.111560 - 0.128216I$ $a = 1.138050 + 0.133066I$ $b = 0.300334 - 1.303030I$	$-6.72567 - 5.75996I$	$7.32314 + 3.54445I$
$u = -0.934946 + 0.641378I$ $a = -0.922641 - 0.641680I$ $b = 0.090797 - 0.681520I$	$-1.26680 - 2.86899I$	$5.76020 + 1.94310I$
$u = -0.934946 - 0.641378I$ $a = -0.922641 + 0.641680I$ $b = 0.090797 + 0.681520I$	$-1.26680 + 2.86899I$	$5.76020 - 1.94310I$
$u = -1.036700 + 0.513296I$ $a = 0.468566 + 0.238758I$ $b = 0.355938 - 0.949026I$	$-9.06115 - 1.11837I$	$4.93303 + 2.48933I$
$u = -1.036700 - 0.513296I$ $a = 0.468566 - 0.238758I$ $b = 0.355938 + 0.949026I$	$-9.06115 + 1.11837I$	$4.93303 - 2.48933I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.958291 + 0.699514I$ $a = 1.42433 - 1.41318I$ $b = -0.04365 - 2.87306I$	$-4.10408 + 4.62202I$	$2.53243 - 5.37025I$
$u = 0.958291 - 0.699514I$ $a = 1.42433 + 1.41318I$ $b = -0.04365 + 2.87306I$	$-4.10408 - 4.62202I$	$2.53243 + 5.37025I$
$u = 0.878474 + 0.799434I$ $a = -2.09357 + 2.58300I$ $b = 0.51980 + 3.81456I$	$-17.4916 + 2.9871I$	$-0.26712 - 2.67515I$
$u = 0.878474 - 0.799434I$ $a = -2.09357 - 2.58300I$ $b = 0.51980 - 3.81456I$	$-17.4916 - 2.9871I$	$-0.26712 + 2.67515I$
$u = 1.021530 + 0.658784I$ $a = -0.388047 + 0.431455I$ $b = -0.012993 + 0.931220I$	$1.70279 + 6.25040I$	$12.13050 - 4.97456I$
$u = 1.021530 - 0.658784I$ $a = -0.388047 - 0.431455I$ $b = -0.012993 - 0.931220I$	$1.70279 - 6.25040I$	$12.13050 + 4.97456I$
$u = -0.993451 + 0.702180I$ $a = 2.14762 + 0.79394I$ $b = 0.45485 + 2.36980I$	$-2.94898 - 7.68050I$	$3.12136 + 6.96771I$
$u = -0.993451 - 0.702180I$ $a = 2.14762 - 0.79394I$ $b = 0.45485 - 2.36980I$	$-2.94898 + 7.68050I$	$3.12136 - 6.96771I$
$u = -0.263163 + 0.716876I$ $a = -1.150580 + 0.479086I$ $b = 0.686293 - 0.231426I$	$-11.29280 - 3.30354I$	$1.02998 + 2.25929I$
$u = -0.263163 - 0.716876I$ $a = -1.150580 - 0.479086I$ $b = 0.686293 + 0.231426I$	$-11.29280 + 3.30354I$	$1.02998 - 2.25929I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.039590 + 0.718934I$ $a = -2.70067 - 0.80992I$ $b = -1.22822 - 3.37949I$	$-12.2131 - 12.1134I$	$2.82361 + 6.65221I$
$u = -1.039590 - 0.718934I$ $a = -2.70067 + 0.80992I$ $b = -1.22822 + 3.37949I$	$-12.2131 + 12.1134I$	$2.82361 - 6.65221I$
$u = 0.515516$ $a = -0.450972$ $b = 0.317465$	0.694754	14.6380
$u = -0.169379 + 0.388841I$ $a = 0.57383 - 1.55781I$ $b = -0.578952 - 0.351526I$	$-1.66775 - 0.90576I$	$-1.19698 + 2.88649I$
$u = -0.169379 - 0.388841I$ $a = 0.57383 + 1.55781I$ $b = -0.578952 + 0.351526I$	$-1.66775 + 0.90576I$	$-1.19698 - 2.88649I$



$$\text{II. } I_2^u = \langle -u^5 + u^3 - u^2 + b - u, -u^7 + u^5 - u^4 - u^3 + a - 1, u^8 - u^7 - u^6 + 2u^5 + u^4 - 2u^3 + 2u - 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^2 + 1 \\ -u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^4 - u^2 + 1 \\ u^4 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^7 - u^5 + u^4 + u^3 + 1 \\ u^5 - u^3 + u^2 + u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^6 - u^4 + 2u^2 - 1 \\ -u^7 + u^6 + 2u^5 - u^4 - 2u^3 + 2u^2 + 2u - 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^7 - u^6 - u^5 + 2u^4 + u^3 - 2u^2 + 2 \\ u^7 - u^6 - u^5 + u^4 + u^3 - u^2 - u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^6 + u^4 - 2u^2 + 1 \\ u^7 - u^6 - 2u^5 + u^4 + 2u^3 - 2u^2 - 2u + 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^7 - u^5 + u^4 + u^3 + 1 \\ u^5 - u^3 + u^2 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

(ii) Obstruction class = 1

$$\text{(iii) Cusp Shapes} = 2u^7 + u^6 - 5u^5 + 5u^3 - u^2 - 4u + 5$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$(u - 1)^8$
$c_3, c_7$	$u^8$
$c_4$	$(u + 1)^8$
$c_5$	$u^8 - u^7 - 3u^6 + 2u^5 + 3u^4 - 2u - 1$
$c_6$	$u^8 + u^7 - u^6 - 2u^5 + u^4 + 2u^3 - 2u - 1$
$c_8, c_{12}$	$u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1$
$c_9$	$u^8 + 3u^7 + 7u^6 + 10u^5 + 11u^4 + 10u^3 + 6u^2 + 4u + 1$
$c_{10}$	$u^8 - u^7 - u^6 + 2u^5 + u^4 - 2u^3 + 2u - 1$
$c_{11}$	$u^8 - 3u^7 + 7u^6 - 10u^5 + 11u^4 - 10u^3 + 6u^2 - 4u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y - 1)^8$
$c_3, c_7$	$y^8$
$c_5, c_8, c_{12}$	$y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1$
$c_6, c_{10}$	$y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1$
$c_9, c_{11}$	$y^8 + 5y^7 + 11y^6 + 6y^5 - 17y^4 - 34y^3 - 22y^2 - 4y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.570868 + 0.730671I$ $a = 0.325934 + 0.693334I$ $b = 0.972127 + 0.565636I$	$-0.604279 - 1.131230I$	$1.47926 + 0.84929I$
$u = 0.570868 - 0.730671I$ $a = 0.325934 - 0.693334I$ $b = 0.972127 - 0.565636I$	$-0.604279 + 1.131230I$	$1.47926 - 0.84929I$
$u = -0.855237 + 0.665892I$ $a = -1.03462 - 0.99451I$ $b = 0.39611 - 1.88650I$	$-3.80435 - 2.57849I$	$2.50535 + 3.23297I$
$u = -0.855237 - 0.665892I$ $a = -1.03462 + 0.99451I$ $b = 0.39611 + 1.88650I$	$-3.80435 + 2.57849I$	$2.50535 - 3.23297I$
$u = -1.09818$ $a = 0.801005$ $b = -0.165005$	4.85780	7.45240
$u = 1.031810 + 0.655470I$ $a = -0.842429 - 0.289836I$ $b = -0.699541 + 1.033710I$	$0.73474 + 6.44354I$	$3.27544 - 5.90525I$
$u = 1.031810 - 0.655470I$ $a = -0.842429 + 0.289836I$ $b = -0.699541 - 1.033710I$	$0.73474 - 6.44354I$	$3.27544 + 5.90525I$
$u = 0.603304$ $a = 1.30123$ $b = 0.827616$	$-0.799899$	3.02750

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u-1)^8)(u^{35} + 49u^{34} + \dots + 102u + 1)$
$c_2$	$((u-1)^8)(u^{35} - 9u^{34} + \dots - 14u + 1)$
$c_3, c_7$	$u^8(u^{35} - u^{34} + \dots - 640u + 256)$
$c_4$	$((u+1)^8)(u^{35} - 9u^{34} + \dots - 14u + 1)$
$c_5$	$(u^8 - u^7 - 3u^6 + 2u^5 + 3u^4 - 2u - 1)(u^{35} - 2u^{34} + \dots + 108u - 36)$
$c_6$	$(u^8 + u^7 + \dots - 2u - 1)(u^{35} + 2u^{34} + \dots - 2u - 1)$
$c_8$	$(u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1)(u^{35} - 2u^{34} + \dots + 108u - 36)$
$c_9$	$(u^8 + 3u^7 + 7u^6 + 10u^5 + 11u^4 + 10u^3 + 6u^2 + 4u + 1)$ $\cdot (u^{35} - 12u^{34} + \dots + 2u - 1)$
$c_{10}$	$(u^8 - u^7 + \dots + 2u - 1)(u^{35} + 2u^{34} + \dots - 2u - 1)$
$c_{11}$	$(u^8 - 3u^7 + 7u^6 - 10u^5 + 11u^4 - 10u^3 + 6u^2 - 4u + 1)$ $\cdot (u^{35} - 12u^{34} + \dots + 2u - 1)$
$c_{12}$	$(u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1)(u^{35} + 36u^{33} + \dots - 4u + 1)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y - 1)^8)(y^{35} - 117y^{34} + \dots + 6150y - 1)$
$c_2, c_4$	$((y - 1)^8)(y^{35} - 49y^{34} + \dots + 102y - 1)$
$c_3, c_7$	$y^8(y^{35} + 51y^{34} + \dots + 835584y - 65536)$
$c_5, c_8$	$(y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1)$ $\cdot (y^{35} - 12y^{34} + \dots + 6840y - 1296)$
$c_6, c_{10}$	$(y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1)$ $\cdot (y^{35} - 12y^{34} + \dots + 2y - 1)$
$c_9, c_{11}$	$(y^8 + 5y^7 + 11y^6 + 6y^5 - 17y^4 - 34y^3 - 22y^2 - 4y + 1)$ $\cdot (y^{35} + 24y^{34} + \dots + 2y - 1)$
$c_{12}$	$(y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1)$ $\cdot (y^{35} + 72y^{34} + \dots + 2y - 1)$