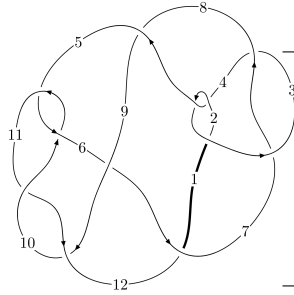
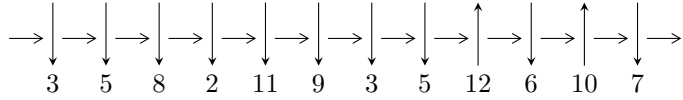


12n<sub>0172</sub> (K12n<sub>0172</sub>)



A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**

$$6,10 \xrightarrow{c_{10}} 11 \xrightarrow{c_{11}} 12 \xrightarrow{c_5} 3,5 \xrightarrow{c_2} 2 \xrightarrow{c_1} 1 \xrightarrow{c_4} 4 \xrightarrow{c_9} 9 \xrightarrow{c_6} 7 \xrightarrow{c_8} 8 \twoheadrightarrow c_3, c_7, c_{12}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -u^{19} - 2u^{18} + \dots + b + 1, -u^{19} - u^{18} + \dots + a - 1, u^{20} + 2u^{19} + \dots - 3u - 1 \rangle$$

$$I_2^u = \langle u^7 + u^5 + 2u^3 + u^2 + b + u, u^6 + u^4 + 2u^2 + a + u + 1, u^9 - u^8 + 2u^7 - u^6 + 3u^5 - u^4 + 2u^3 + u + 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 29 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle -u^{19} - 2u^{18} + \dots + b + 1, -u^{19} - u^{18} + \dots + a - 1, u^{20} + 2u^{19} + \dots - 3u - 1 \rangle \quad \mathbf{I.}$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} u^2 + 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^{19} + u^{18} + \dots + u^2 + 1 \\ u^{19} + 2u^{18} + \dots - 4u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 2u^{19} + 2u^{18} + \dots - 4u^3 + 1 \\ 2u^{19} + 4u^{18} + \dots - 6u - 2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^{16} - 3u^{14} - 7u^{12} - 10u^{10} - 11u^8 - 8u^6 - 4u^4 + 1 \\ -u^{16} - 2u^{14} - 4u^{12} - 4u^{10} - 2u^8 + 2u^4 + 2u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -3u^{19} - 3u^{18} + \dots + u^2 + 2u \\ -3u^{19} - 6u^{18} + \dots + 9u + 3 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^4 + u^2 + 1 \\ u^4 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^9 - 2u^7 - 3u^5 - 2u^3 - u \\ -u^9 - u^7 - u^5 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^8 - u^6 - u^4 + 1 \\ -u^{10} - 2u^8 - 3u^6 - 2u^4 - u^2 \end{pmatrix}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = 4u^{19} + 2u^{18} + 12u^{17} + 2u^{16} + 32u^{15} + 51u^{13} - 13u^{12} + 70u^{11} - 37u^{10} + 68u^9 - 56u^8 + 60u^7 - 63u^6 + 33u^5 - 47u^4 + 20u^3 - 17u^2 + 4u - 10$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{20} + 42u^{19} + \dots + 23u + 1$
$c_2, c_4$	$u^{20} - 10u^{19} + \dots + 5u - 1$
$c_3, c_7$	$u^{20} - u^{19} + \dots + 512u + 512$
$c_5, c_{10}$	$u^{20} + 2u^{19} + \dots - 3u - 1$
$c_6$	$u^{20} - 10u^{19} + \dots + 85u - 43$
$c_8, c_{12}$	$u^{20} + 2u^{19} + \dots - 3u - 1$
$c_9, c_{11}$	$u^{20} - 6u^{19} + \dots + 3u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{20} - 198y^{19} + \dots - 639y + 1$
$c_2, c_4$	$y^{20} - 42y^{19} + \dots - 23y + 1$
$c_3, c_7$	$y^{20} - 57y^{19} + \dots + 1310720y + 262144$
$c_5, c_{10}$	$y^{20} + 6y^{19} + \dots - 3y + 1$
$c_6$	$y^{20} - 18y^{19} + \dots - 4731y + 1849$
$c_8, c_{12}$	$y^{20} - 42y^{19} + \dots - 3y + 1$
$c_9, c_{11}$	$y^{20} + 18y^{19} + \dots - 91y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.124469 + 0.908169I$ $a = -0.110121 - 0.528184I$ $b = -0.493387 + 0.034266I$	$1.75893 + 1.54466I$	$-2.08831 - 4.86880I$
$u = -0.124469 - 0.908169I$ $a = -0.110121 + 0.528184I$ $b = -0.493387 - 0.034266I$	$1.75893 - 1.54466I$	$-2.08831 + 4.86880I$
$u = -0.654133 + 0.871364I$ $a = 0.515368 - 0.661677I$ $b = -0.239442 - 0.881898I$	$-0.94442 + 2.54047I$	$-4.33649 - 2.91190I$
$u = -0.654133 - 0.871364I$ $a = 0.515368 + 0.661677I$ $b = -0.239442 + 0.881898I$	$-0.94442 - 2.54047I$	$-4.33649 + 2.91190I$
$u = 0.783905 + 0.795880I$ $a = 0.264051 + 0.040665I$ $b = -0.174627 - 0.242030I$	$-3.99381 + 0.03901I$	$-12.11070 - 0.38222I$
$u = 0.783905 - 0.795880I$ $a = 0.264051 - 0.040665I$ $b = -0.174627 + 0.242030I$	$-3.99381 - 0.03901I$	$-12.11070 + 0.38222I$
$u = 0.284303 + 1.108040I$ $a = -1.021690 - 0.407132I$ $b = -0.160650 + 1.247820I$	$-15.4921 - 3.6755I$	$-8.75395 + 3.01938I$
$u = 0.284303 - 1.108040I$ $a = -1.021690 + 0.407132I$ $b = -0.160650 - 1.247820I$	$-15.4921 + 3.6755I$	$-8.75395 - 3.01938I$
$u = -0.903441 + 0.739223I$ $a = 0.70712 - 2.78319I$ $b = -1.41856 - 3.03717I$	$15.9851 - 3.3273I$	$-13.99686 + 0.12457I$
$u = -0.903441 - 0.739223I$ $a = 0.70712 + 2.78319I$ $b = -1.41856 + 3.03717I$	$15.9851 + 3.3273I$	$-13.99686 - 0.12457I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.806281$ $a = 1.15429$ $b = -0.930683$	-19.2190	-14.0620
$u = -0.803779 + 0.892292I$ $a = -1.76637 + 2.05694I$ $b = 0.41562 + 3.22944I$	$-6.89324 + 3.01130I$	$-13.76983 - 2.67964I$
$u = -0.803779 - 0.892292I$ $a = -1.76637 - 2.05694I$ $b = 0.41562 - 3.22944I$	$-6.89324 - 3.01130I$	$-13.76983 + 2.67964I$
$u = 0.745691 + 0.953776I$ $a = -0.209775 + 0.092781I$ $b = 0.244919 + 0.130892I$	$-3.50610 - 5.81808I$	$-10.51658 + 5.66339I$
$u = 0.745691 - 0.953776I$ $a = -0.209775 - 0.092781I$ $b = 0.244919 - 0.130892I$	$-3.50610 + 5.81808I$	$-10.51658 - 5.66339I$
$u = -0.784642 + 1.031280I$ $a = 2.58208 - 1.12211I$ $b = 0.86881 - 3.54329I$	$16.8999 + 9.5713I$	$-12.72981 - 4.75135I$
$u = -0.784642 - 1.031280I$ $a = 2.58208 + 1.12211I$ $b = 0.86881 + 3.54329I$	$16.8999 - 9.5713I$	$-12.72981 + 4.75135I$
$u = 0.216278 + 0.660670I$ $a = 0.396693 + 1.247630I$ $b = 0.738473 - 0.531917I$	$-1.26262 - 0.98137I$	$-9.38815 + 0.54437I$
$u = 0.216278 - 0.660670I$ $a = 0.396693 - 1.247630I$ $b = 0.738473 + 0.531917I$	$-1.26262 + 0.98137I$	$-9.38815 - 0.54437I$
$u = -0.325708$ $a = 1.13099$ $b = 0.368372$	-0.688798	-14.5570

$$\text{II. } I_2^u = \langle u^7 + u^5 + 2u^3 + u^2 + b + u, u^6 + u^4 + 2u^2 + a + u + 1, u^9 - u^8 + 2u^7 - u^6 + 3u^5 - u^4 + 2u^3 + u + 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} u^2 + 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^6 - u^4 - 2u^2 - u - 1 \\ -u^7 - u^5 - 2u^3 - u^2 - u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u^6 - u^4 - 2u^2 - 2u - 1 \\ -u^7 - u^5 - 3u^3 - u^2 - 2u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u \\ -u^3 - u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^6 - u^4 - 2u^2 - u - 1 \\ -u^7 - u^5 - 2u^3 - u^2 - u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^4 + u^2 + 1 \\ u^4 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^8 - u^6 - u^4 + 1 \\ -u^8 + u^7 - u^6 + 2u^5 - u^4 + 2u^3 + 2u + 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^8 - u^6 - u^4 + 1 \\ -u^8 + u^7 - u^6 + 2u^5 - u^4 + 2u^3 + 2u + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

$$\text{(iii) Cusp Shapes} = 4u^7 - 4u^6 + 3u^5 - 3u^4 + 6u^3 - 3u^2 - u - 13$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$(u - 1)^9$
$c_3, c_7$	$u^9$
$c_4$	$(u + 1)^9$
$c_5$	$u^9 + u^8 + 2u^7 + u^6 + 3u^5 + u^4 + 2u^3 + u - 1$
$c_6$	$u^9 - 5u^8 + 12u^7 - 15u^6 + 9u^5 + u^4 - 4u^3 + 2u^2 + u - 1$
$c_8, c_{12}$	$u^9 - u^8 - 2u^7 + 3u^6 + u^5 - 3u^4 + 2u^3 - u + 1$
$c_9$	$u^9 + 3u^8 + 8u^7 + 13u^6 + 17u^5 + 17u^4 + 12u^3 + 6u^2 + u - 1$
$c_{10}$	$u^9 - u^8 + 2u^7 - u^6 + 3u^5 - u^4 + 2u^3 + u + 1$
$c_{11}$	$u^9 - 3u^8 + 8u^7 - 13u^6 + 17u^5 - 17u^4 + 12u^3 - 6u^2 + u + 1$



(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y - 1)^9$
$c_3, c_7$	$y^9$
$c_5, c_{10}$	$y^9 + 3y^8 + 8y^7 + 13y^6 + 17y^5 + 17y^4 + 12y^3 + 6y^2 + y - 1$
$c_6$	$y^9 - y^8 + 12y^7 - 7y^6 + 37y^5 + y^4 - 10y^2 + 5y - 1$
$c_8, c_{12}$	$y^9 - 5y^8 + 12y^7 - 15y^6 + 9y^5 + y^4 - 4y^3 + 2y^2 + y - 1$
$c_9, c_{11}$	$y^9 + 7y^8 + 20y^7 + 25y^6 + 5y^5 - 15y^4 + 22y^2 + 13y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.140343 + 0.966856I$ $a = 0.770941 - 0.258974I$ $b = 0.142194 + 0.781734I$	$0.13850 + 2.09337I$	$-6.69021 - 3.87975I$
$u = -0.140343 - 0.966856I$ $a = 0.770941 + 0.258974I$ $b = 0.142194 - 0.781734I$	$0.13850 - 2.09337I$	$-6.69021 + 3.87975I$
$u = -0.628449 + 0.875112I$ $a = 0.147409 - 0.367985I$ $b = 0.229389 + 0.360259I$	$-2.26187 + 2.45442I$	$-12.49381 - 3.35442I$
$u = -0.628449 - 0.875112I$ $a = 0.147409 + 0.367985I$ $b = 0.229389 - 0.360259I$	$-2.26187 - 2.45442I$	$-12.49381 + 3.35442I$
$u = 0.796005 + 0.733148I$ $a = -0.24323 - 1.73417I$ $b = 1.07779 - 1.55873I$	$-6.01628 + 1.33617I$	$-13.53709 - 1.22905I$
$u = 0.796005 - 0.733148I$ $a = -0.24323 + 1.73417I$ $b = 1.07779 + 1.55873I$	$-6.01628 - 1.33617I$	$-13.53709 + 1.22905I$
$u = 0.728966 + 0.986295I$ $a = -1.62529 - 0.46000I$ $b = -0.73109 - 1.93833I$	$-5.24306 - 7.08493I$	$-12.02676 + 6.64241I$
$u = 0.728966 - 0.986295I$ $a = -1.62529 + 0.46000I$ $b = -0.73109 + 1.93833I$	$-5.24306 + 7.08493I$	$-12.02676 - 6.64241I$
$u = -0.512358$ $a = -1.09967$ $b = 0.563422$	$-2.84338$	$-14.5040$

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u - 1)^9)(u^{20} + 42u^{19} + \dots + 23u + 1)$
$c_2$	$((u - 1)^9)(u^{20} - 10u^{19} + \dots + 5u - 1)$
$c_3, c_7$	$u^9(u^{20} - u^{19} + \dots + 512u + 512)$
$c_4$	$((u + 1)^9)(u^{20} - 10u^{19} + \dots + 5u - 1)$
$c_5$	$(u^9 + u^8 + \dots + u - 1)(u^{20} + 2u^{19} + \dots - 3u - 1)$
$c_6$	$(u^9 - 5u^8 + 12u^7 - 15u^6 + 9u^5 + u^4 - 4u^3 + 2u^2 + u - 1)$ $\cdot (u^{20} - 10u^{19} + \dots + 85u - 43)$
$c_8, c_{12}$	$(u^9 - u^8 + \dots - u + 1)(u^{20} + 2u^{19} + \dots - 3u - 1)$
$c_9$	$(u^9 + 3u^8 + 8u^7 + 13u^6 + 17u^5 + 17u^4 + 12u^3 + 6u^2 + u - 1)$ $\cdot (u^{20} - 6u^{19} + \dots + 3u + 1)$
$c_{10}$	$(u^9 - u^8 + \dots + u + 1)(u^{20} + 2u^{19} + \dots - 3u - 1)$
$c_{11}$	$(u^9 - 3u^8 + 8u^7 - 13u^6 + 17u^5 - 17u^4 + 12u^3 - 6u^2 + u + 1)$ $\cdot (u^{20} - 6u^{19} + \dots + 3u + 1)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y - 1)^9)(y^{20} - 198y^{19} + \dots - 639y + 1)$
$c_2, c_4$	$((y - 1)^9)(y^{20} - 42y^{19} + \dots - 23y + 1)$
$c_3, c_7$	$y^9(y^{20} - 57y^{19} + \dots + 1310720y + 262144)$
$c_5, c_{10}$	$(y^9 + 3y^8 + 8y^7 + 13y^6 + 17y^5 + 17y^4 + 12y^3 + 6y^2 + y - 1)$ $\cdot (y^{20} + 6y^{19} + \dots - 3y + 1)$
$c_6$	$(y^9 - y^8 + 12y^7 - 7y^6 + 37y^5 + y^4 - 10y^2 + 5y - 1)$ $\cdot (y^{20} - 18y^{19} + \dots - 4731y + 1849)$
$c_8, c_{12}$	$(y^9 - 5y^8 + 12y^7 - 15y^6 + 9y^5 + y^4 - 4y^3 + 2y^2 + y - 1)$ $\cdot (y^{20} - 42y^{19} + \dots - 3y + 1)$
$c_9, c_{11}$	$(y^9 + 7y^8 + 20y^7 + 25y^6 + 5y^5 - 15y^4 + 22y^2 + 13y - 1)$ $\cdot (y^{20} + 18y^{19} + \dots - 91y + 1)$