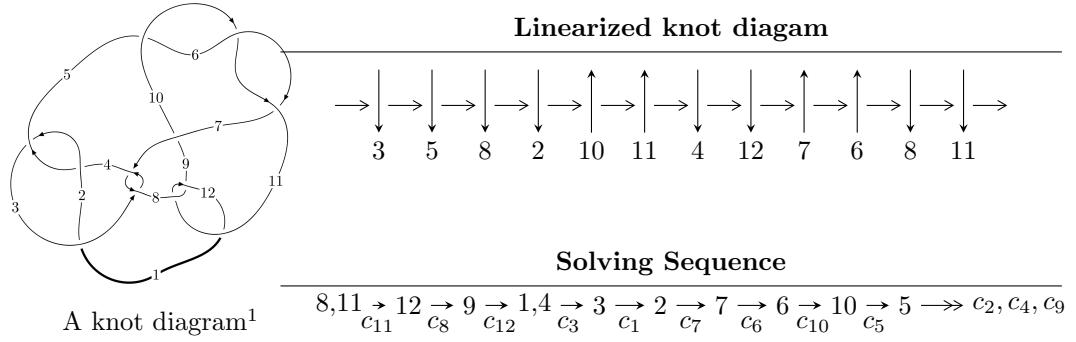


$12n_{0189}$  ( $K12n_{0189}$ )



### Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$\begin{aligned}
 I_1^u &= \langle 2.49444 \times 10^{61} u^{47} + 5.42508 \times 10^{61} u^{46} + \dots + 2.55281 \times 10^{62} b - 1.56879 \times 10^{62}, \\
 &\quad - 4.23145 \times 10^{61} u^{47} - 5.04595 \times 10^{62} u^{46} + \dots + 3.57394 \times 10^{63} a - 9.28768 \times 10^{63}, \\
 &\quad u^{48} + 4u^{47} + \dots + 59u - 7 \rangle \\
 I_2^u &= \langle -2a^2b + b^2 - 2ba - 2a^2 - 4b - a - 3, a^3 + a^2 + 2a + 1, u - 1 \rangle \\
 I_3^u &= \langle -a^2 + b - a - 2, a^3 + a^2 + 2a + 1, u + 1 \rangle
 \end{aligned}$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 57 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle 2.49 \times 10^{61}u^{47} + 5.43 \times 10^{61}u^{46} + \dots + 2.55 \times 10^{62}b - 1.57 \times 10^{62}, -4.23 \times 10^{61}u^{47} - 5.05 \times 10^{62}u^{46} + \dots + 3.57 \times 10^{63}a - 9.29 \times 10^{63}, u^{48} + 4u^{47} + \dots + 59u - 7 \rangle$$

(i) **Arc colorings**

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u \\ -u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 + 1 \\ u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.0118397u^{47} + 0.141187u^{46} + \dots + 28.4662u + 2.59872 \\ -0.0977134u^{47} - 0.212514u^{46} + \dots - 12.5106u + 0.614532 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.0118397u^{47} + 0.141187u^{46} + \dots + 28.4662u + 2.59872 \\ -0.0177507u^{47} + 0.0119124u^{46} + \dots - 7.05758u - 0.0422666 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.0759396u^{47} - 0.266607u^{46} + \dots - 5.41635u - 1.07005 \\ -0.0304639u^{47} - 0.113488u^{46} + \dots + 3.98647u + 0.0378137 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -0.135632u^{47} - 0.469213u^{46} + \dots + 8.97335u + 2.53780 \\ -0.0896845u^{47} - 0.258891u^{46} + \dots + 1.36877u - 0.441443 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -0.0459476u^{47} - 0.210322u^{46} + \dots + 7.60458u + 2.97924 \\ -0.0896845u^{47} - 0.258891u^{46} + \dots + 1.36877u - 0.441443 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0.157505u^{47} + 0.559805u^{46} + \dots + 0.744089u - 1.23298 \\ 0.0268891u^{47} - 0.0213891u^{46} + \dots + 0.825649u - 0.103367 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0.0759396u^{47} + 0.266607u^{46} + \dots + 5.41635u + 1.07005 \\ 0.0604385u^{47} + 0.190773u^{46} + \dots - 4.38559u + 0.332492 \end{pmatrix}$$

(ii) **Obstruction class** = -1

(iii) **Cusp Shapes** =  $-0.523196u^{47} - 1.69630u^{46} + \dots - 33.3905u - 3.88072$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^{48} + 28u^{47} + \cdots + 2u + 1$
$c_2, c_4$	$u^{48} - 4u^{47} + \cdots + 2u + 1$
$c_3, c_7$	$u^{48} + 2u^{47} + \cdots + 8u - 1$
$c_5, c_6, c_{10}$	$u^{48} - 3u^{47} + \cdots - 8u - 8$
$c_8, c_{11}$	$u^{48} + 4u^{47} + \cdots + 59u - 7$
$c_9$	$u^{48} + 9u^{47} + \cdots - 6632u - 1192$
$c_{12}$	$u^{48} + 54u^{47} + \cdots + 4853u + 49$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{48} - 12y^{47} + \cdots - 250y + 1$
$c_2, c_4$	$y^{48} - 28y^{47} + \cdots - 2y + 1$
$c_3, c_7$	$y^{48} + 12y^{47} + \cdots - 42y + 1$
$c_5, c_6, c_{10}$	$y^{48} - 41y^{47} + \cdots - 1984y + 64$
$c_8, c_{11}$	$y^{48} - 54y^{47} + \cdots - 4853y + 49$
$c_9$	$y^{48} + 43y^{47} + \cdots - 58172992y + 1420864$
$c_{12}$	$y^{48} - 110y^{47} + \cdots - 30351829y + 2401$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.03474$		
$a = -0.149518$	1.67136	-181.820
$b = 15.9108$		
$u = -1.10270$		
$a = 0.558674$	-2.51979	7.01870
$b = -1.21115$		
$u = 0.402947 + 0.795966I$		
$a = -1.253190 + 0.172447I$	4.33668 - 4.44228I	2.15682 + 4.76767I
$b = 1.24412 - 0.98353I$		
$u = 0.402947 - 0.795966I$		
$a = -1.253190 - 0.172447I$	4.33668 + 4.44228I	2.15682 - 4.76767I
$b = 1.24412 + 0.98353I$		
$u = 0.549733 + 0.682678I$		
$a = 0.009774 - 1.180920I$	0.44919 - 3.43575I	-2.94781 + 4.14151I
$b = 1.026340 - 0.304021I$		
$u = 0.549733 - 0.682678I$		
$a = 0.009774 + 1.180920I$	0.44919 + 3.43575I	-2.94781 - 4.14151I
$b = 1.026340 + 0.304021I$		
$u = 0.607419 + 0.971287I$		
$a = -0.314066 - 0.576245I$	0.79074 + 2.62631I	0. - 8.02541I
$b = 0.709420 + 0.166838I$		
$u = 0.607419 - 0.971287I$		
$a = -0.314066 + 0.576245I$	0.79074 - 2.62631I	0. + 8.02541I
$b = 0.709420 - 0.166838I$		
$u = 0.529099 + 1.030160I$		
$a = 1.023660 - 0.202973I$	1.27119 - 9.09207I	0. + 7.97032I
$b = -1.33032 + 0.89512I$		
$u = 0.529099 - 1.030160I$		
$a = 1.023660 + 0.202973I$	1.27119 + 9.09207I	0. - 7.97032I
$b = -1.33032 - 0.89512I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.606029 + 0.575291I$		
$a = 1.095020 + 0.107629I$	$0.227251 - 0.982213I$	$-2.97456 + 4.05134I$
$b = -1.62264 + 1.09670I$		
$u = 0.606029 - 0.575291I$		
$a = 1.095020 - 0.107629I$	$0.227251 + 0.982213I$	$-2.97456 - 4.05134I$
$b = -1.62264 - 1.09670I$		
$u = 0.717418 + 0.339614I$		
$a = -0.411980 + 0.676361I$	$3.05609 + 0.03450I$	$1.61725 - 0.04014I$
$b = -1.010750 - 0.101614I$		
$u = 0.717418 - 0.339614I$		
$a = -0.411980 - 0.676361I$	$3.05609 - 0.03450I$	$1.61725 + 0.04014I$
$b = -1.010750 + 0.101614I$		
$u = 0.774869 + 0.143809I$		
$a = 0.06554 - 1.56635I$	$1.81458 + 2.58829I$	$4.64199 + 0.62738I$
$b = 0.115818 - 0.297169I$		
$u = 0.774869 - 0.143809I$		
$a = 0.06554 + 1.56635I$	$1.81458 - 2.58829I$	$4.64199 - 0.62738I$
$b = 0.115818 + 0.297169I$		
$u = -0.705342 + 0.225017I$		
$a = 0.374486 + 0.783406I$	$-2.89686 + 0.77640I$	$-12.15233 + 3.40612I$
$b = -0.02299 + 1.57993I$		
$u = -0.705342 - 0.225017I$		
$a = 0.374486 - 0.783406I$	$-2.89686 - 0.77640I$	$-12.15233 - 3.40612I$
$b = -0.02299 - 1.57993I$		
$u = 0.701884$		
$a = -0.734892$	$3.11349$	$3.07850$
$b = -1.24205$		
$u = -0.856659 + 0.985459I$		
$a = 0.636632 + 0.001328I$	$-2.80642 + 4.10771I$	$0$
$b = -0.672001 - 0.732654I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.856659 - 0.985459I$		
$a = 0.636632 - 0.001328I$	$-2.80642 - 4.10771I$	0
$b = -0.672001 + 0.732654I$		
$u = -1.312340 + 0.022176I$		
$a = 0.159774 - 1.198270I$	$3.83816 + 3.33221I$	0
$b = -0.14705 - 1.53594I$		
$u = -1.312340 - 0.022176I$		
$a = 0.159774 + 1.198270I$	$3.83816 - 3.33221I$	0
$b = -0.14705 + 1.53594I$		
$u = -0.382101 + 0.518429I$		
$a = -1.066440 - 0.036820I$	$-0.164421 + 1.292600I$	$-1.96598 - 5.14574I$
$b = 0.434772 + 0.642582I$		
$u = -0.382101 - 0.518429I$		
$a = -1.066440 + 0.036820I$	$-0.164421 - 1.292600I$	$-1.96598 + 5.14574I$
$b = 0.434772 - 0.642582I$		
$u = -1.52813 + 0.03469I$		
$a = 0.749487 + 0.430849I$	$-3.85416 + 1.06668I$	0
$b = -0.082283 + 0.460779I$		
$u = -1.52813 - 0.03469I$		
$a = 0.749487 - 0.430849I$	$-3.85416 - 1.06668I$	0
$b = -0.082283 - 0.460779I$		
$u = 1.49504 + 0.38475I$		
$a = 0.007881 + 0.362733I$	$2.74848 + 0.89020I$	0
$b = -0.372006 - 0.161707I$		
$u = 1.49504 - 0.38475I$		
$a = 0.007881 - 0.362733I$	$2.74848 - 0.89020I$	0
$b = -0.372006 + 0.161707I$		
$u = -1.52900 + 0.29105I$		
$a = 0.706345 - 0.671808I$	$-2.00876 + 8.44086I$	0
$b = -1.06205 - 1.60612I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.52900 - 0.29105I$		
$a = 0.706345 + 0.671808I$	$-2.00876 - 8.44086I$	0
$b = -1.06205 + 1.60612I$		
$u = 1.55775 + 0.15557I$		
$a = 0.721994 + 0.568337I$	$-6.88003 - 3.75478I$	0
$b = -0.497955 + 1.196310I$		
$u = 1.55775 - 0.15557I$		
$a = 0.721994 - 0.568337I$	$-6.88003 + 3.75478I$	0
$b = -0.497955 - 1.196310I$		
$u = -1.58783 + 0.13072I$		
$a = -0.527458 + 0.690762I$	$-7.23928 + 3.38751I$	0
$b = 0.89707 + 1.86682I$		
$u = -1.58783 - 0.13072I$		
$a = -0.527458 - 0.690762I$	$-7.23928 - 3.38751I$	0
$b = 0.89707 - 1.86682I$		
$u = -1.58112 + 0.20855I$		
$a = -0.673201 - 0.601686I$	$-6.71802 + 6.70728I$	0
$b = -0.115011 - 0.480919I$		
$u = -1.58112 - 0.20855I$		
$a = -0.673201 + 0.601686I$	$-6.71802 - 6.70728I$	0
$b = -0.115011 + 0.480919I$		
$u = 1.61634 + 0.04594I$		
$a = -0.607271 + 0.648913I$	$-10.99530 - 1.69093I$	0
$b = 0.278472 + 1.264590I$		
$u = 1.61634 - 0.04594I$		
$a = -0.607271 - 0.648913I$	$-10.99530 + 1.69093I$	0
$b = 0.278472 - 1.264590I$		
$u = -1.59092 + 0.39585I$		
$a = -0.731570 + 0.550707I$	$-5.5392 + 14.3589I$	0
$b = 1.26602 + 1.60368I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.59092 - 0.39585I$		
$a = -0.731570 - 0.550707I$	$-5.5392 - 14.3589I$	0
$b = 1.26602 - 1.60368I$		
$u = 1.68738 + 0.31170I$		
$a = -0.685391 - 0.453228I$	$-11.1104 - 9.0785I$	0
$b = 0.64974 - 1.33823I$		
$u = 1.68738 - 0.31170I$		
$a = -0.685391 + 0.453228I$	$-11.1104 + 9.0785I$	0
$b = 0.64974 + 1.33823I$		
$u = -1.74551 + 0.16291I$		
$a = -0.577118 + 0.344131I$	$-8.76602 + 2.97970I$	0
$b = 0.165920 + 0.739123I$		
$u = -1.74551 - 0.16291I$		
$a = -0.577118 - 0.344131I$	$-8.76602 - 2.97970I$	0
$b = 0.165920 - 0.739123I$		
$u = -0.088616 + 0.165371I$		
$a = -5.53925 + 3.96896I$	$7.99255 - 2.76208I$	$5.58674 + 2.99226I$
$b = 0.242833 - 1.027870I$		
$u = -0.088616 - 0.165371I$		
$a = -5.53925 - 3.96896I$	$7.99255 + 2.76208I$	$5.58674 - 2.99226I$
$b = 0.242833 + 1.027870I$		
$u = 0.0931699$		
$a = 5.14127$	-1.24876	-7.95330
$b = -0.648531$		

$$\text{II. } I_2^u = \langle -2a^2b + b^2 - 2ba - 2a^2 - 4b - a - 3, a^3 + a^2 + 2a + 1, u - 1 \rangle$$

(i) Arc colorings

$$a_8 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$a_3 = \begin{pmatrix} a \\ b-a \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -a^2 \\ -ba + a^2 + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} a^2 \\ ba + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -ba + a^2 - 1 \\ ba + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -a^2b - 2ba + a^2 - b - 1 \\ 2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -a^2 \\ -ba - 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-4a^2 - 4a - 8$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_7$	$(u^3 - u^2 + 2u - 1)^2$
$c_2$	$(u^3 + u^2 - 1)^2$
$c_3$	$(u^3 + u^2 + 2u + 1)^2$
$c_4$	$(u^3 - u^2 + 1)^2$
$c_5, c_6, c_9$ $c_{10}$	$(u^2 - 2)^3$
$c_8, c_{12}$	$(u + 1)^6$
$c_{11}$	$(u - 1)^6$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_3, c_7$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_2, c_4$	$(y^3 - y^2 + 2y - 1)^2$
$c_5, c_6, c_9$ $c_{10}$	$(y - 2)^6$
$c_8, c_{11}, c_{12}$	$(y - 1)^6$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$		
$a = -0.215080 + 1.307140I$	$6.31400 + 2.82812I$	$-0.49024 - 2.97945I$
$b = -0.050766 - 0.308532I$		
$u = 1.00000$		
$a = -0.215080 + 1.307140I$	$6.31400 + 2.82812I$	$-0.49024 - 2.97945I$
$b = 0.29589 + 1.79826I$		
$u = 1.00000$		
$a = -0.215080 - 1.307140I$	$6.31400 - 2.82812I$	$-0.49024 + 2.97945I$
$b = -0.050766 + 0.308532I$		
$u = 1.00000$		
$a = -0.215080 - 1.307140I$	$6.31400 - 2.82812I$	$-0.49024 + 2.97945I$
$b = 0.29589 - 1.79826I$		
$u = 1.00000$		
$a = -0.569840$	2.17641	-7.01950
$b = -0.726894$		
$u = 1.00000$		
$a = -0.569840$	2.17641	-7.01950
$b = 4.23665$		

$$\text{III. } I_3^u = \langle -a^2 + b - a - 2, a^3 + a^2 + 2a + 1, u + 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_8 &= \begin{pmatrix} 0 \\ -1 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ a_9 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_1 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ a_4 &= \begin{pmatrix} a \\ a^2 + a + 2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} a \\ a^2 + 2 \end{pmatrix} \\ a_2 &= \begin{pmatrix} -a^2 \\ a^2 + 2 \end{pmatrix} \\ a_7 &= \begin{pmatrix} -a^2 \\ 0 \end{pmatrix} \\ a_6 &= \begin{pmatrix} -a^2 \\ 0 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_5 &= \begin{pmatrix} -a^2 \\ 0 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-12a^2 - 10a - 32$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_3$	$u^3 - u^2 + 2u - 1$
$c_2$	$u^3 + u^2 - 1$
$c_4$	$u^3 - u^2 + 1$
$c_5, c_6, c_9$ $c_{10}$	$u^3$
$c_7$	$u^3 + u^2 + 2u + 1$
$c_8$	$(u - 1)^3$
$c_{11}, c_{12}$	$(u + 1)^3$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_3, c_7$	$y^3 + 3y^2 + 2y - 1$
$c_2, c_4$	$y^3 - y^2 + 2y - 1$
$c_5, c_6, c_9$ $c_{10}$	$y^3$
$c_8, c_{11}, c_{12}$	$(y - 1)^3$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.00000$		
$a = -0.215080 + 1.307140I$	$1.37919 + 2.82812I$	$-9.90089 - 6.32406I$
$b = 0.122561 + 0.744862I$		
$u = -1.00000$		
$a = -0.215080 - 1.307140I$	$1.37919 - 2.82812I$	$-9.90089 + 6.32406I$
$b = 0.122561 - 0.744862I$		
$u = -1.00000$		
$a = -0.569840$	$-2.75839$	$-30.1980$
$b = 1.75488$		

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u^3 - u^2 + 2u - 1)^3)(u^{48} + 28u^{47} + \dots + 2u + 1)$
$c_2$	$((u^3 + u^2 - 1)^3)(u^{48} - 4u^{47} + \dots + 2u + 1)$
$c_3$	$(u^3 - u^2 + 2u - 1)(u^3 + u^2 + 2u + 1)^2(u^{48} + 2u^{47} + \dots + 8u - 1)$
$c_4$	$((u^3 - u^2 + 1)^3)(u^{48} - 4u^{47} + \dots + 2u + 1)$
$c_5, c_6, c_{10}$	$u^3(u^2 - 2)^3(u^{48} - 3u^{47} + \dots - 8u - 8)$
$c_7$	$((u^3 - u^2 + 2u - 1)^2)(u^3 + u^2 + 2u + 1)(u^{48} + 2u^{47} + \dots + 8u - 1)$
$c_8$	$((u - 1)^3)(u + 1)^6(u^{48} + 4u^{47} + \dots + 59u - 7)$
$c_9$	$u^3(u^2 - 2)^3(u^{48} + 9u^{47} + \dots - 6632u - 1192)$
$c_{11}$	$((u - 1)^6)(u + 1)^3(u^{48} + 4u^{47} + \dots + 59u - 7)$
$c_{12}$	$((u + 1)^9)(u^{48} + 54u^{47} + \dots + 4853u + 49)$

## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y^3 + 3y^2 + 2y - 1)^3)(y^{48} - 12y^{47} + \dots - 250y + 1)$
$c_2, c_4$	$((y^3 - y^2 + 2y - 1)^3)(y^{48} - 28y^{47} + \dots - 2y + 1)$
$c_3, c_7$	$((y^3 + 3y^2 + 2y - 1)^3)(y^{48} + 12y^{47} + \dots - 42y + 1)$
$c_5, c_6, c_{10}$	$y^3(y - 2)^6(y^{48} - 41y^{47} + \dots - 1984y + 64)$
$c_8, c_{11}$	$((y - 1)^9)(y^{48} - 54y^{47} + \dots - 4853y + 49)$
$c_9$	$y^3(y - 2)^6(y^{48} + 43y^{47} + \dots - 5.81730 \times 10^7 y + 1420864)$
$c_{12}$	$((y - 1)^9)(y^{48} - 110y^{47} + \dots - 3.03518 \times 10^7 y + 2401)$