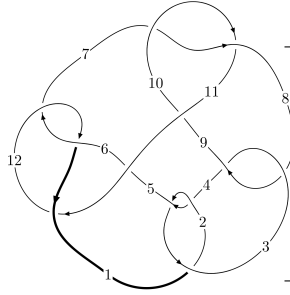
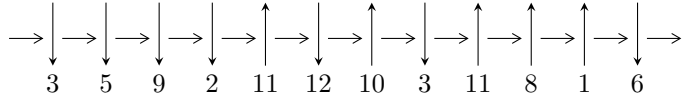


12n₀₂₂₁ (K12n₀₂₂₁)



A knot diagram¹

Linearized knot diagram



Solving Sequence

$$2,5 \xrightarrow{c_2} 3,11 \xrightarrow{c_5} 6 \xrightarrow{c_1} 1 \xrightarrow{c_4} 4,9 \xrightarrow{c_8} 8 \xrightarrow{c_{10}} 10 \xrightarrow{c_{12}} 12 \xrightarrow{c_6} 7 \longrightarrow c_3, c_7, c_9, c_{11}$$

Ideals for irreducible components² of X_{par}

$$I_1^u = \langle 2258808925u^{16} + 15860359921u^{15} + \dots + 96479313856d - 115431185160, \\ 154139657205u^{16} + 1193150529938u^{15} + \dots + 2508462160256c - 2116899433348, \\ 4410667u^{16} + 32637095u^{15} + \dots + 83243584b - 126220048, \\ 7888753u^{16} + 58699357u^{15} + \dots + 83243584a - 107418368, u^{17} + 8u^{16} + \dots - 8u - 16 \rangle$$

$$I_2^u = \langle d - a, c - a, b - a, a^2 - a + 1, u - 1 \rangle$$

$$I_3^u = \langle d + 1, c, b - 1, a - 1, u - 1 \rangle$$

$$I_4^u = \langle da - ca + 1, c^2 - c + 1, b - a, u - 1 \rangle$$

$$I_1^v = \langle c, d - a - 1, b, a^2 + a + 1, v - 1 \rangle$$

* 4 irreducible components of $\dim_{\mathbb{C}} = 0$, with total 22 representations.

* 1 irreducible components of $\dim_{\mathbb{C}} = 1$

¹The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

²All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle 2.26 \times 10^9 u^{16} + 1.59 \times 10^{10} u^{15} + \dots + 9.65 \times 10^{10} d - 1.15 \times 10^{11}, 1.54 \times 10^{11} u^{16} + 1.19 \times 10^{12} u^{15} + \dots + 2.51 \times 10^{12} c - 2.12 \times 10^{12}, 4.41 \times 10^6 u^{16} + 3.26 \times 10^7 u^{15} + \dots + 8.32 \times 10^7 b - 1.26 \times 10^8, 7.89 \times 10^6 u^{16} + 5.87 \times 10^7 u^{15} + \dots + 8.32 \times 10^7 a - 1.07 \times 10^8, u^{17} + 8u^{16} + \dots - 8u - 16 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -0.0614479u^{16} - 0.475650u^{15} + \dots - 2.87951u + 0.843903 \\ -0.0234124u^{16} - 0.164391u^{15} + \dots + 0.757508u + 1.19643 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0.0269638u^{16} + 0.213054u^{15} + \dots + 1.26938u + 0.117878 \\ -0.00435471u^{16} - 0.0521688u^{15} + \dots + 0.124403u - 0.745012 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 + 1 \\ -u^4 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u \\ u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -0.0947671u^{16} - 0.705152u^{15} + \dots + 0.385893u + 1.29041 \\ -0.0529851u^{16} - 0.392067u^{15} + \dots - 0.532273u + 1.51627 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -0.115939u^{16} - 0.867332u^{15} + \dots + 0.946013u + 1.95892 \\ -0.0618930u^{16} - 0.460152u^{15} + \dots - 0.813462u + 1.63140 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -0.118012u^{16} - 0.895172u^{15} + \dots - 0.266541u + 1.78486 \\ -0.0519178u^{16} - 0.373867u^{15} + \dots + 0.197985u + 1.94957 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -0.0656989u^{16} - 0.494197u^{15} + \dots - 2.08468u + 1.58103 \\ -0.0178046u^{16} - 0.118986u^{15} + \dots + 0.894693u + 1.10503 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0.0678023u^{16} + 0.515689u^{15} + \dots + 1.99680u - 0.449277 \\ 0.0201677u^{16} + 0.140474u^{15} + \dots + 0.214188u - 1.03624 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -\frac{3288027181}{156778885016} u^{16} - \frac{220303787499}{627115540064} u^{15} + \dots - \frac{1274298730399}{156778885016} u - \frac{92162275708}{19597360627}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1	$u^{17} - 6u^{16} + \dots + 32u + 256$
c_2, c_4	$u^{17} - 8u^{16} + \dots - 8u + 16$
c_3, c_8	$u^{17} + u^{16} + \dots - 1024u + 512$
c_5	$u^{17} + 14u^{16} + \dots + 6768u + 2592$
c_6, c_{12}	$u^{17} - 5u^{16} + \dots - 11u^2 + 4$
c_7, c_{10}	$u^{17} + 8u^{16} + \dots - 8u + 16$
c_9	$u^{17} - 34u^{16} + \dots + 6176u - 256$
c_{11}	$u^{17} - 15u^{16} + \dots + 88u + 16$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1	$y^{17} + 66y^{16} + \dots + 2613760y - 65536$
c_2, c_4	$y^{17} + 6y^{16} + \dots + 32y - 256$
c_3, c_8	$y^{17} + 81y^{16} + \dots - 524288y - 262144$
c_5	$y^{17} - 66y^{16} + \dots + 36764928y - 6718464$
c_6, c_{12}	$y^{17} + 15y^{16} + \dots + 88y - 16$
c_7, c_{10}	$y^{17} - 34y^{16} + \dots + 6176y - 256$
c_9	$y^{17} - 94y^{16} + \dots + 7397888y - 65536$
c_{11}	$y^{17} - 21y^{16} + \dots + 36640y - 256$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.789321$ $a = 0.386224$ $b = -0.304855$ $c = 0.374974$ $d = -0.586930$	-1.13318	-9.61860
$u = 1.281020 + 0.078323I$ $a = 0.027802 - 0.314358I$ $b = -0.060236 + 0.400521I$ $c = 0.519378 - 0.243814I$ $d = 0.992769 + 0.281518I$	$-0.72956 - 1.37071I$	$-0.698150 + 0.213889I$
$u = 1.281020 - 0.078323I$ $a = 0.027802 + 0.314358I$ $b = -0.060236 - 0.400521I$ $c = 0.519378 + 0.243814I$ $d = 0.992769 - 0.281518I$	$-0.72956 + 1.37071I$	$-0.698150 - 0.213889I$
$u = -0.709544 + 0.075286I$ $a = 0.27087 + 2.81412I$ $b = 0.40406 + 1.97635I$ $c = 0.41875 + 1.62653I$ $d = 0.386110 + 1.184900I$	$-0.79868 + 2.33972I$	$0.33078 - 5.26516I$
$u = -0.709544 - 0.075286I$ $a = 0.27087 - 2.81412I$ $b = 0.40406 - 1.97635I$ $c = 0.41875 - 1.62653I$ $d = 0.386110 - 1.184900I$	$-0.79868 - 2.33972I$	$0.33078 + 5.26516I$
$u = 0.491842 + 0.197993I$ $a = -0.746901 + 0.354453I$ $b = 0.437536 - 0.026454I$ $c = -0.68906 + 1.41559I$ $d = 1.212590 - 0.601554I$	$0.77904 - 2.74622I$	$-2.48507 + 7.16740I$

Solutions to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.491842 - 0.197993I$ $a = -0.746901 - 0.354453I$ $b = 0.437536 + 0.026454I$ $c = -0.68906 - 1.41559I$ $d = 1.212590 + 0.601554I$	$0.77904 + 2.74622I$	$-2.48507 - 7.16740I$
$u = -0.118015 + 0.350813I$ $a = -0.50791 + 1.75422I$ $b = 0.555461 + 0.385207I$ $c = 1.154330 + 0.052946I$ $d = 0.083184 + 0.147223I$	$1.75773 + 0.71028I$	$3.71531 + 0.02644I$
$u = -0.118015 - 0.350813I$ $a = -0.50791 - 1.75422I$ $b = 0.555461 - 0.385207I$ $c = 1.154330 - 0.052946I$ $d = 0.083184 - 0.147223I$	$1.75773 - 0.71028I$	$3.71531 - 0.02644I$
$u = -1.65818 + 0.90820I$ $a = 0.976512 + 0.609189I$ $b = 2.17250 + 0.12328I$ $c = -0.94081 - 1.33349I$ $d = -5.94499 - 2.01193I$	$18.4182 + 12.9335I$	$-1.01650 - 5.27491I$
$u = -1.65818 - 0.90820I$ $a = 0.976512 - 0.609189I$ $b = 2.17250 - 0.12328I$ $c = -0.94081 + 1.33349I$ $d = -5.94499 + 2.01193I$	$18.4182 - 12.9335I$	$-1.01650 + 5.27491I$
$u = -1.62328 + 1.28695I$ $a = -0.780321 - 0.614432I$ $b = -2.05742 + 0.00684I$ $c = 0.67125 + 1.78783I$ $d = 8.16028 + 1.30179I$	$15.3110 + 5.6503I$	$-2.10303 - 1.68119I$

Solutions to I_1^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.62328 - 1.28695I$ $a = -0.780321 + 0.614432I$ $b = -2.05742 - 0.00684I$ $c = 0.67125 - 1.78783I$ $d = 8.16028 - 1.30179I$	$15.3110 - 5.6503I$	$-2.10303 + 1.68119I$
$u = -0.77580 + 2.21598I$ $a = -0.394911 - 0.622157I$ $b = -1.68506 + 0.39245I$ $c = -1.69156 - 0.56408I$ $d = 4.42924 + 7.49395I$	$9.63429 + 3.26152I$	$0.10201 - 1.44169I$
$u = -0.77580 - 2.21598I$ $a = -0.394911 + 0.622157I$ $b = -1.68506 - 0.39245I$ $c = -1.69156 + 0.56408I$ $d = 4.42924 - 7.49395I$	$9.63429 - 3.26152I$	$0.10201 + 1.44169I$
$u = -1.28271 + 2.40373I$ $a = 0.461749 + 0.538038I$ $b = 1.88559 - 0.41977I$ $c = -0.25477 - 2.11813I$ $d = -12.0257 + 7.8857I$	$-17.4865 - 1.7702I$	$-60.10 + 0.657690I$
$u = -1.28271 - 2.40373I$ $a = 0.461749 - 0.538038I$ $b = 1.88559 + 0.41977I$ $c = -0.25477 + 2.11813I$ $d = -12.0257 - 7.8857I$	$-17.4865 + 1.7702I$	$-60.10 - 0.657690I$

$$\text{II. } I_2^u = \langle d - a, c - a, b - a, a^2 - a + 1, u - 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} a \\ a \end{pmatrix}$$

$$a_6 = \begin{pmatrix} a - 1 \\ a \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a \\ a \end{pmatrix}$$

$$a_8 = \begin{pmatrix} a \\ a \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} a \\ a \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} a \\ a \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = $-4a - 7$

(iv) **u**-Polynomials at the component

Crossings	u -Polynomials at each crossing
c_1, c_2	$(u - 1)^2$
c_3, c_7, c_8 c_9, c_{10}	u^2
c_4	$(u + 1)^2$
c_5, c_{11}, c_{12}	$u^2 + u + 1$
c_6	$u^2 - u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_2, c_4	$(y - 1)^2$
c_3, c_7, c_8 c_9, c_{10}	y^2
c_5, c_6, c_{11} c_{12}	$y^2 + y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_2^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$		
$a = 0.500000 + 0.866025I$		
$b = 0.500000 + 0.866025I$	$-1.64493 + 2.02988I$	$-9.00000 - 3.46410I$
$c = 0.500000 + 0.866025I$		
$d = 0.500000 + 0.866025I$		
$u = 1.00000$		
$a = 0.500000 - 0.866025I$		
$b = 0.500000 - 0.866025I$	$-1.64493 - 2.02988I$	$-9.00000 + 3.46410I$
$c = 0.500000 - 0.866025I$		
$d = 0.500000 - 0.866025I$		

$$\text{III. } I_3^u = \langle d + 1, c, b - 1, a - 1, u - 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = 0

(iv) **u**-Polynomials at the component

Crossings	u -Polynomials at each crossing
c_1, c_2, c_{10}	$u - 1$
c_3, c_5, c_6 c_8, c_{11}, c_{12}	u
c_4, c_7, c_9	$u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_2, c_4 c_7, c_9, c_{10}	$y - 1$
c_3, c_5, c_6 c_8, c_{11}, c_{12}	y

(vi) Complex Volumes and Cusp Shapes

Solutions to I_3^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$		
$a = 1.00000$		
$b = 1.00000$	0	0
$c = 0$		
$d = -1.00000$		

$$\text{IV. } I_4^u = \langle da - ca + 1, c^2 - c + 1, b - a, u - 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} c \\ d \end{pmatrix}$$

$$a_6 = \begin{pmatrix} c - 1 \\ dc + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a \\ a \end{pmatrix}$$

$$a_8 = \begin{pmatrix} a \\ a \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} c + a \\ d + a \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} c \\ d - c \end{pmatrix}$$

$$a_7 = \begin{pmatrix} c \\ d \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes = $d^2 - 2dc + a^2 - 3c - 1$

(iv) u-Polynomials at the component : It cannot be defined for a positive dimension component.

(v) Riley Polynomials at the component : It cannot be defined for a positive dimension component.

(iv) Complex Volumes and Cusp Shapes

Solution to I_4^u	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = \dots$		
$a = \dots$		
$b = \dots$	$-2.02988I$	$-0.06692 - 3.42770I$
$c = \dots$		
$d = \dots$		

$$\mathbf{V. } I_1^v = \langle c, d - a - 1, b, a^2 + a + 1, v - 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ a + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ -a \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} a \\ 0 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} a \\ a + 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} a + 1 \\ a + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ -a - 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = $-4a + 1$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
c_1, c_2, c_3 c_4, c_8	u^2
c_5, c_{12}	$u^2 - u + 1$
c_6, c_{11}	$u^2 + u + 1$
c_7, c_9	$(u + 1)^2$
c_{10}	$(u - 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
c_1, c_2, c_3 c_4, c_8	y^2
c_5, c_6, c_{11} c_{12}	$y^2 + y + 1$
c_7, c_9, c_{10}	$(y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to I_1^v	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$v = 1.00000$ $a = -0.500000 + 0.866025I$ $b = 0$ $c = 0$ $d = 0.500000 + 0.866025I$	$1.64493 + 2.02988I$	$3.00000 - 3.46410I$
$v = 1.00000$ $a = -0.500000 - 0.866025I$ $b = 0$ $c = 0$ $d = 0.500000 - 0.866025I$	$1.64493 - 2.02988I$	$3.00000 + 3.46410I$

VI. u-Polynomials

Crossings	u-Polynomials at each crossing
c_1	$u^2(u-1)^3(u^{17} - 6u^{16} + \dots + 32u + 256)$
c_2	$u^2(u-1)^3(u^{17} - 8u^{16} + \dots - 8u + 16)$
c_3, c_8	$u^5(u^{17} + u^{16} + \dots - 1024u + 512)$
c_4	$u^2(u+1)^3(u^{17} - 8u^{16} + \dots - 8u + 16)$
c_5	$u(u^2 - u + 1)(u^2 + u + 1)(u^{17} + 14u^{16} + \dots + 6768u + 2592)$
c_6, c_{12}	$u(u^2 - u + 1)(u^2 + u + 1)(u^{17} - 5u^{16} + \dots - 11u^2 + 4)$
c_7	$u^2(u+1)^3(u^{17} + 8u^{16} + \dots - 8u + 16)$
c_9	$u^2(u+1)^3(u^{17} - 34u^{16} + \dots + 6176u - 256)$
c_{10}	$u^2(u-1)^3(u^{17} + 8u^{16} + \dots - 8u + 16)$
c_{11}	$u(u^2 + u + 1)^2(u^{17} - 15u^{16} + \dots + 88u + 16)$

VII. Riley Polynomials

Crossings	Riley Polynomials at each crossing
c_1	$y^2(y-1)^3(y^{17} + 66y^{16} + \dots + 2613760y - 65536)$
c_2, c_4	$y^2(y-1)^3(y^{17} + 6y^{16} + \dots + 32y - 256)$
c_3, c_8	$y^5(y^{17} + 81y^{16} + \dots - 524288y - 262144)$
c_5	$y(y^2 + y + 1)^2(y^{17} - 66y^{16} + \dots + 3.67649 \times 10^7 y - 6718464)$
c_6, c_{12}	$y(y^2 + y + 1)^2(y^{17} + 15y^{16} + \dots + 88y - 16)$
c_7, c_{10}	$y^2(y-1)^3(y^{17} - 34y^{16} + \dots + 6176y - 256)$
c_9	$y^2(y-1)^3(y^{17} - 94y^{16} + \dots + 7397888y - 65536)$
c_{11}	$y(y^2 + y + 1)^2(y^{17} - 21y^{16} + \dots + 36640y - 256)$