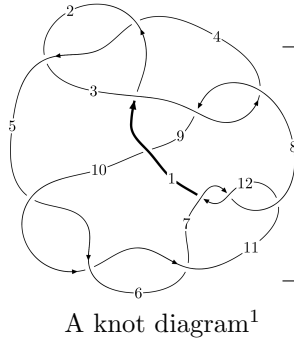
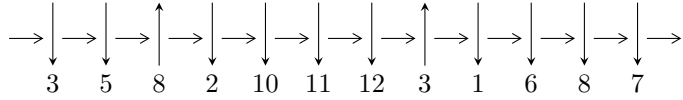


12n<sub>0237</sub> (K12n<sub>0237</sub>)



**Linearized knot diagram**



**Solving Sequence**

$$8, 11 \xrightarrow{c_{11}} 4, 12 \xrightarrow{c_3} 3 \xrightarrow{c_8} 9 \xrightarrow{c_7} 7 \xrightarrow{c_{12}} 1 \xrightarrow{c_1} 2 \xrightarrow{c_6} 6 \xrightarrow{c_{10}} 10 \xrightarrow{c_5} 5 \rightsquigarrow c_2, c_4, c_9$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle u^{41} + u^{40} + \dots + b + 2u, u^{41} + u^{40} + \dots + a - 4u, u^{42} + 2u^{41} + \dots + u + 1 \rangle$$

$$I_2^u = \langle -u^4 + u^3 - 2u^2 + b + u, a, u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 48 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle u^{41} + u^{40} + \dots + b + 2u, u^{41} + u^{40} + \dots + a - 4u, u^{42} + 2u^{41} + \dots + u + 1 \rangle \quad \mathbf{I.}$$

(i) Arc colorings

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^{41} - u^{40} + \dots + 2u^2 + 4u \\ -u^{41} - u^{40} + \dots - 8u^2 - 2u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^{41} - u^{40} + \dots + 2u^2 + 4u \\ -u^{41} - 2u^{40} + \dots - 2u - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^{12} + 5u^{10} + 9u^8 + 4u^6 - 6u^4 - 5u^2 + 1 \\ u^{14} + 6u^{12} + 13u^{10} + 10u^8 - 4u^6 - 8u^4 - u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^9 + 4u^7 + 5u^5 - 3u \\ -u^{37} - u^{36} + \dots + 8u^2 + 2u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^3 + 2u \\ u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^6 - 3u^4 - 2u^2 + 1 \\ -u^6 - 2u^4 - u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^9 - 4u^7 - 5u^5 + 3u \\ -u^9 - 3u^7 - 3u^5 + u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $-4u^{41} - 8u^{40} + \dots + 15u - 9$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{42} + 13u^{41} + \dots + 28u + 1$
$c_2, c_4$	$u^{42} - 7u^{41} + \dots - 8u + 1$
$c_3, c_8$	$u^{42} + u^{41} + \dots + 192u + 64$
$c_5, c_6, c_{10}$	$u^{42} - 2u^{41} + \dots - 55u + 17$
$c_7, c_{11}, c_{12}$	$u^{42} + 2u^{41} + \dots + u + 1$
$c_9$	$u^{42} + 2u^{41} + \dots + 5u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{42} + 39y^{41} + \dots - 212y + 1$
$c_2, c_4$	$y^{42} - 13y^{41} + \dots - 28y + 1$
$c_3, c_8$	$y^{42} - 39y^{41} + \dots - 90112y + 4096$
$c_5, c_6, c_{10}$	$y^{42} - 38y^{41} + \dots - 4011y + 289$
$c_7, c_{11}, c_{12}$	$y^{42} + 34y^{41} + \dots - 19y + 1$
$c_9$	$y^{42} + 46y^{41} + \dots - 19y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.884997$ $a = -0.685840$ $b = -0.554744$	-8.59934	-6.83080
$u = -0.860411 + 0.091340I$ $a = 1.60817 + 0.83486I$ $b = 0.281208 + 1.248210I$	$-1.28738 + 8.63776I$	$-11.30852 - 5.50570I$
$u = -0.860411 - 0.091340I$ $a = 1.60817 - 0.83486I$ $b = 0.281208 - 1.248210I$	$-1.28738 - 8.63776I$	$-11.30852 + 5.50570I$
$u = -0.819362 + 0.102352I$ $a = -1.76027 - 0.48960I$ $b = -0.358516 - 0.802687I$	$0.15821 + 2.18866I$	$-9.63326 - 1.25581I$
$u = -0.819362 - 0.102352I$ $a = -1.76027 + 0.48960I$ $b = -0.358516 + 0.802687I$	$0.15821 - 2.18866I$	$-9.63326 + 1.25581I$
$u = -0.818580$ $a = 1.24094$ $b = -0.564691$	-7.20526	-12.4480
$u = 0.813444 + 0.036008I$ $a = -0.240335 - 1.364030I$ $b = -0.236106 - 1.145350I$	$-5.56172 - 2.39562I$	$-13.13795 + 3.14651I$
$u = 0.813444 - 0.036008I$ $a = -0.240335 + 1.364030I$ $b = -0.236106 + 1.145350I$	$-5.56172 + 2.39562I$	$-13.13795 - 3.14651I$
$u = -0.360418 + 1.144850I$ $a = 0.461015 + 1.052370I$ $b = 0.396025 + 0.874829I$	$3.34850 + 2.08589I$	$-6.37713 - 2.84313I$
$u = -0.360418 - 1.144850I$ $a = 0.461015 - 1.052370I$ $b = 0.396025 - 0.874829I$	$3.34850 - 2.08589I$	$-6.37713 + 2.84313I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.039780 + 1.228480I$ $a = -0.408011 + 0.443145I$ $b = 0.69289 + 2.43964I$	$1.48060 - 0.96666I$	$-6.26750 - 1.40491I$
$u = 0.039780 - 1.228480I$ $a = -0.408011 - 0.443145I$ $b = 0.69289 - 2.43964I$	$1.48060 + 0.96666I$	$-6.26750 + 1.40491I$
$u = -0.193377 + 1.231330I$ $a = 0.262816 + 0.483914I$ $b = 0.802069 + 0.546478I$	$2.73948 + 2.47038I$	$-3.06420 - 3.92417I$
$u = -0.193377 - 1.231330I$ $a = 0.262816 - 0.483914I$ $b = 0.802069 - 0.546478I$	$2.73948 - 2.47038I$	$-3.06420 + 3.92417I$
$u = -0.415470 + 1.179020I$ $a = -0.775903 - 0.927476I$ $b = -0.095078 - 0.588213I$	$2.05152 - 4.06193I$	$-8.19749 + 0.I$
$u = -0.415470 - 1.179020I$ $a = -0.775903 + 0.927476I$ $b = -0.095078 + 0.588213I$	$2.05152 + 4.06193I$	$-8.19749 + 0.I$
$u = 0.357741 + 1.238750I$ $a = -0.744213 - 0.408723I$ $b = 0.418778 - 1.322100I$	$-1.85166 - 1.82090I$	$-9.63780 + 0.I$
$u = 0.357741 - 1.238750I$ $a = -0.744213 + 0.408723I$ $b = 0.418778 + 1.322100I$	$-1.85166 + 1.82090I$	$-9.63780 + 0.I$
$u = -0.079247 + 1.295310I$ $a = 0.637223 - 0.258877I$ $b = 1.01766 - 1.34640I$	$4.02758 + 2.15699I$	0
$u = -0.079247 - 1.295310I$ $a = 0.637223 + 0.258877I$ $b = 1.01766 + 1.34640I$	$4.02758 - 2.15699I$	0

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.365234 + 1.269190I$ $a = -0.284658 - 0.718402I$ $b = -0.34989 - 2.35592I$	$-3.26541 + 4.25757I$	0
$u = -0.365234 - 1.269190I$ $a = -0.284658 + 0.718402I$ $b = -0.34989 + 2.35592I$	$-3.26541 - 4.25757I$	0
$u = 0.517463 + 0.417209I$ $a = 1.47848 - 1.77950I$ $b = 0.187293 - 1.152780I$	$4.73868 - 4.97358I$	$-7.62188 + 6.30595I$
$u = 0.517463 - 0.417209I$ $a = 1.47848 + 1.77950I$ $b = 0.187293 + 1.152780I$	$4.73868 + 4.97358I$	$-7.62188 - 6.30595I$
$u = 0.443990 + 0.494284I$ $a = -1.67760 + 1.53589I$ $b = -0.210344 + 0.860490I$	$5.02884 + 1.46246I$	$-6.53686 + 0.89586I$
$u = 0.443990 - 0.494284I$ $a = -1.67760 - 1.53589I$ $b = -0.210344 - 0.860490I$	$5.02884 - 1.46246I$	$-6.53686 - 0.89586I$
$u = 0.418536 + 1.277910I$ $a = 0.134196 - 0.433061I$ $b = 0.636951 - 0.103857I$	$-4.63050 - 4.66445I$	0
$u = 0.418536 - 1.277910I$ $a = 0.134196 + 0.433061I$ $b = 0.636951 + 0.103857I$	$-4.63050 + 4.66445I$	0
$u = 0.361948 + 1.295330I$ $a = 0.843712 + 0.183840I$ $b = 0.34096 + 1.49506I$	$-1.40855 - 6.62685I$	0
$u = 0.361948 - 1.295330I$ $a = 0.843712 - 0.183840I$ $b = 0.34096 - 1.49506I$	$-1.40855 + 6.62685I$	0

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.361210 + 1.333990I$ $a = 0.133728 + 1.145250I$ $b = 1.68733 + 2.67654I$	$4.66389 + 6.44264I$	0
$u = -0.361210 - 1.333990I$ $a = 0.133728 - 1.145250I$ $b = 1.68733 - 2.67654I$	$4.66389 - 6.44264I$	0
$u = 0.108456 + 1.380140I$ $a = 0.156921 - 1.186400I$ $b = 1.07791 - 4.11722I$	$10.88470 - 0.27094I$	0
$u = 0.108456 - 1.380140I$ $a = 0.156921 + 1.186400I$ $b = 1.07791 + 4.11722I$	$10.88470 + 0.27094I$	0
$u = 0.146369 + 1.377200I$ $a = 0.141022 + 1.192190I$ $b = -0.84503 + 4.22749I$	$10.39390 - 7.18776I$	0
$u = 0.146369 - 1.377200I$ $a = 0.141022 - 1.192190I$ $b = -0.84503 - 4.22749I$	$10.39390 + 7.18776I$	0
$u = -0.385158 + 1.335160I$ $a = 0.110632 - 1.155620I$ $b = -1.67195 - 2.97617I$	$3.18564 + 13.11100I$	0
$u = -0.385158 - 1.335160I$ $a = 0.110632 + 1.155620I$ $b = -1.67195 + 2.97617I$	$3.18564 - 13.11100I$	0
$u = -0.492418$ $a = -0.979682$ $b = -0.240678$	$-0.983238$	$-9.79060$
$u = -0.283222 + 0.253637I$ $a = -1.15749 + 1.43241I$ $b = -0.099702 + 0.442777I$	$-0.614323 + 0.919516I$	$-9.08610 - 7.37537I$



Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.283222 - 0.253637I$		
$a = -1.15749 - 1.43241I$	$-0.614323 - 0.919516I$	$-9.08610 + 7.37537I$
$b = -0.099702 - 0.442777I$		
$u = 0.256764$		
$a = 1.58568$	$-2.02811$	$1.75710$
$b = -0.984824$		

$$\text{II. } I_2^u = \langle -u^4 + u^3 - 2u^2 + b + u, a, u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1 \rangle$$

(i) Arc colorings

$$a_8 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0 \\ u^4 - u^3 + 2u^2 - u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0 \\ u^4 - u^3 + 2u^2 - u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^2 + 1 \\ 2u^4 - u^3 + 4u^2 - u \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^3 + 2u \\ u^3 + u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^5 - 2u^3 - u \\ -u^5 + u^4 - 2u^3 + u^2 - u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^2 - 1 \\ -u^4 - 2u^2 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-5u^4 + 6u^3 - 11u^2 + 6u - 17$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$(u - 1)^6$
$c_3, c_8$	$u^6$
$c_4$	$(u + 1)^6$
$c_5, c_6, c_9$	$u^6 - u^5 - 3u^4 + 2u^3 + 2u^2 + u - 1$
$c_7$	$u^6 + u^5 + 3u^4 + 2u^3 + 2u^2 + u - 1$
$c_{10}$	$u^6 + u^5 - 3u^4 - 2u^3 + 2u^2 - u - 1$
$c_{11}, c_{12}$	$u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y - 1)^6$
$c_3, c_8$	$y^6$
$c_5, c_6, c_9$ $c_{10}$	$y^6 - 7y^5 + 17y^4 - 16y^3 + 6y^2 - 5y + 1$
$c_7, c_{11}, c_{12}$	$y^6 + 5y^5 + 9y^4 + 4y^3 - 6y^2 - 5y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.873214$ $a = 0$ $b = 0.567375$	$-9.30502$	$-19.0600$
$u = -0.138835 + 1.234450I$ $a = 0$ $b = -1.35607 + 0.92119I$	$1.31531 + 1.97241I$	$-8.22189 - 4.83849I$
$u = -0.138835 - 1.234450I$ $a = 0$ $b = -1.35607 - 0.92119I$	$1.31531 - 1.97241I$	$-8.22189 + 4.83849I$
$u = 0.408802 + 1.276380I$ $a = 0$ $b = -0.354716 - 0.801205I$	$-5.34051 - 4.59213I$	$-15.2853 + 2.7994I$
$u = 0.408802 - 1.276380I$ $a = 0$ $b = -0.354716 + 0.801205I$	$-5.34051 + 4.59213I$	$-15.2853 - 2.7994I$
$u = -0.413150$ $a = 0$ $b = 0.854195$	$-2.38379$	$-21.9250$

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u-1)^6)(u^{42} + 13u^{41} + \dots + 28u + 1)$
$c_2$	$((u-1)^6)(u^{42} - 7u^{41} + \dots - 8u + 1)$
$c_3, c_8$	$u^6(u^{42} + u^{41} + \dots + 192u + 64)$
$c_4$	$((u+1)^6)(u^{42} - 7u^{41} + \dots - 8u + 1)$
$c_5, c_6$	$(u^6 - u^5 - 3u^4 + 2u^3 + 2u^2 + u - 1)(u^{42} - 2u^{41} + \dots - 55u + 17)$
$c_7$	$(u^6 + u^5 + 3u^4 + 2u^3 + 2u^2 + u - 1)(u^{42} + 2u^{41} + \dots + u + 1)$
$c_9$	$(u^6 - u^5 - 3u^4 + 2u^3 + 2u^2 + u - 1)(u^{42} + 2u^{41} + \dots + 5u + 1)$
$c_{10}$	$(u^6 + u^5 - 3u^4 - 2u^3 + 2u^2 - u - 1)(u^{42} - 2u^{41} + \dots - 55u + 17)$
$c_{11}, c_{12}$	$(u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1)(u^{42} + 2u^{41} + \dots + u + 1)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y - 1)^6)(y^{42} + 39y^{41} + \dots - 212y + 1)$
$c_2, c_4$	$((y - 1)^6)(y^{42} - 13y^{41} + \dots - 28y + 1)$
$c_3, c_8$	$y^6(y^{42} - 39y^{41} + \dots - 90112y + 4096)$
$c_5, c_6, c_{10}$	$(y^6 - 7y^5 + 17y^4 - 16y^3 + 6y^2 - 5y + 1)$ $\cdot (y^{42} - 38y^{41} + \dots - 4011y + 289)$
$c_7, c_{11}, c_{12}$	$(y^6 + 5y^5 + \dots - 5y + 1)(y^{42} + 34y^{41} + \dots - 19y + 1)$
$c_9$	$(y^6 - 7y^5 + \dots - 5y + 1)(y^{42} + 46y^{41} + \dots - 19y + 1)$