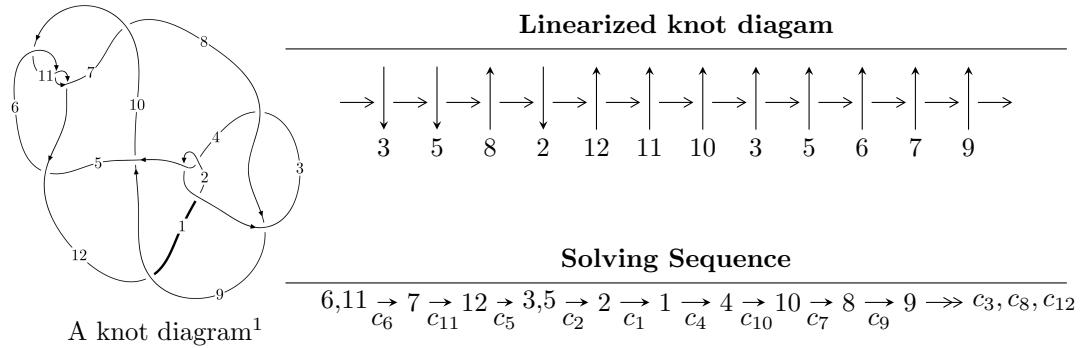


$12n_{0239}$  ( $K12n_{0239}$ )



Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$

$$I_1^u = \langle -u^{28} - u^{27} + \dots + b + u, 3u^{28} + 3u^{27} + \dots + a + 1, u^{29} + 2u^{28} + \dots - u + 1 \rangle$$

$$I_2^u = \langle -u^3 + b + u + 1, -u^6 + 3u^4 + u^3 - 2u^2 + a - u - 2, u^8 - u^7 - 3u^6 + 2u^5 + 3u^4 - 2u - 1 \rangle$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 37 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle -u^{28} - u^{27} + \dots + b + u, \ 3u^{28} + 3u^{27} + \dots + a + 1, \ u^{29} + 2u^{28} + \dots - u + 1 \rangle^{\text{I.}}$$

(i) Arc colorings

$$\begin{aligned} a_6 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_7 &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} u \\ -u^3 + u \end{pmatrix} \\ a_3 &= \begin{pmatrix} -3u^{28} - 3u^{27} + \dots - u - 1 \\ u^{28} + u^{27} + \dots + 6u^2 - u \end{pmatrix} \\ a_5 &= \begin{pmatrix} -u^4 + u^2 + 1 \\ u^6 - 2u^4 + u^2 \end{pmatrix} \\ a_2 &= \begin{pmatrix} -2u^{28} - 2u^{27} + \dots - 3u - 1 \\ u^{28} + u^{27} + \dots + 5u^2 - u \end{pmatrix} \\ a_1 &= \begin{pmatrix} -u^{21} + 8u^{19} + \dots - 6u^3 - u \\ u^{23} - 9u^{21} + \dots + 4u^3 - u \end{pmatrix} \\ a_4 &= \begin{pmatrix} -u^{28} - u^{27} + \dots - 5u^2 - 3u \\ u^{28} + u^{27} + \dots + 8u^3 + 5u^2 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} -u \\ u \end{pmatrix} \\ a_8 &= \begin{pmatrix} -u^4 + u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix} \\ a_9 &= \begin{pmatrix} u^{11} - 4u^9 + 4u^7 + 2u^5 - 3u^3 - 2u \\ -u^{13} + 5u^{11} - 9u^9 + 6u^7 - u^3 + u \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$\begin{aligned} &= -5u^{28} - 4u^{27} + 53u^{26} + 33u^{25} - 247u^{24} - 102u^{23} + 634u^{22} + 96u^{21} - 879u^{20} + \\ &214u^{19} + 362u^{18} - 658u^{17} + 771u^{16} + 486u^{15} - 1175u^{14} + 371u^{13} + 217u^{12} - 670u^{11} + \\ &671u^{10} - 12u^9 - 364u^8 + 380u^7 - 164u^6 - 26u^5 + 109u^4 - 112u^3 + 24u^2 - 15u - 1 \end{aligned}$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^{29} + 45u^{28} + \cdots + 63u + 1$
$c_2, c_4$	$u^{29} - 9u^{28} + \cdots - 15u + 1$
$c_3, c_8$	$u^{29} + u^{28} + \cdots - 640u + 256$
$c_5, c_7$	$u^{29} + 6u^{28} + \cdots - 27u - 7$
$c_6, c_{10}, c_{11}$	$u^{29} - 2u^{28} + \cdots - u - 1$
$c_9$	$u^{29} + 2u^{28} + \cdots + 847u - 505$
$c_{12}$	$u^{29} + 30u^{27} + \cdots - u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{29} - 113y^{28} + \cdots + 3195y - 1$
$c_2, c_4$	$y^{29} - 45y^{28} + \cdots + 63y - 1$
$c_3, c_8$	$y^{29} + 51y^{28} + \cdots + 835584y - 65536$
$c_5, c_7$	$y^{29} + 24y^{28} + \cdots + 239y - 49$
$c_6, c_{10}, c_{11}$	$y^{29} - 24y^{28} + \cdots - y - 1$
$c_9$	$y^{29} + 24y^{28} + \cdots - 2572161y - 255025$
$c_{12}$	$y^{29} + 60y^{28} + \cdots - y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.116211 + 0.866176I$		
$a = 0.518979 - 0.390879I$	$-18.1545 + 6.4457I$	$-0.28264 - 3.13315I$
$b = 0.83718 - 3.00348I$		
$u = 0.116211 - 0.866176I$		
$a = 0.518979 + 0.390879I$	$-18.1545 - 6.4457I$	$-0.28264 + 3.13315I$
$b = 0.83718 + 3.00348I$		
$u = 0.025688 + 0.825088I$		
$a = -0.594569 + 0.459352I$	$-6.93251 + 1.75256I$	$-0.94314 - 1.34488I$
$b = -0.23014 + 2.32002I$		
$u = 0.025688 - 0.825088I$		
$a = -0.594569 - 0.459352I$	$-6.93251 - 1.75256I$	$-0.94314 + 1.34488I$
$b = -0.23014 - 2.32002I$		
$u = 1.19941$		
$a = 2.02803$	0.978770	8.30090
$b = -1.96066$		
$u = 1.149060 + 0.431164I$		
$a = -2.80477 - 0.74741I$	$-14.9885 - 1.7991I$	$2.59660 - 0.52377I$
$b = 1.17842 + 2.47370I$		
$u = 1.149060 - 0.431164I$		
$a = -2.80477 + 0.74741I$	$-14.9885 + 1.7991I$	$2.59660 + 0.52377I$
$b = 1.17842 - 2.47370I$		
$u = -0.078085 + 0.755576I$		
$a = 0.230600 - 0.278548I$	$-2.53613 - 2.06791I$	$5.52324 + 3.00073I$
$b = -0.073065 - 0.703159I$		
$u = -0.078085 - 0.755576I$		
$a = 0.230600 + 0.278548I$	$-2.53613 + 2.06791I$	$5.52324 - 3.00073I$
$b = -0.073065 + 0.703159I$		
$u = -1.215610 + 0.296484I$		
$a = 0.992404 - 0.146386I$	$0.91968 - 1.73508I$	$8.51016 + 0.61510I$
$b = -0.306342 + 0.431513I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.215610 - 0.296484I$		
$a = 0.992404 + 0.146386I$	$0.91968 + 1.73508I$	$8.51016 - 0.61510I$
$b = -0.306342 - 0.431513I$		
$u = 0.497421 + 0.544735I$		
$a = -1.52270 + 0.49021I$	$-12.40890 + 1.96182I$	$2.44090 - 3.22875I$
$b = 0.327073 - 0.572505I$		
$u = 0.497421 - 0.544735I$		
$a = -1.52270 - 0.49021I$	$-12.40890 - 1.96182I$	$2.44090 + 3.22875I$
$b = 0.327073 + 0.572505I$		
$u = -1.274370 + 0.082675I$		
$a = 0.269484 + 1.345140I$	$2.70011 - 2.01756I$	$7.88389 + 3.97312I$
$b = 0.022151 - 0.326489I$		
$u = -1.274370 - 0.082675I$		
$a = 0.269484 - 1.345140I$	$2.70011 + 2.01756I$	$7.88389 - 3.97312I$
$b = 0.022151 + 0.326489I$		
$u = 1.248070 + 0.369947I$		
$a = 1.89588 + 1.19040I$	$-3.15292 + 2.54446I$	$2.89231 - 2.35754I$
$b = -0.78649 - 2.39370I$		
$u = 1.248070 - 0.369947I$		
$a = 1.89588 - 1.19040I$	$-3.15292 - 2.54446I$	$2.89231 + 2.35754I$
$b = -0.78649 + 2.39370I$		
$u = -1.289030 + 0.369909I$		
$a = -2.15007 + 1.65647I$	$-2.83662 - 6.04967I$	$3.35930 + 4.53104I$
$b = 0.23955 - 2.15882I$		
$u = -1.289030 - 0.369909I$		
$a = -2.15007 - 1.65647I$	$-2.83662 + 6.04967I$	$3.35930 - 4.53104I$
$b = 0.23955 + 2.15882I$		
$u = 1.35324$		
$a = -0.560180$	$5.98164$	$16.6760$
$b = 0.713936$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.321100 + 0.324348I$		
$a = -0.492619 - 0.546231I$	$1.85790 + 5.97624I$	$10.99426 - 4.76972I$
$b = 0.144058 + 0.921802I$		
$u = 1.321100 - 0.324348I$		
$a = -0.492619 + 0.546231I$	$1.85790 - 5.97624I$	$10.99426 + 4.76972I$
$b = 0.144058 - 0.921802I$		
$u = -1.350820 + 0.384218I$		
$a = 1.84401 - 2.84917I$	$-13.5440 - 10.9372I$	$3.78628 + 5.28747I$
$b = 0.46306 + 3.28611I$		
$u = -1.350820 - 0.384218I$		
$a = 1.84401 + 2.84917I$	$-13.5440 + 10.9372I$	$3.78628 - 5.28747I$
$b = 0.46306 - 3.28611I$		
$u = -1.41186 + 0.13721I$		
$a = -0.04816 - 1.43092I$	$-6.30066 - 4.18073I$	$6.94767 + 2.86022I$
$b = -0.645077 + 1.183340I$		
$u = -1.41186 - 0.13721I$		
$a = -0.04816 + 1.43092I$	$-6.30066 + 4.18073I$	$6.94767 - 2.86022I$
$b = -0.645077 - 1.183340I$		
$u = -0.378524$		
$a = 0.615391$	$0.641421$	$15.6290$
$b = 0.176038$		
$u = 0.175172 + 0.291603I$		
$a = 0.31991 - 2.02255I$	$-1.62334 + 0.70173I$	$-1.51173 - 3.13517I$
$b = -0.635045 + 0.442420I$		
$u = 0.175172 - 0.291603I$		
$a = 0.31991 + 2.02255I$	$-1.62334 - 0.70173I$	$-1.51173 + 3.13517I$
$b = -0.635045 - 0.442420I$		

$$\text{II. } I_2^u = \langle -u^3 + b + u + 1, -u^6 + 3u^4 + u^3 - 2u^2 + a - u - 2, u^8 - u^7 - 3u^6 + 2u^5 + 3u^4 - 2u - 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^6 - 3u^4 - u^3 + 2u^2 + u + 2 \\ u^3 - u - 1 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^4 + u^2 + 1 \\ u^6 - 2u^4 + u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^6 - 2u^4 - u^3 + u^2 + u + 1 \\ -u^6 + 2u^4 + u^3 - u^2 - u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^4 - u^2 - 1 \\ -u^6 + 2u^4 - u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^6 - 3u^4 - u^3 + 2u^2 + u + 2 \\ u^3 - u - 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^4 + u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^4 + u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $u^7 + 2u^6 - 2u^5 - 8u^4 - 3u^3 + 7u^2 + 8u + 7$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$(u - 1)^8$
$c_3, c_8$	$u^8$
$c_4$	$(u + 1)^8$
$c_5, c_7$	$u^8 + 3u^7 + 7u^6 + 10u^5 + 11u^4 + 10u^3 + 6u^2 + 4u + 1$
$c_6$	$u^8 - u^7 - 3u^6 + 2u^5 + 3u^4 - 2u - 1$
$c_9, c_{12}$	$u^8 - u^7 - u^6 + 2u^5 + u^4 - 2u^3 + 2u - 1$
$c_{10}, c_{11}$	$u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_4$	$(y - 1)^8$
$c_3, c_8$	$y^8$
$c_5, c_7$	$y^8 + 5y^7 + 11y^6 + 6y^5 - 17y^4 - 34y^3 - 22y^2 - 4y + 1$
$c_6, c_{10}, c_{11}$	$y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1$
$c_9, c_{12}$	$y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.180120 + 0.268597I$		
$a = 1.53392 - 0.14090I$	$-0.604279 - 1.131230I$	$3.90459 + 0.80511I$
$b = -1.20799 + 0.83423I$		
$u = -1.180120 - 0.268597I$		
$a = 1.53392 + 0.14090I$	$-0.604279 + 1.131230I$	$3.90459 - 0.80511I$
$b = -1.20799 - 0.83423I$		
$u = -0.108090 + 0.747508I$		
$a = -0.322641 + 0.144481I$	$-3.80435 - 2.57849I$	$-0.21961 + 3.88175I$
$b = -0.711982 - 1.138990I$		
$u = -0.108090 - 0.747508I$		
$a = -0.322641 - 0.144481I$	$-3.80435 + 2.57849I$	$-0.21961 - 3.88175I$
$b = -0.711982 + 1.138990I$		
$u = 1.37100$		
$a = 0.595007$	4.85780	7.82890
$b = 0.205997$		
$u = 1.334530 + 0.318930I$		
$a = -0.47742 - 1.64247I$	$0.73474 + 6.44354I$	$4.50908 - 6.04101I$
$b = -0.365014 + 1.352640I$		
$u = 1.334530 - 0.318930I$		
$a = -0.47742 + 1.64247I$	$0.73474 - 6.44354I$	$4.50908 + 6.04101I$
$b = -0.365014 - 1.352640I$		
$u = -0.463640$		
$a = 1.93726$	-0.799899	4.78300
$b = -0.636025$		

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u - 1)^8)(u^{29} + 45u^{28} + \dots + 63u + 1)$
$c_2$	$((u - 1)^8)(u^{29} - 9u^{28} + \dots - 15u + 1)$
$c_3, c_8$	$u^8(u^{29} + u^{28} + \dots - 640u + 256)$
$c_4$	$((u + 1)^8)(u^{29} - 9u^{28} + \dots - 15u + 1)$
$c_5, c_7$	$(u^8 + 3u^7 + 7u^6 + 10u^5 + 11u^4 + 10u^3 + 6u^2 + 4u + 1) \cdot (u^{29} + 6u^{28} + \dots - 27u - 7)$
$c_6$	$(u^8 - u^7 - 3u^6 + 2u^5 + 3u^4 - 2u - 1)(u^{29} - 2u^{28} + \dots - u - 1)$
$c_9$	$(u^8 - u^7 + \dots + 2u - 1)(u^{29} + 2u^{28} + \dots + 847u - 505)$
$c_{10}, c_{11}$	$(u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1)(u^{29} - 2u^{28} + \dots - u - 1)$
$c_{12}$	$(u^8 - u^7 + \dots + 2u - 1)(u^{29} + 30u^{27} + \dots - u + 1)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y - 1)^8)(y^{29} - 113y^{28} + \dots + 3195y - 1)$
$c_2, c_4$	$((y - 1)^8)(y^{29} - 45y^{28} + \dots + 63y - 1)$
$c_3, c_8$	$y^8(y^{29} + 51y^{28} + \dots + 835584y - 65536)$
$c_5, c_7$	$(y^8 + 5y^7 + 11y^6 + 6y^5 - 17y^4 - 34y^3 - 22y^2 - 4y + 1) \cdot (y^{29} + 24y^{28} + \dots + 239y - 49)$
$c_6, c_{10}, c_{11}$	$(y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1) \cdot (y^{29} - 24y^{28} + \dots - y - 1)$
$c_9$	$(y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1) \cdot (y^{29} + 24y^{28} + \dots - 2572161y - 255025)$
$c_{12}$	$(y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1) \cdot (y^{29} + 60y^{28} + \dots - y - 1)$