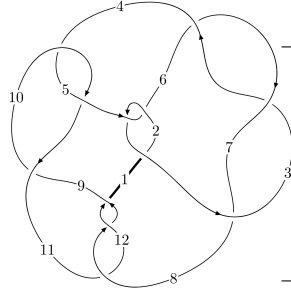
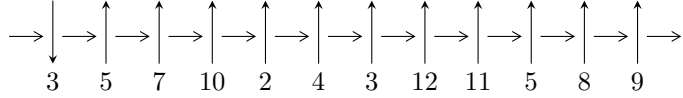


12n<sub>0329</sub> (K12n<sub>0329</sub>)



A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**

$$3,5 \xrightarrow{c_2} 2 \xrightarrow{c_5} 6 \xrightarrow{c_1} 1,11 \xrightarrow{c_{10}} 10 \xrightarrow{c_4} 4 \xrightarrow{c_6} 7 \xrightarrow{c_7} 8 \xrightarrow{c_9} 9 \xrightarrow{c_{12}} 12 \twoheadrightarrow c_3, c_8, c_{11}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle 47u^{14} + 142u^{13} + \dots + 256b + 305, 49u^{14} + 88u^{13} + \dots + 64a + 45, u^{15} + u^{14} + \dots + 4u + 1 \rangle$$

$$I_2^u = \langle -a^4u - a^4 + a^3u + a^3 - 2a^2u - 2a^2 + au + b - u - 1, a^5 - a^4 + 2a^3 - a^2 + a - 1, u^2 + 1 \rangle$$

$$I_3^u = \langle 10u^5 + 9u^4 - 4u^3 + 144u^2 + 107b - 160u + 346, 92u^5 - 174u^4 + 648u^3 - 965u^2 + 1819a + 1310u - 733, u^6 - 3u^5 + 10u^4 - 14u^3 + 22u^2 - 10u + 17 \rangle$$

$$I_4^u = \langle 2b - 1, a, u + 1 \rangle$$

\* 4 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 32 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\mathbf{I. } I_1^u = \langle 47u^{14} + 142u^{13} + \dots + 256b + 305, 49u^{14} + 88u^{13} + \dots + 64a + 45, u^{15} + u^{14} + \dots + 4u + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^2 + 1 \\ u^2 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -0.765625u^{14} - 1.37500u^{13} + \dots - 8.10938u - 0.703125 \\ -0.183594u^{14} - 0.554688u^{13} + \dots - 6.01953u - 1.19141 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -0.765625u^{14} - 1.37500u^{13} + \dots - 8.10938u - 0.703125 \\ 0.238281u^{14} - 0.0546875u^{13} + \dots - 2.81641u - 0.582031 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -\frac{1}{32}u^{14} - \frac{1}{32}u^{13} + \dots - \frac{1}{32}u + 1 \\ -u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} \frac{1}{32}u^{13} + \frac{1}{32}u^{12} + \dots + \frac{17}{8}u + \frac{1}{32} \\ u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} \frac{1}{32}u^{13} + \frac{1}{32}u^{12} + \dots + \frac{25}{8}u + \frac{1}{32} \\ u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -1.42188u^{14} - 1.21875u^{13} + \dots - 2.07813u + 1.29688 \\ -0.230469u^{14} - 0.210938u^{13} + \dots - 2.34766u + 0.0429688 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -0.0312500u^{14} - 0.500000u^{13} + \dots + 0.593750u + 1.15625 \\ -0.0351563u^{14} - 0.320313u^{13} + \dots - 3.38672u - 0.589844 \end{pmatrix}$$

(ii) Obstruction class = -1

$$\mathbf{(iii) } \text{Cusp Shapes} = \frac{1851}{512}u^{14} + \frac{499}{256}u^{13} + \dots + \frac{2553}{512}u + \frac{5749}{512}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{15} + 27u^{14} + \dots - 10u - 1$
$c_2, c_3, c_5$ $c_6, c_7$	$u^{15} - u^{14} + \dots + 4u - 1$
$c_4, c_{10}$	$u^{15} - 3u^{14} + \dots + 18u - 8$
$c_8, c_{11}, c_{12}$	$u^{15} + 2u^{14} + \dots + 13u - 4$
$c_9$	$u^{15} - 3u^{14} + \dots + 244u - 64$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{15} - 105y^{14} + \dots + 150y - 1$
$c_2, c_3, c_5$ $c_6, c_7$	$y^{15} + 27y^{14} + \dots - 10y - 1$
$c_4, c_{10}$	$y^{15} - 3y^{14} + \dots + 244y - 64$
$c_8, c_{11}, c_{12}$	$y^{15} - 12y^{14} + \dots + 209y - 16$
$c_9$	$y^{15} + 65y^{14} + \dots + 27664y - 4096$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.085190 + 0.639113I$ $a = -0.187470 - 0.679349I$ $b = 0.244487 - 0.095273I$	$3.76428 + 0.87599I$	$12.27688 - 4.04131I$
$u = -1.085190 - 0.639113I$ $a = -0.187470 + 0.679349I$ $b = 0.244487 + 0.095273I$	$3.76428 - 0.87599I$	$12.27688 + 4.04131I$
$u = -0.171837 + 0.650095I$ $a = 1.10255 + 1.42436I$ $b = 0.682015 + 0.663583I$	$3.90578 - 5.04152I$	$15.1629 + 7.2560I$
$u = -0.171837 - 0.650095I$ $a = 1.10255 - 1.42436I$ $b = 0.682015 - 0.663583I$	$3.90578 + 5.04152I$	$15.1629 - 7.2560I$
$u = -0.113165 + 0.510319I$ $a = -1.23811 - 1.27126I$ $b = -0.714447 - 0.300419I$	$-1.07843 - 2.01114I$	$7.99911 + 6.03699I$
$u = -0.113165 - 0.510319I$ $a = -1.23811 + 1.27126I$ $b = -0.714447 + 0.300419I$	$-1.07843 + 2.01114I$	$7.99911 - 6.03699I$
$u = 0.139618 + 0.358203I$ $a = 1.15825 + 1.37391I$ $b = 0.824625 - 0.322737I$	$1.56037 + 0.76584I$	$8.88156 - 1.45117I$
$u = 0.139618 - 0.358203I$ $a = 1.15825 - 1.37391I$ $b = 0.824625 + 0.322737I$	$1.56037 - 0.76584I$	$8.88156 + 1.45117I$
$u = -0.301931$ $a = 1.66090$ $b = 0.268883$	$0.626145$	$16.4510$
$u = 0.61910 + 1.98975I$ $a = -0.734091 + 0.400986I$ $b = -2.19546 + 0.22693I$	$-16.5790 + 11.3907I$	$8.72673 - 4.72209I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.61910 - 1.98975I$ $a = -0.734091 - 0.400986I$ $b = -2.19546 - 0.22693I$	$-16.5790 - 11.3907I$	$8.72673 + 4.72209I$
$u = -0.10073 + 2.21378I$ $a = -0.558378 + 0.607390I$ $b = -1.76407 + 0.10317I$	$-16.6861 - 2.0181I$	$8.39824 + 0.80099I$
$u = -0.10073 - 2.21378I$ $a = -0.558378 - 0.607390I$ $b = -1.76407 - 0.10317I$	$-16.6861 + 2.0181I$	$8.39824 - 0.80099I$
$u = 0.36316 + 2.28705I$ $a = 0.626797 - 0.473627I$ $b = 2.03841 - 0.08740I$	$18.2203 + 4.7904I$	$6.45412 - 1.91248I$
$u = 0.36316 - 2.28705I$ $a = 0.626797 + 0.473627I$ $b = 2.03841 + 0.08740I$	$18.2203 - 4.7904I$	$6.45412 + 1.91248I$

$$\text{II. } I_2^u = \langle -a^4u + a^3u + \cdots - 2a^2 - 1, a^5 - a^4 + 2a^3 - a^2 + a - 1, u^2 + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} a^4u + a^4 - a^3u - a^3 + 2a^2u + 2a^2 - au + u + 1 \\ a \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} a^4u + a^4 - a^3u - a^3 + 2a^2u + 2a^2 - au - a + u + 1 \\ a \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -a^2u \\ 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -a^2 + u \\ -u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -a^2 \\ -u \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a^3 + a \\ a^4u + a^4 - a^3u + 2a^2u + 2a^2 + u + 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} a^4 - a^3 + a^2 + 1 \\ a^4u + a^4 - a^3 + 2a^2u + 2a^2 + u + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $-4a^3 + 4a^2 - 4a + 8$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$(u - 1)^{10}$
$c_2, c_3, c_5$ $c_6, c_7$	$(u^2 + 1)^5$
$c_4, c_{10}$	$u^{10} - 3u^8 + 4u^6 - u^4 - u^2 + 1$
$c_8$	$(u^5 - u^4 - 2u^3 + u^2 + u + 1)^2$
$c_9$	$(u^5 + 3u^4 + 4u^3 + u^2 - u - 1)^2$
$c_{11}, c_{12}$	$(u^5 + u^4 - 2u^3 - u^2 + u - 1)^2$



(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$(y - 1)^{10}$
$c_2, c_3, c_5$ $c_6, c_7$	$(y + 1)^{10}$
$c_4, c_{10}$	$(y^5 - 3y^4 + 4y^3 - y^2 - y + 1)^2$
$c_8, c_{11}, c_{12}$	$(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2$
$c_9$	$(y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.000000I$ $a = -0.339110 + 0.822375I$ $b = 0.271616 - 0.645450I$	$-2.96077 + 1.53058I$	$4.51511 - 4.43065I$
$u = 1.000000I$ $a = -0.339110 - 0.822375I$ $b = -1.80694 - 0.21165I$	$-2.96077 - 1.53058I$	$4.51511 + 4.43065I$
$u = 1.000000I$ $a = 0.766826$ $b = 2.07090 + 1.30408I$	$-0.888787$	$5.48110$
$u = 1.000000I$ $a = 0.455697 + 1.200150I$ $b = 1.46044 + 0.74843I$	$2.58269 - 4.40083I$	$8.74431 + 3.49859I$
$u = 1.000000I$ $a = 0.455697 - 1.200150I$ $b = 0.003972 - 0.195404I$	$2.58269 + 4.40083I$	$8.74431 - 3.49859I$
$u = -1.000000I$ $a = -0.339110 + 0.822375I$ $b = -1.80694 + 0.21165I$	$-2.96077 + 1.53058I$	$4.51511 - 4.43065I$
$u = -1.000000I$ $a = -0.339110 - 0.822375I$ $b = 0.271616 + 0.645450I$	$-2.96077 - 1.53058I$	$4.51511 + 4.43065I$
$u = -1.000000I$ $a = 0.766826$ $b = 2.07090 - 1.30408I$	$-0.888787$	$5.48110$
$u = -1.000000I$ $a = 0.455697 + 1.200150I$ $b = 0.003972 + 0.195404I$	$2.58269 - 4.40083I$	$8.74431 + 3.49859I$
$u = -1.000000I$ $a = 0.455697 - 1.200150I$ $b = 1.46044 - 0.74843I$	$2.58269 + 4.40083I$	$8.74431 - 3.49859I$

$$\text{III. } I_3^u = \langle 10u^5 + 9u^4 + \cdots + 107b + 346, 92u^5 - 174u^4 + \cdots + 1819a - 733, u^6 - 3u^5 + 10u^4 - 14u^3 + 22u^2 - 10u + 17 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^2 + 1 \\ u^2 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -0.0505772u^5 + 0.0956570u^4 + \cdots - 0.720176u + 0.402969 \\ -0.0934579u^5 - 0.0841121u^4 + \cdots + 1.49533u - 3.23364 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -0.0505772u^5 + 0.0956570u^4 + \cdots - 0.720176u + 0.402969 \\ -0.112150u^5 + 0.299065u^4 + \cdots + 1.79439u - 2.28037 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -0.0170423u^5 - 0.0329852u^4 + \cdots + 0.213854u - 0.483782 \\ -0.0280374u^5 + 0.0747664u^4 + \cdots - 0.551402u + 2.42991 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0.0588235u^5 - 0.176471u^4 + \cdots + 1.29412u - 0.588235 \\ -0.0841121u^5 + 0.224299u^4 + \cdots - 1.65421u + 0.289720 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -0.0252886u^5 + 0.0478285u^4 + \cdots - 0.360088u - 0.298516 \\ -0.0841121u^5 + 0.224299u^4 + \cdots - 1.65421u + 0.289720 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -0.109951u^5 + 0.0775151u^4 + \cdots - 0.652556u + 0.136888 \\ -0.149533u^5 + 0.0654206u^4 + \cdots + 1.39252u - 2.37383 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0.0340847u^5 + 0.0659703u^4 + \cdots - 0.427708u + 0.967565 \\ u - 2 \end{pmatrix}$$

(ii) Obstruction class = -1

$$\text{(iii) Cusp Shapes} = \frac{60}{107}u^5 - \frac{160}{107}u^4 + \frac{404}{107}u^3 - \frac{420}{107}u^2 + \frac{324}{107}u + \frac{1006}{107}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^6 + 11u^5 + 60u^4 + 218u^3 + 544u^2 + 648u + 289$
$c_2, c_3, c_5$ $c_6, c_7$	$u^6 + 3u^5 + 10u^4 + 14u^3 + 22u^2 + 10u + 17$
$c_4, c_{10}$	$(u^3 + u^2 + 2u + 1)^2$
$c_8, c_{11}, c_{12}$	$(u^3 + u^2 - 1)^2$
$c_9$	$(u^3 + 3u^2 + 2u - 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^6 - y^5 - 108y^4 + 4078y^3 + 48088y^2 - 105472y + 83521$
$c_2, c_3, c_5$ $c_6, c_7$	$y^6 + 11y^5 + 60y^4 + 218y^3 + 544y^2 + 648y + 289$
$c_4, c_{10}$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_8, c_{11}, c_{12}$	$(y^3 - y^2 + 2y - 1)^2$
$c_9$	$(y^3 - 5y^2 + 10y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.162359 + 1.038790I$ $a = -0.083694 - 0.535481I$ $b = -2.03980 + 1.82295I$	-2.17641	13.01951 + 0.I
$u = -0.162359 - 1.038790I$ $a = -0.083694 + 0.535481I$ $b = -2.03980 - 1.82295I$	-2.17641	13.01951 + 0.I
$u = 1.23597 + 1.45071I$ $a = 0.595267 + 0.358893I$ $b = 1.109500 + 0.002038I$	-6.31400 + 2.82812I	6.49024 - 2.97945I
$u = 1.23597 - 1.45071I$ $a = 0.595267 - 0.358893I$ $b = 1.109500 - 0.002038I$	-6.31400 - 2.82812I	6.49024 + 2.97945I
$u = 0.42639 + 2.01299I$ $a = -0.599808 - 0.233897I$ $b = -1.56970 - 0.18054I$	-6.31400 - 2.82812I	6.49024 + 2.97945I
$u = 0.42639 - 2.01299I$ $a = -0.599808 + 0.233897I$ $b = -1.56970 + 0.18054I$	-6.31400 + 2.82812I	6.49024 - 2.97945I

$$\text{IV. } I_4^u = \langle 2b - 1, a, u + 1 \rangle$$

(i) Arc colorings

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = 9.75

(iv) **u**-Polynomials at the component

Crossings	<b>u</b> -Polynomials at each crossing
$c_1, c_2, c_3$ $c_8$	$u + 1$
$c_4, c_9, c_{10}$	$u$
$c_5, c_6, c_7$ $c_{11}, c_{12}$	$u - 1$



(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_3$ $c_5, c_6, c_7$ $c_8, c_{11}, c_{12}$	$y - 1$
$c_4, c_9, c_{10}$	$y$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_4^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.00000$		
$a = 0$	3.28987	9.75000
$b = 0.500000$		

### V. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u-1)^{10})(u+1)(u^6 + 11u^5 + \dots + 648u + 289)$ $\cdot (u^{15} + 27u^{14} + \dots - 10u - 1)$
$c_2, c_3$	$(u+1)(u^2+1)^5(u^6 + 3u^5 + 10u^4 + 14u^3 + 22u^2 + 10u + 17)$ $\cdot (u^{15} - u^{14} + \dots + 4u - 1)$
$c_4, c_{10}$	$u(u^3 + u^2 + 2u + 1)^2(u^{10} - 3u^8 + 4u^6 - u^4 - u^2 + 1)$ $\cdot (u^{15} - 3u^{14} + \dots + 18u - 8)$
$c_5, c_6, c_7$	$(u-1)(u^2+1)^5(u^6 + 3u^5 + 10u^4 + 14u^3 + 22u^2 + 10u + 17)$ $\cdot (u^{15} - u^{14} + \dots + 4u - 1)$
$c_8$	$(u+1)(u^3 + u^2 - 1)^2(u^5 - u^4 - 2u^3 + u^2 + u + 1)^2$ $\cdot (u^{15} + 2u^{14} + \dots + 13u - 4)$
$c_9$	$u(u^3 + 3u^2 + 2u - 1)^2(u^5 + 3u^4 + 4u^3 + u^2 - u - 1)^2$ $\cdot (u^{15} - 3u^{14} + \dots + 244u - 64)$
$c_{11}, c_{12}$	$(u-1)(u^3 + u^2 - 1)^2(u^5 + u^4 - 2u^3 - u^2 + u - 1)^2$ $\cdot (u^{15} + 2u^{14} + \dots + 13u - 4)$

## VI. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y-1)^{11})(y^6 - y^5 + \dots - 105472y + 83521)$ $\cdot (y^{15} - 105y^{14} + \dots + 150y - 1)$
$c_2, c_3, c_5$ $c_6, c_7$	$(y-1)(y+1)^{10}(y^6 + 11y^5 + \dots + 648y + 289)$ $\cdot (y^{15} + 27y^{14} + \dots - 10y - 1)$
$c_4, c_{10}$	$y(y^3 + 3y^2 + 2y - 1)^2(y^5 - 3y^4 + 4y^3 - y^2 - y + 1)^2$ $\cdot (y^{15} - 3y^{14} + \dots + 244y - 64)$
$c_8, c_{11}, c_{12}$	$(y-1)(y^3 - y^2 + 2y - 1)^2(y^5 - 5y^4 + 8y^3 - 3y^2 - y - 1)^2$ $\cdot (y^{15} - 12y^{14} + \dots + 209y - 16)$
$c_9$	$y(y^3 - 5y^2 + 10y - 1)^2(y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)^2$ $\cdot (y^{15} + 65y^{14} + \dots + 27664y - 4096)$