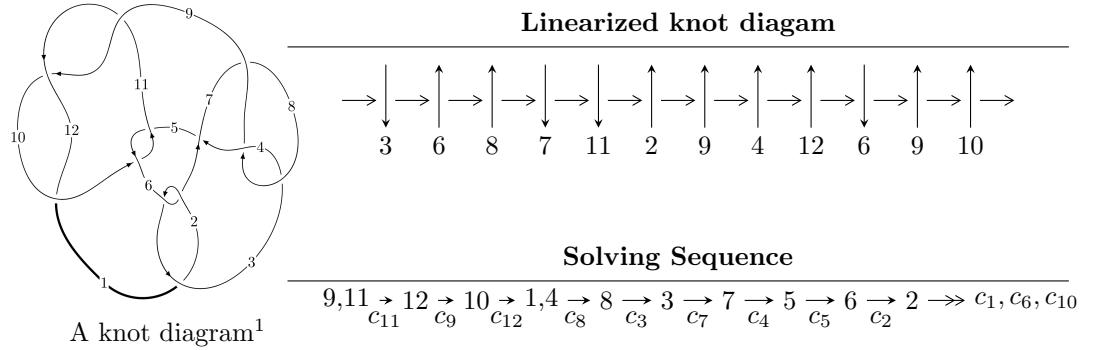


## $12n_{0336}$ ( $K12n_{0336}$ )



### Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$\begin{aligned}
 I_1^u &= \langle -1706550177091u^{18} - 23512487300441u^{17} + \dots + 39336273597696b - 69157159108721, \\
 &\quad - 32608816754461u^{18} - 456051534293894u^{17} + \dots + 19668136798848a - 1093408920959144, \\
 &\quad u^{19} + 14u^{18} + \dots + 52u + 1 \rangle \\
 I_2^u &= \langle -a^8 + a^7 + 3a^6 - 2a^5 - 3a^4 + 2a^3 + b + a + 2, \ a^9 - a^8 - 2a^7 + 3a^6 + a^5 - 3a^4 + 2a^3 - a + 1, \ u - 1 \rangle \\
 I_3^u &= \langle -3a^3u + 2a^3 - a^2u + b + a, \ a^4 - a^2u - 2a^2 + 3u + 5, \ u^2 + u - 1 \rangle
 \end{aligned}$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 36 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\mathbf{I. } I_1^u = \langle -1.71 \times 10^{12}u^{18} - 2.35 \times 10^{13}u^{17} + \dots + 3.93 \times 10^{13}b - 6.92 \times 10^{13}, -3.26 \times 10^{13}u^{18} - 4.56 \times 10^{14}u^{17} + \dots + 1.97 \times 10^{13}a - 1.09 \times 10^{15}, u^{19} + 14u^{18} + \dots + 52u + 1 \rangle$$

(i) Arc colorings

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1.65795u^{18} + 23.1873u^{17} + \dots + 1589.07u + 55.5929 \\ 0.0433836u^{18} + 0.597730u^{17} + \dots + 39.8060u + 1.75810 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} 1.84251u^{18} + 25.7334u^{17} + \dots + 1669.72u + 43.2886 \\ 0.0542335u^{18} + 0.767279u^{17} + \dots + 55.0156u + 1.70577 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.243745u^{18} + 3.43967u^{17} + \dots + 332.641u + 35.7067 \\ -0.0193717u^{18} - 0.267431u^{17} + \dots - 10.7266u + 0.631068 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1.84251u^{18} + 25.7334u^{17} + \dots + 1669.72u + 43.2886 \\ 0.0598519u^{18} + 0.839590u^{17} + \dots + 56.3822u + 1.76749 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0.0513825u^{18} + 0.731152u^{17} + \dots + 83.7307u + 7.87385 \\ -0.00184480u^{18} - 0.0331746u^{17} + \dots - 3.98487u + 0.0723727 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 0.0532273u^{18} + 0.764327u^{17} + \dots + 87.7156u + 7.80147 \\ -0.00184480u^{18} - 0.0331746u^{17} + \dots - 3.98487u + 0.0723727 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.136733u^{18} - 1.87566u^{17} + \dots - 19.9103u + 26.8869 \\ -0.0281326u^{18} - 0.392506u^{17} + \dots - 21.4809u + 0.261960 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -\frac{390713901803}{1639011399904}u^{18} - \frac{21917006139285}{6556045599616}u^{17} + \dots - \frac{430703611764529}{1639011399904}u - \frac{74684759366323}{6556045599616}$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^{19} - 9u^{18} + \cdots + 9432u - 1296$
$c_2, c_6$	$u^{19} - 7u^{18} + \cdots - 36u + 36$
$c_3, c_8$	$u^{19} - 2u^{18} + \cdots - 12u + 9$
$c_4$	$u^{19} - 6u^{18} + \cdots + 19710u - 2349$
$c_5, c_{10}$	$u^{19} + u^{18} + \cdots + 1536u + 512$
$c_7$	$u^{19} - 16u^{18} + \cdots + 540u - 81$
$c_9, c_{11}, c_{12}$	$u^{19} + 14u^{18} + \cdots + 52u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{19} - y^{18} + \cdots + 159068448y - 1679616$
$c_2, c_6$	$y^{19} - 9y^{18} + \cdots + 9432y - 1296$
$c_3, c_8$	$y^{19} - 16y^{18} + \cdots + 540y - 81$
$c_4$	$y^{19} + 56y^{18} + \cdots + 419265396y - 5517801$
$c_5, c_{10}$	$y^{19} + 69y^{18} + \cdots + 15204352y - 262144$
$c_7$	$y^{19} - 20y^{18} + \cdots + 220968y - 6561$
$c_9, c_{11}, c_{12}$	$y^{19} - 72y^{18} + \cdots + 696y - 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.090810 + 0.087492I$		
$a = 0.864947 + 0.453435I$	$0.12746 + 2.07953I$	$0.79710 + 6.78619I$
$b = 1.25966 - 4.27188I$		
$u = 1.090810 - 0.087492I$		
$a = 0.864947 - 0.453435I$	$0.12746 - 2.07953I$	$0.79710 - 6.78619I$
$b = 1.25966 + 4.27188I$		
$u = 1.12421$		
$a = -0.369206$	2.16627	2.38650
$b = -0.262337$		
$u = -0.866404 + 0.903603I$		
$a = 0.403452 + 1.041090I$	$4.84224 + 5.75329I$	$6.23581 - 2.60069I$
$b = -1.260210 - 0.033364I$		
$u = -0.866404 - 0.903603I$		
$a = 0.403452 - 1.041090I$	$4.84224 - 5.75329I$	$6.23581 + 2.60069I$
$b = -1.260210 + 0.033364I$		
$u = 0.264112 + 0.511238I$		
$a = 0.998671 + 0.095546I$	$0.335625 + 1.223470I$	$4.13138 - 4.81848I$
$b = 0.655201 - 0.830329I$		
$u = 0.264112 - 0.511238I$		
$a = 0.998671 - 0.095546I$	$0.335625 - 1.223470I$	$4.13138 + 4.81848I$
$b = 0.655201 + 0.830329I$		
$u = -1.54015 + 0.03306I$		
$a = -0.616311 + 0.224492I$	$8.24788 - 1.49342I$	$14.3201 - 0.4966I$
$b = 0.39526 - 1.71073I$		
$u = -1.54015 - 0.03306I$		
$a = -0.616311 - 0.224492I$	$8.24788 + 1.49342I$	$14.3201 + 0.4966I$
$b = 0.39526 + 1.71073I$		
$u = -0.272498$		
$a = -3.42293$	1.52167	5.90680
$b = -0.561462$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.0292040 + 0.0188465I$		
$a = 11.6667 + 24.4338I$	$-1.74489 + 2.05954I$	$-4.07768 - 4.15264I$
$b = 0.653657 + 0.621796I$		
$u = -0.0292040 - 0.0188465I$		
$a = 11.6667 - 24.4338I$	$-1.74489 - 2.05954I$	$-4.07768 + 4.15264I$
$b = 0.653657 - 0.621796I$		
$u = -2.00138 + 0.76457I$		
$a = -0.689796 + 0.112485I$	$-15.5608 - 12.7812I$	$6.72063 + 5.11794I$
$b = -0.56693 - 3.57760I$		
$u = -2.00138 - 0.76457I$		
$a = -0.689796 - 0.112485I$	$-15.5608 + 12.7812I$	$6.72063 - 5.11794I$
$b = -0.56693 + 3.57760I$		
$u = -2.42283 + 0.72006I$		
$a = 0.017324 - 0.538247I$	$-18.5440 - 5.6494I$	$4.00000 + 0.I$
$b = 2.30004 + 3.34538I$		
$u = -2.42283 - 0.72006I$		
$a = 0.017324 + 0.538247I$	$-18.5440 + 5.6494I$	$4.00000 + 0.I$
$b = 2.30004 - 3.34538I$		
$u = -3.92517 + 0.42402I$		
$a = 0.495462 - 0.122568I$	$-11.60890 - 1.83503I$	0
$b = -4.38040 + 4.23334I$		
$u = -3.92517 - 0.42402I$		
$a = 0.495462 + 0.122568I$	$-11.60890 + 1.83503I$	0
$b = -4.38040 - 4.23334I$		
$u = 4.00873$		
$a = 0.511273$	11.4850	0
$b = -5.28877$		

$$\text{II. } I_2^u = \langle -a^8 + a^7 + 3a^6 - 2a^5 - 3a^4 + 2a^3 + b + a + 2, a^9 - a^8 - 2a^7 + 3a^6 + a^5 - 3a^4 + 2a^3 - a + 1, u - 1 \rangle$$

(i) **Arc colorings**

$$\begin{aligned} a_9 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_1 &= \begin{pmatrix} 0 \\ -1 \end{pmatrix} \\ a_4 &= \begin{pmatrix} a \\ a^8 - a^7 - 3a^6 + 2a^5 + 3a^4 - 2a^3 - a - 2 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -a^2 \\ a^7 + a^6 - 2a^5 - a^4 + 2a^3 + a^2 + a + 2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} -a^3 + a \\ 2a^8 - 5a^6 + a^5 + 5a^4 - a^3 + a^2 + a - 2 \end{pmatrix} \\ a_7 &= \begin{pmatrix} -a^2 \\ a^7 + a^6 - 2a^5 - a^4 + 2a^3 + a + 2 \end{pmatrix} \\ a_5 &= \begin{pmatrix} a^5 - a^3 + a \\ 0 \end{pmatrix} \\ a_6 &= \begin{pmatrix} a^5 - a^3 + a \\ 0 \end{pmatrix} \\ a_2 &= \begin{pmatrix} a^6 - 2a^4 + a^2 \\ 2a^8 - 5a^6 + a^5 + 5a^4 - a^3 + a^2 + a - 2 \end{pmatrix} \end{aligned}$$

(ii) **Obstruction class = 1**

(iii) **Cusp Shapes** =  $6a^8 - 3a^7 - 10a^6 + 8a^5 + 2a^4 - 8a^3 + 12a^2 + 6$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_4$	$u^9 - 3u^8 + 8u^7 - 13u^6 + 17u^5 - 17u^4 + 12u^3 - 6u^2 + u + 1$
$c_2$	$u^9 - u^8 + 2u^7 - u^6 + 3u^5 - u^4 + 2u^3 + u + 1$
$c_3$	$u^9 - u^8 - 2u^7 + 3u^6 + u^5 - 3u^4 + 2u^3 - u + 1$
$c_5, c_{10}$	$u^9$
$c_6$	$u^9 + u^8 + 2u^7 + u^6 + 3u^5 + u^4 + 2u^3 + u - 1$
$c_7$	$u^9 + 5u^8 + 12u^7 + 15u^6 + 9u^5 - u^4 - 4u^3 - 2u^2 + u + 1$
$c_8$	$u^9 + u^8 - 2u^7 - 3u^6 + u^5 + 3u^4 + 2u^3 - u - 1$
$c_9$	$(u + 1)^9$
$c_{11}, c_{12}$	$(u - 1)^9$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$y^9 + 7y^8 + 20y^7 + 25y^6 + 5y^5 - 15y^4 + 22y^2 + 13y - 1$
$c_2, c_6$	$y^9 + 3y^8 + 8y^7 + 13y^6 + 17y^5 + 17y^4 + 12y^3 + 6y^2 + y - 1$
$c_3, c_8$	$y^9 - 5y^8 + 12y^7 - 15y^6 + 9y^5 + y^4 - 4y^3 + 2y^2 + y - 1$
$c_5, c_{10}$	$y^9$
$c_7$	$y^9 - y^8 + 12y^7 - 7y^6 + 37y^5 + y^4 - 10y^2 + 5y - 1$
$c_9, c_{11}, c_{12}$	$(y - 1)^9$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.00000$		
$a = 0.772920 + 0.510351I$	$-0.13850 + 2.09337I$	$11.64247 + 5.88316I$
$b = -3.25219 + 0.42284I$		
$u = 1.00000$		
$a = 0.772920 - 0.510351I$	$-0.13850 - 2.09337I$	$11.64247 - 5.88316I$
$b = -3.25219 - 0.42284I$		
$u = 1.00000$		
$a = -0.825933$	2.84338	15.4610
$b = 0.106533$		
$u = 1.00000$		
$a = -1.173910 + 0.391555I$	$6.01628 - 1.33617I$	$7.94914 + 0.75351I$
$b = -0.121911 - 0.782086I$		
$u = 1.00000$		
$a = -1.173910 - 0.391555I$	$6.01628 + 1.33617I$	$7.94914 - 0.75351I$
$b = -0.121911 + 0.782086I$		
$u = 1.00000$		
$a = 0.141484 + 0.739668I$	$2.26187 - 2.45442I$	$4.75622 + 3.91612I$
$b = -0.217279 - 0.962736I$		
$u = 1.00000$		
$a = 0.141484 - 0.739668I$	$2.26187 + 2.45442I$	$4.75622 - 3.91612I$
$b = -0.217279 + 0.962736I$		
$u = 1.00000$		
$a = 1.172470 + 0.500383I$	$5.24306 + 7.08493I$	$7.92182 - 8.89461I$
$b = 0.038112 - 1.195250I$		
$u = 1.00000$		
$a = 1.172470 - 0.500383I$	$5.24306 - 7.08493I$	$7.92182 + 8.89461I$
$b = 0.038112 + 1.195250I$		

$$\text{III. } I_3^u = \langle -3a^3u + 2a^3 - a^2u + b + a, a^4 - a^2u - 2a^2 + 3u + 5, u^2 + u - 1 \rangle$$

(i) Arc colorings

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 1 \\ u-1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ -u+1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u \\ -u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} a \\ 3a^3u - 2a^3 + a^2u - a \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -a^2u \\ a^3u - a^3 + 3a^2u - a^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} a^3u - a^3 + a \\ -a^3u + a^3 - a + u + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -a^2u \\ a^3u - a^3 + a^2u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ 3a^3u - 2a^3 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -3a^3u + 2a^3 \\ 3a^3u - 2a^3 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} a^3u - a^3 + a + u \\ -a^3u + a^3 - a + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $4a^2u - 4a^2 + 8$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$(u - 1)^8$
$c_2, c_6$	$(u^2 + 1)^4$
$c_3, c_4, c_8$	$(u^4 - u^2 + 1)^2$
$c_5, c_{10}$	$(u^4 + 3u^2 + 1)^2$
$c_7$	$(u^2 + u + 1)^4$
$c_9$	$(u^2 - u - 1)^4$
$c_{11}, c_{12}$	$(u^2 + u - 1)^4$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$(y - 1)^8$
$c_2, c_6$	$(y + 1)^8$
$c_3, c_4, c_8$	$(y^2 - y + 1)^4$
$c_5, c_{10}$	$(y^2 + 3y + 1)^4$
$c_7$	$(y^2 + y + 1)^4$
$c_9, c_{11}, c_{12}$	$(y^2 - 3y + 1)^4$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.618034$		
$a = 1.40126 + 0.80902I$	$-0.65797 + 2.02988I$	$6.00000 - 3.46410I$
$b = -0.592242 - 0.025792I$		
$u = 0.618034$		
$a = 1.40126 - 0.80902I$	$-0.65797 - 2.02988I$	$6.00000 + 3.46410I$
$b = -0.592242 + 0.025792I$		
$u = 0.618034$		
$a = -1.40126 + 0.80902I$	$-0.65797 - 2.02988I$	$6.00000 + 3.46410I$
$b = 2.21028 - 2.82831I$		
$u = 0.618034$		
$a = -1.40126 - 0.80902I$	$-0.65797 + 2.02988I$	$6.00000 - 3.46410I$
$b = 2.21028 + 2.82831I$		
$u = -1.61803$		
$a = -0.535233 + 0.309017I$	$7.23771 + 2.02988I$	$6.00000 - 3.46410I$
$b = 0.226216 - 1.391820I$		
$u = -1.61803$		
$a = -0.535233 - 0.309017I$	$7.23771 - 2.02988I$	$6.00000 + 3.46410I$
$b = 0.226216 + 1.391820I$		
$u = -1.61803$		
$a = 0.535233 + 0.309017I$	$7.23771 - 2.02988I$	$6.00000 + 3.46410I$
$b = -0.84425 - 2.46228I$		
$u = -1.61803$		
$a = 0.535233 - 0.309017I$	$7.23771 + 2.02988I$	$6.00000 - 3.46410I$
$b = -0.84425 + 2.46228I$		

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u - 1)^8)(u^9 - 3u^8 + \dots + u + 1)$ $\cdot (u^{19} - 9u^{18} + \dots + 9432u - 1296)$
$c_2$	$(u^2 + 1)^4(u^9 - u^8 + 2u^7 - u^6 + 3u^5 - u^4 + 2u^3 + u + 1)$ $\cdot (u^{19} - 7u^{18} + \dots - 36u + 36)$
$c_3$	$(u^4 - u^2 + 1)^2(u^9 - u^8 - 2u^7 + 3u^6 + u^5 - 3u^4 + 2u^3 - u + 1)$ $\cdot (u^{19} - 2u^{18} + \dots - 12u + 9)$
$c_4$	$(u^4 - u^2 + 1)^2$ $\cdot (u^9 - 3u^8 + 8u^7 - 13u^6 + 17u^5 - 17u^4 + 12u^3 - 6u^2 + u + 1)$ $\cdot (u^{19} - 6u^{18} + \dots + 19710u - 2349)$
$c_5, c_{10}$	$u^9(u^4 + 3u^2 + 1)^2(u^{19} + u^{18} + \dots + 1536u + 512)$
$c_6$	$(u^2 + 1)^4(u^9 + u^8 + 2u^7 + u^6 + 3u^5 + u^4 + 2u^3 + u - 1)$ $\cdot (u^{19} - 7u^{18} + \dots - 36u + 36)$
$c_7$	$((u^2 + u + 1)^4)(u^9 + 5u^8 + \dots + u + 1)$ $\cdot (u^{19} - 16u^{18} + \dots + 540u - 81)$
$c_8$	$(u^4 - u^2 + 1)^2(u^9 + u^8 - 2u^7 - 3u^6 + u^5 + 3u^4 + 2u^3 - u - 1)$ $\cdot (u^{19} - 2u^{18} + \dots - 12u + 9)$
$c_9$	$((u + 1)^9)(u^2 - u - 1)^4(u^{19} + 14u^{18} + \dots + 52u + 1)$
$c_{11}, c_{12}$	$((u - 1)^9)(u^2 + u - 1)^4(u^{19} + 14u^{18} + \dots + 52u + 1)$

## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$(y - 1)^8(y^9 + 7y^8 + 20y^7 + 25y^6 + 5y^5 - 15y^4 + 22y^2 + 13y - 1) \cdot (y^{19} - y^{18} + \dots + 159068448y - 1679616)$
$c_2, c_6$	$((y + 1)^8)(y^9 + 3y^8 + \dots + y - 1) \cdot (y^{19} - 9y^{18} + \dots + 9432y - 1296)$
$c_3, c_8$	$((y^2 - y + 1)^4)(y^9 - 5y^8 + \dots + y - 1) \cdot (y^{19} - 16y^{18} + \dots + 540y - 81)$
$c_4$	$((y^2 - y + 1)^4)(y^9 + 7y^8 + \dots + 13y - 1) \cdot (y^{19} + 56y^{18} + \dots + 419265396y - 5517801)$
$c_5, c_{10}$	$y^9(y^2 + 3y + 1)^4(y^{19} + 69y^{18} + \dots + 1.52044 \times 10^7y - 262144)$
$c_7$	$(y^2 + y + 1)^4(y^9 - y^8 + 12y^7 - 7y^6 + 37y^5 + y^4 - 10y^2 + 5y - 1) \cdot (y^{19} - 20y^{18} + \dots + 220968y - 6561)$
$c_9, c_{11}, c_{12}$	$((y - 1)^9)(y^2 - 3y + 1)^4(y^{19} - 72y^{18} + \dots + 696y - 1)$