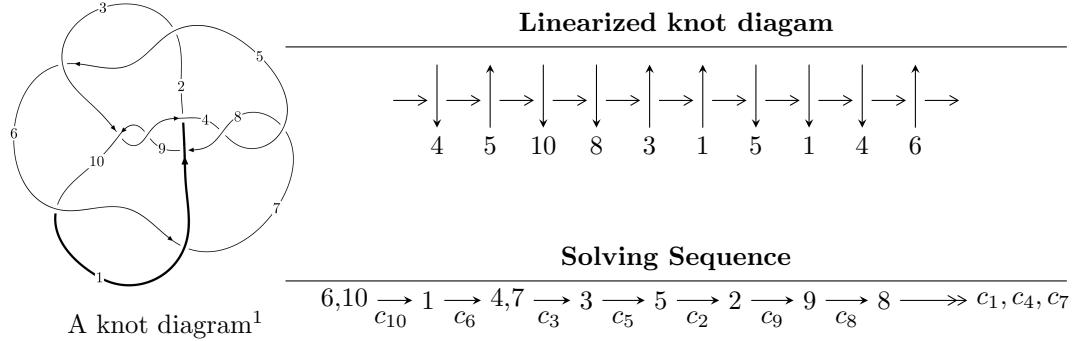


10<sub>158</sub> ( $K10n_{41}$ )



### Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$\begin{aligned}
 I_1^u &= \langle 17u^{11} + 20u^{10} - 54u^9 - 54u^8 + 116u^7 + 92u^6 - 101u^5 - 48u^4 - u^3 + 16u^2 + 37b + 5u - 10, \\
 &\quad - 17u^{11} - 20u^{10} + 54u^9 + 54u^8 - 116u^7 - 92u^6 + 101u^5 + 48u^4 + u^3 - 16u^2 + 37a - 5u - 27, \\
 &\quad u^{12} + u^{11} - 3u^{10} - 3u^9 + 7u^8 + 6u^7 - 6u^6 - 6u^5 + 3u^4 + 3u^3 + 1 \rangle \\
 I_2^u &= \langle 2201978u^{15} - 5194678u^{14} + \dots + 19920857b + 76411393, \\
 &\quad - 7295235u^{15} + 80490581u^{14} + \dots + 378496283a - 1195533446, u^{16} - u^{15} + \dots + 2u + 19 \rangle \\
 I_3^u &= \langle -u^4 + u^3 + u^2 + b - u, u^4 - u^3 - u^2 + a + u + 1, u^6 - u^5 - 2u^4 + 2u^3 + u^2 - u + 1 \rangle
 \end{aligned}$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 34 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle 17u^{11} + 20u^{10} + \cdots + 37b - 10, -17u^{11} - 20u^{10} + \cdots + 37a - 27, u^{12} + u^{11} + \cdots + 3u^3 + 1 \rangle$$

(i) **Arc colorings**

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.459459u^{11} + 0.540541u^{10} + \cdots + 0.135135u + 0.729730 \\ -0.459459u^{11} - 0.540541u^{10} + \cdots - 0.135135u + 0.270270 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ -0.459459u^{11} - 0.540541u^{10} + \cdots - 0.135135u + 0.270270 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ 0.0810811u^{11} - 0.0810811u^{10} + \cdots + 0.729730u - 0.459459 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u^2 + 1 \\ -0.621622u^{11} - 0.378378u^{10} + \cdots - 0.594595u + 0.189189 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -0.945946u^{11} - 0.0540541u^{10} + \cdots + 0.486486u + 0.0270270 \\ 1.40541u^{11} + 0.594595u^{10} + \cdots - 0.351351u + 0.702703 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -0.324324u^{11} + 0.324324u^{10} + \cdots + 1.08108u - 0.162162 \\ 0.918919u^{11} + 0.0810811u^{10} + \cdots + 0.270270u + 0.459459 \end{pmatrix}$$

(ii) **Obstruction class** = -1

(iii) **Cusp Shapes**

$$= -\frac{50}{37}u^{11} - \frac{135}{37}u^{10} + \frac{87}{37}u^9 + \frac{383}{37}u^8 - \frac{191}{37}u^7 - \frac{843}{37}u^6 + \frac{25}{37}u^5 + \frac{657}{37}u^4 + \frac{127}{37}u^3 - \frac{404}{37}u^2 - \frac{117}{37}u - \frac{25}{37}$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_8$	$u^{12} - u^{11} + \cdots - 3u + 1$
$c_2, c_5, c_6$ $c_{10}$	$u^{12} + u^{11} - 3u^{10} - 3u^9 + 7u^8 + 6u^7 - 6u^6 - 6u^5 + 3u^4 + 3u^3 + 1$
$c_3, c_9$	$u^{12} + 9u^{11} + \cdots + 96u + 16$
$c_4, c_7$	$u^{12} - 6u^{11} + \cdots - 10u + 4$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_8$	$y^{12} - 15y^{11} + \cdots + 7y + 1$
$c_2, c_5, c_6$ $c_{10}$	$y^{12} - 7y^{11} + \cdots + 6y^2 + 1$
$c_3, c_9$	$y^{12} + 5y^{11} + \cdots + 896y + 256$
$c_4, c_7$	$y^{12} + 6y^{11} + \cdots + 20y + 16$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.932110 + 0.403591I$		
$a = 0.86095 - 1.68780I$	$7.74885 - 1.69313I$	$-0.90926 + 4.65688I$
$b = 0.13905 + 1.68780I$		
$u = -0.932110 - 0.403591I$		
$a = 0.86095 + 1.68780I$	$7.74885 + 1.69313I$	$-0.90926 - 4.65688I$
$b = 0.13905 - 1.68780I$		
$u = 0.964469 + 0.359565I$		
$a = -0.466137 - 0.540935I$	$-1.16607 + 4.31349I$	$1.88826 - 4.73148I$
$b = 1.46614 + 0.54093I$		
$u = 0.964469 - 0.359565I$		
$a = -0.466137 + 0.540935I$	$-1.16607 - 4.31349I$	$1.88826 + 4.73148I$
$b = 1.46614 - 0.54093I$		
$u = -0.581296 + 0.573734I$		
$a = -0.118591 + 0.442092I$	$-3.10204 + 1.08202I$	$-2.61157 + 1.33940I$
$b = 1.118590 - 0.442092I$		
$u = -0.581296 - 0.573734I$		
$a = -0.118591 - 0.442092I$	$-3.10204 - 1.08202I$	$-2.61157 - 1.33940I$
$b = 1.118590 + 0.442092I$		
$u = 1.157820 + 0.740786I$		
$a = 0.319275 + 1.332450I$	$-0.09105 + 5.46102I$	$-0.77116 - 3.85424I$
$b = 0.68073 - 1.33245I$		
$u = 1.157820 - 0.740786I$		
$a = 0.319275 - 1.332450I$	$-0.09105 - 5.46102I$	$-0.77116 + 3.85424I$
$b = 0.68073 + 1.33245I$		
$u = 0.256008 + 0.492477I$		
$a = 0.735049 + 0.459069I$	$-0.090701 + 1.098140I$	$-1.42722 - 6.18957I$
$b = 0.264951 - 0.459069I$		
$u = 0.256008 - 0.492477I$		
$a = 0.735049 - 0.459069I$	$-0.090701 - 1.098140I$	$-1.42722 + 6.18957I$
$b = 0.264951 + 0.459069I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.36489 + 0.70235I$		
$a = 0.169451 - 1.349320I$	$1.63582 - 12.27120I$	$1.33095 + 7.21681I$
$b = 0.83055 + 1.34932I$		
$u = -1.36489 - 0.70235I$		
$a = 0.169451 + 1.349320I$	$1.63582 + 12.27120I$	$1.33095 - 7.21681I$
$b = 0.83055 - 1.34932I$		

II.

$$I_2^u = \langle 2.20 \times 10^6 u^{15} - 5.19 \times 10^6 u^{14} + \dots + 1.99 \times 10^7 b + 7.64 \times 10^7, -7.30 \times 10^6 u^{15} + 8.05 \times 10^7 u^{14} + \dots + 3.78 \times 10^8 a - 1.20 \times 10^9, u^{16} - u^{15} + \dots + 2u + 19 \rangle$$

(i) **Arc colorings**

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.0192743u^{15} - 0.212659u^{14} + \dots + 0.532412u + 3.15864 \\ -0.110536u^{15} + 0.260766u^{14} + \dots + 2.55924u - 3.83575 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -0.0912621u^{15} + 0.0481069u^{14} + \dots + 3.09165u - 0.677108 \\ -0.110536u^{15} + 0.260766u^{14} + \dots + 2.55924u - 3.83575 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -0.0769980u^{15} + 0.306127u^{14} + \dots - 1.09454u + 0.395782 \\ 0.245201u^{15} - 0.534471u^{14} + \dots - 4.85223u + 6.14044 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.489701u^{15} + 0.863952u^{14} + \dots + 6.26646u - 3.26339 \\ 0.294483u^{15} - 0.508467u^{14} + \dots - 6.40955u + 4.64021 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -0.212645u^{15} + 0.307616u^{14} + \dots + 2.68411u - 2.66285 \\ -0.110536u^{15} + 0.260766u^{14} + \dots + 2.55924u - 2.83575 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -0.507128u^{15} + 0.816083u^{14} + \dots + 9.09366u - 7.30305 \\ 0.0690837u^{15} - 0.0661749u^{14} + \dots - 2.60797u + 1.22994 \end{pmatrix}$$

(ii) **Obstruction class = -1**

(iii) **Cusp Shapes** =  $-\frac{187108568}{378496283}u^{15} + \frac{381114024}{378496283}u^{14} + \dots + \frac{1298769712}{378496283}u - \frac{136693666}{19920857}$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_8$	$u^{16} - 3u^{15} + \cdots - 10u + 1$
$c_2, c_5, c_6$ $c_{10}$	$u^{16} - u^{15} + \cdots + 2u + 19$
$c_3, c_9$	$(u^2 - u + 1)^8$
$c_4, c_7$	$(u^4 + u^3 + u^2 + 1)^4$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_8$	$y^{16} - 5y^{15} + \cdots + 88y + 1$
$c_2, c_5, c_6$ $c_{10}$	$y^{16} - 9y^{15} + \cdots - 1980y + 361$
$c_3, c_9$	$(y^2 + y + 1)^8$
$c_4, c_7$	$(y^4 + y^3 + 3y^2 + 2y + 1)^4$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.921978 + 0.154671I$		
$a = -1.18718 + 0.84702I$	$5.14581 - 0.61478I$	$3.82674 - 1.44464I$
$b = -0.500000 - 0.866025I$		
$u = -0.921978 - 0.154671I$		
$a = -1.18718 - 0.84702I$	$5.14581 + 0.61478I$	$3.82674 + 1.44464I$
$b = -0.500000 + 0.866025I$		
$u = -1.000120 + 0.458209I$		
$a = 0.23948 + 2.07179I$	$-1.85594 - 5.19385I$	$0.17326 + 6.02890I$
$b = -0.500000 - 0.866025I$		
$u = -1.000120 - 0.458209I$		
$a = 0.23948 - 2.07179I$	$-1.85594 + 5.19385I$	$0.17326 - 6.02890I$
$b = -0.500000 + 0.866025I$		
$u = 0.740779 + 0.385723I$		
$a = 0.60451 - 2.36642I$	$-1.85594 - 1.13408I$	$0.173262 - 0.899303I$
$b = -0.500000 + 0.866025I$		
$u = 0.740779 - 0.385723I$		
$a = 0.60451 + 2.36642I$	$-1.85594 + 1.13408I$	$0.173262 + 0.899303I$
$b = -0.500000 - 0.866025I$		
$u = 0.656157 + 1.071140I$		
$a = 0.546203 + 0.202750I$	$-1.85594 + 1.13408I$	$0.173262 + 0.899303I$
$b = -0.500000 - 0.866025I$		
$u = 0.656157 - 1.071140I$		
$a = 0.546203 - 0.202750I$	$-1.85594 - 1.13408I$	$0.173262 - 0.899303I$
$b = -0.500000 + 0.866025I$		
$u = -0.291942 + 1.325280I$		
$a = 0.328801 - 0.073667I$	$-1.85594 + 5.19385I$	$0.17326 - 6.02890I$
$b = -0.500000 + 0.866025I$		
$u = -0.291942 - 1.325280I$		
$a = 0.328801 + 0.073667I$	$-1.85594 - 5.19385I$	$0.17326 + 6.02890I$
$b = -0.500000 - 0.866025I$		

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.126160 + 0.776883I$		
$a = -0.398018 - 0.407492I$	$5.14581 + 3.44499I$	$3.82674 - 8.37284I$
$b = -0.500000 + 0.866025I$		
$u = 1.126160 - 0.776883I$		
$a = -0.398018 + 0.407492I$	$5.14581 - 3.44499I$	$3.82674 + 8.37284I$
$b = -0.500000 - 0.866025I$		
$u = -1.367540 + 0.181274I$		
$a = -0.383277 + 1.317030I$	$5.14581 - 3.44499I$	$3.82674 + 8.37284I$
$b = -0.500000 - 0.866025I$		
$u = -1.367540 - 0.181274I$		
$a = -0.383277 - 1.317030I$	$5.14581 + 3.44499I$	$3.82674 - 8.37284I$
$b = -0.500000 + 0.866025I$		
$u = 1.55848 + 0.24344I$		
$a = -0.092631 - 0.872701I$	$5.14581 + 0.61478I$	$3.82674 + 1.44464I$
$b = -0.500000 + 0.866025I$		
$u = 1.55848 - 0.24344I$		
$a = -0.092631 + 0.872701I$	$5.14581 - 0.61478I$	$3.82674 - 1.44464I$
$b = -0.500000 - 0.866025I$		

**III.**

$$I_3^u = \langle -u^4 + u^3 + u^2 + b - u, \ u^4 - u^3 - u^2 + a + u + 1, \ u^6 - u^5 - 2u^4 + 2u^3 + u^2 - u + 1 \rangle$$

(i) **Arc colorings**

$$a_6 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^4 + u^3 + u^2 - u - 1 \\ u^4 - u^3 - u^2 + u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -1 \\ u^4 - u^3 - u^2 + u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u \\ u^5 - u^4 - u^3 + u^2 + u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^2 - 1 \\ -u^2 + 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^4 - u^3 - 2u^2 + 2u + 1 \\ u^2 - u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^4 - u^3 - u^2 + 2u \\ -u^4 + 2u^2 - u \end{pmatrix}$$

(ii) **Obstruction class = 1**

(iii) **Cusp Shapes** =  $-2u^5 - u^4 + 6u^3 + 3u^2 - 7u + 3$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_8$	$u^6 - u^5 - 2u^3 + 2u^2 + 1$
$c_2, c_6$	$u^6 + u^5 - 2u^4 - 2u^3 + u^2 + u + 1$
$c_3$	$u^6 + 2u^4 + 2u^3 + u + 1$
$c_4$	$u^6 - u^5 + 3u^4 - 2u^3 + 3u^2 + 1$
$c_5, c_{10}$	$u^6 - u^5 - 2u^4 + 2u^3 + u^2 - u + 1$
$c_7$	$u^6 + u^5 + 3u^4 + 2u^3 + 3u^2 + 1$
$c_9$	$u^6 + 2u^4 - 2u^3 - u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_8$	$y^6 - y^5 - 2y^3 + 4y^2 + 4y + 1$
$c_2, c_5, c_6$ $c_{10}$	$y^6 - 5y^5 + 10y^4 - 8y^3 + y^2 + y + 1$
$c_3, c_9$	$y^6 + 4y^5 + 4y^4 - 2y^3 - y + 1$
$c_4, c_7$	$y^6 + 5y^5 + 11y^4 + 16y^3 + 15y^2 + 6y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.099190 + 0.287563I$		
$a = -0.69782 + 1.52185I$	$8.54916 - 1.24964I$	$7.95941 - 0.00232I$
$b = -0.30218 - 1.52185I$		
$u = -1.099190 - 0.287563I$		
$a = -0.69782 - 1.52185I$	$8.54916 + 1.24964I$	$7.95941 + 0.00232I$
$b = -0.30218 + 1.52185I$		
$u = 0.264925 + 0.576623I$		
$a = -1.74836 - 0.18113I$	$-2.37427 + 2.84527I$	$-1.26269 - 3.26816I$
$b = 0.748359 + 0.181129I$		
$u = 0.264925 - 0.576623I$		
$a = -1.74836 + 0.18113I$	$-2.37427 - 2.84527I$	$-1.26269 + 3.26816I$
$b = 0.748359 - 0.181129I$		
$u = 1.334260 + 0.378781I$		
$a = -0.553818 - 0.708238I$	$5.33965 + 2.32699I$	$5.80328 - 1.20156I$
$b = -0.446182 + 0.708238I$		
$u = 1.334260 - 0.378781I$		
$a = -0.553818 + 0.708238I$	$5.33965 - 2.32699I$	$5.80328 + 1.20156I$
$b = -0.446182 - 0.708238I$		

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_8$	$(u^6 - u^5 - 2u^3 + 2u^2 + 1)(u^{12} - u^{11} + \dots - 3u + 1)$ $\cdot (u^{16} - 3u^{15} + \dots - 10u + 1)$
$c_2, c_6$	$(u^6 + u^5 - 2u^4 - 2u^3 + u^2 + u + 1)$ $\cdot (u^{12} + u^{11} - 3u^{10} - 3u^9 + 7u^8 + 6u^7 - 6u^6 - 6u^5 + 3u^4 + 3u^3 + 1)$ $\cdot (u^{16} - u^{15} + \dots + 2u + 19)$
$c_3$	$((u^2 - u + 1)^8)(u^6 + 2u^4 + 2u^3 + u + 1)(u^{12} + 9u^{11} + \dots + 96u + 16)$
$c_4$	$(u^4 + u^3 + u^2 + 1)^4(u^6 - u^5 + 3u^4 - 2u^3 + 3u^2 + 1)$ $\cdot (u^{12} - 6u^{11} + \dots - 10u + 4)$
$c_5, c_{10}$	$(u^6 - u^5 - 2u^4 + 2u^3 + u^2 - u + 1)$ $\cdot (u^{12} + u^{11} - 3u^{10} - 3u^9 + 7u^8 + 6u^7 - 6u^6 - 6u^5 + 3u^4 + 3u^3 + 1)$ $\cdot (u^{16} - u^{15} + \dots + 2u + 19)$
$c_7$	$(u^4 + u^3 + u^2 + 1)^4(u^6 + u^5 + 3u^4 + 2u^3 + 3u^2 + 1)$ $\cdot (u^{12} - 6u^{11} + \dots - 10u + 4)$
$c_9$	$((u^2 - u + 1)^8)(u^6 + 2u^4 - 2u^3 - u + 1)(u^{12} + 9u^{11} + \dots + 96u + 16)$

## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_8$	$(y^6 - y^5 - 2y^3 + 4y^2 + 4y + 1)(y^{12} - 15y^{11} + \dots + 7y + 1)$ $\cdot (y^{16} - 5y^{15} + \dots + 88y + 1)$
$c_2, c_5, c_6$ $c_{10}$	$(y^6 - 5y^5 + 10y^4 - 8y^3 + y^2 + y + 1)(y^{12} - 7y^{11} + \dots + 6y^2 + 1)$ $\cdot (y^{16} - 9y^{15} + \dots - 1980y + 361)$
$c_3, c_9$	$(y^2 + y + 1)^8(y^6 + 4y^5 + 4y^4 - 2y^3 - y + 1)$ $\cdot (y^{12} + 5y^{11} + \dots + 896y + 256)$
$c_4, c_7$	$(y^4 + y^3 + 3y^2 + 2y + 1)^4(y^6 + 5y^5 + 11y^4 + 16y^3 + 15y^2 + 6y + 1)$ $\cdot (y^{12} + 6y^{11} + \dots + 20y + 16)$