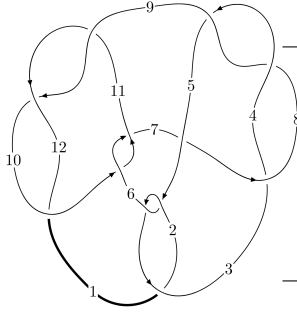
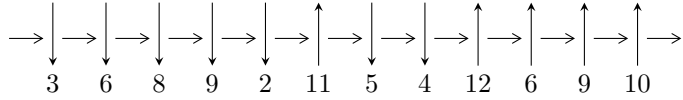


12n<sub>0346</sub> (K12n<sub>0346</sub>)

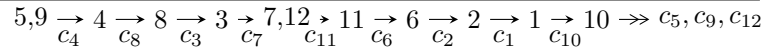


A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**



**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -8563315867395u^{19} + 12169590007513u^{18} + \dots + 13082431761068b + 64802000815324, \\ 52899877847623u^{19} - 79722292100065u^{18} + \dots + 13082431761068a - 413103210378158, \\ u^{20} - 2u^{19} + \dots - 12u + 4 \rangle$$

$$I_2^u = \langle u^7 - u^6 - 2u^5 + 3u^4 - 2u^2 + b + 4u - 2, -u^7 - u^6 + 3u^5 + 2u^4 - 3u^3 + a - 2, \\ u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1 \rangle$$

$$I_3^u = \langle -au + b - u - 1, 2a^2 + au - 1, u^2 - 2 \rangle$$

$$I_1^v = \langle a, b - v + 2, v^2 - 3v + 1 \rangle$$

\* 4 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 34 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle -8.56 \times 10^{12} u^{19} + 1.22 \times 10^{13} u^{18} + \dots + 1.31 \times 10^{13} b + 6.48 \times 10^{13}, 5.29 \times 10^{13} u^{19} - 7.97 \times 10^{13} u^{18} + \dots + 1.31 \times 10^{13} a - 4.13 \times 10^{14}, u^{20} - 2u^{19} + \dots - 12u + 4 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^3 + 2u \\ -u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -4.04358u^{19} + 6.09384u^{18} + \dots - 25.5560u + 31.5769 \\ 0.654566u^{19} - 0.930224u^{18} + \dots + 6.84847u - 4.95336 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -4.04358u^{19} + 6.09384u^{18} + \dots - 25.5560u + 31.5769 \\ -0.367222u^{19} + 0.611916u^{18} + \dots - 0.897037u + 3.01992 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -6.21544u^{19} + 9.28273u^{18} + \dots - 48.2193u + 51.0252 \\ 0.348398u^{19} - 0.661202u^{18} + \dots + 3.63448u - 3.80249 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -4.90774u^{19} + 7.58167u^{18} + \dots - 38.9378u + 42.2351 \\ 2.23801u^{19} - 3.26288u^{18} + \dots + 16.4563u - 17.7254 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -5.35239u^{19} + 8.30016u^{18} + \dots - 43.5388u + 45.8952 \\ 2.16712u^{19} - 3.30945u^{18} + \dots + 15.7609u - 18.5509 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} 5.60383u^{19} - 8.48450u^{18} + \dots + 48.3882u - 47.1067 \\ -2.25347u^{19} + 3.54770u^{18} + \dots - 15.9192u + 19.1670 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= \frac{8651471282093}{3270607940267} u^{19} - \frac{12603254258761}{3270607940267} u^{18} + \dots - \frac{78299168927818}{3270607940267} u - \frac{83998088848016}{3270607940267}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{20} + 24u^{18} + \dots + 3807u + 81$
$c_2, c_5$	$u^{20} + 4u^{19} + \dots - 93u - 9$
$c_3, c_4, c_8$	$u^{20} + 2u^{19} + \dots + 12u + 4$
$c_6, c_{10}$	$u^{20} + 2u^{19} + \dots + 3200u + 256$
$c_7$	$u^{20} - 6u^{19} + \dots - 6212u - 964$
$c_9, c_{11}, c_{12}$	$u^{20} + 12u^{19} + \dots - 58u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{20} + 48y^{19} + \dots - 10684791y + 6561$
$c_2, c_5$	$y^{20} + 24y^{18} + \dots - 3807y + 81$
$c_3, c_4, c_8$	$y^{20} - 14y^{19} + \dots - 432y + 16$
$c_6, c_{10}$	$y^{20} - 60y^{19} + \dots - 5160960y + 65536$
$c_7$	$y^{20} + 58y^{19} + \dots - 44905072y + 929296$
$c_9, c_{11}, c_{12}$	$y^{20} - 44y^{19} + \dots - 2922y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.311540 + 0.820503I$ $a = -1.183960 + 0.104977I$ $b = -0.936205 - 0.261735I$	$1.94026 - 0.82547I$	$0.912486 + 0.761117I$
$u = -0.311540 - 0.820503I$ $a = -1.183960 - 0.104977I$ $b = -0.936205 + 0.261735I$	$1.94026 + 0.82547I$	$0.912486 - 0.761117I$
$u = 0.806269 + 0.099052I$ $a = -0.029427 - 0.611307I$ $b = 0.383913 - 0.245700I$	$-1.290640 + 0.060456I$	$-5.65835 + 0.55419I$
$u = 0.806269 - 0.099052I$ $a = -0.029427 + 0.611307I$ $b = 0.383913 + 0.245700I$	$-1.290640 - 0.060456I$	$-5.65835 - 0.55419I$
$u = 1.33638$ $a = -1.16154$ $b = 1.16836$	$3.62783$	$3.71420$
$u = -1.301610 + 0.401831I$ $a = 0.508418 - 0.295516I$ $b = -0.439577 + 0.851076I$	$-1.51005 + 5.52302I$	$-5.54282 - 3.24717I$
$u = -1.301610 - 0.401831I$ $a = 0.508418 + 0.295516I$ $b = -0.439577 - 0.851076I$	$-1.51005 - 5.52302I$	$-5.54282 + 3.24717I$
$u = 1.037800 + 0.883330I$ $a = 0.77758 - 1.27910I$ $b = -2.72069 - 2.90147I$	$4.14315 - 3.36874I$	$0.04800 + 2.58345I$
$u = 1.037800 - 0.883330I$ $a = 0.77758 + 1.27910I$ $b = -2.72069 + 2.90147I$	$4.14315 + 3.36874I$	$0.04800 - 2.58345I$
$u = -1.38998$ $a = -0.0133698$ $b = -1.23390$	$-6.53389$	$-14.0580$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.307583 + 1.370650I$ $a = 0.05460 + 1.81689I$ $b = -1.41200 + 5.33955I$	$-16.9450 + 5.1999I$	$0.48479 - 2.11695I$
$u = -0.307583 - 1.370650I$ $a = 0.05460 - 1.81689I$ $b = -1.41200 - 5.33955I$	$-16.9450 - 5.1999I$	$0.48479 + 2.11695I$
$u = 1.43456$ $a = 0.671677$ $b = 7.72000$	$-4.98064$	$60.9990$
$u = 0.486279$ $a = -3.47218$ $b = 1.37914$	$6.88313$	$-9.17840$
$u = 0.388073$ $a = 0.457271$ $b = 0.718784$	$-1.01688$	$-12.8430$
$u = 1.58364 + 0.49918I$ $a = -1.064100 + 0.688577I$ $b = 3.34498 + 2.38478I$	$16.4234 - 11.7755I$	$-1.93237 + 4.54638I$
$u = 1.58364 - 0.49918I$ $a = -1.064100 - 0.688577I$ $b = 3.34498 - 2.38478I$	$16.4234 + 11.7755I$	$-1.93237 - 4.54638I$
$u = -0.283054$ $a = -3.06452$ $b = -0.370034$	$1.22670$	$10.7000$
$u = -1.49310 + 0.84801I$ $a = 1.22823 + 0.86872I$ $b = -3.91160 + 3.62679I$	$19.0199 + 2.7480I$	$-0.478669 - 0.983748I$
$u = -1.49310 - 0.84801I$ $a = 1.22823 - 0.86872I$ $b = -3.91160 - 3.62679I$	$19.0199 - 2.7480I$	$-0.478669 + 0.983748I$

$$\text{II. } I_2^u = \langle u^7 - u^6 - 2u^5 + 3u^4 - 2u^2 + b + 4u - 2, -u^7 - u^6 + 3u^5 + 2u^4 - 3u^3 + a - 2, u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ -u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -u^2 + 1 \\ u^4 - 2u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^3 + 2u \\ -u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} u^7 + u^6 - 3u^5 - 2u^4 + 3u^3 + 2 \\ -u^7 + u^6 + 2u^5 - 3u^4 + 2u^2 - 4u + 2 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} u^7 + u^6 - 3u^5 - 2u^4 + 3u^3 + 2 \\ -u^7 + u^6 + 2u^5 - 3u^4 + 2u^2 - 3u + 2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -u^3 + 2u \\ -u^3 + u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^5 - 2u^3 + u \\ -u^7 + 3u^5 - 2u^3 - u \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^7 + u^6 - 3u^5 - 2u^4 + 3u^3 + 2 \\ -u^7 + u^6 + 2u^5 - 3u^4 + 2u^2 - 3u + 2 \end{pmatrix}$$

(ii) Obstruction class = 1

$$\text{(iii) Cusp Shapes} = 4u^7 - 9u^6 - 10u^5 + 27u^4 - 2u^3 - 18u^2 + 20u - 17$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^8 - 3u^7 + 7u^6 - 10u^5 + 11u^4 - 10u^3 + 6u^2 - 4u + 1$
$c_2$	$u^8 - u^7 - u^6 + 2u^5 + u^4 - 2u^3 + 2u - 1$
$c_3, c_4$	$u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1$
$c_5$	$u^8 + u^7 - u^6 - 2u^5 + u^4 + 2u^3 - 2u - 1$
$c_6, c_{10}$	$u^8$
$c_7$	$u^8 + 3u^7 + 7u^6 + 10u^5 + 11u^4 + 10u^3 + 6u^2 + 4u + 1$
$c_8$	$u^8 - u^7 - 3u^6 + 2u^5 + 3u^4 - 2u - 1$
$c_9$	$(u + 1)^8$
$c_{11}, c_{12}$	$(u - 1)^8$



(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_7$	$y^8 + 5y^7 + 11y^6 + 6y^5 - 17y^4 - 34y^3 - 22y^2 - 4y + 1$
$c_2, c_5$	$y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1$
$c_3, c_4, c_8$	$y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1$
$c_6, c_{10}$	$y^8$
$c_9, c_{11}, c_{12}$	$(y - 1)^8$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.180120 + 0.268597I$ $a = 0.805639 - 0.183365I$ $b = -1.14297 - 0.89911I$	$0.604279 - 1.131230I$	$-1.38132 + 1.25921I$
$u = 1.180120 - 0.268597I$ $a = 0.805639 + 0.183365I$ $b = -1.14297 + 0.89911I$	$0.604279 + 1.131230I$	$-1.38132 - 1.25921I$
$u = 0.108090 + 0.747508I$ $a = 0.189481 - 1.310380I$ $b = -0.02521 - 1.55019I$	$3.80435 - 2.57849I$	$1.74277 + 4.63100I$
$u = 0.108090 - 0.747508I$ $a = 0.189481 + 1.310380I$ $b = -0.02521 + 1.55019I$	$3.80435 + 2.57849I$	$1.74277 - 4.63100I$
$u = -1.37100$ $a = -0.729394$ $b = 6.70204$	$-4.85780$	$-25.4550$
$u = -1.334530 + 0.318930I$ $a = -0.708845 - 0.169402I$ $b = 1.07471 - 1.15185I$	$-0.73474 + 6.44354I$	$-1.71699 - 7.87618I$
$u = -1.334530 - 0.318930I$ $a = -0.708845 + 0.169402I$ $b = 1.07471 + 1.15185I$	$-0.73474 - 6.44354I$	$-1.71699 + 7.87618I$
$u = 0.463640$ $a = 2.15684$ $b = 0.484913$	$0.799899$	$-10.8330$

$$\text{III. } I_3^u = \langle -au + b - u - 1, 2a^2 + au - 1, u^2 - 2 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u \\ -u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 0 \\ -u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} a \\ au + u + 1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} a \\ au + 2a + u + 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -a + \frac{1}{2}u \\ 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -a + \frac{1}{2}u - 1 \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -a + \frac{1}{2}u \\ 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -a + \frac{1}{2}u \\ 2a + u + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = -4

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_5$	$(u - 1)^4$
$c_2$	$(u + 1)^4$
$c_3, c_4, c_7$ $c_8$	$(u^2 - 2)^2$
$c_6, c_{11}, c_{12}$	$(u^2 + u - 1)^2$
$c_9, c_{10}$	$(u^2 - u - 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_5$	$(y - 1)^4$
$c_3, c_4, c_7$ $c_8$	$(y - 2)^4$
$c_6, c_9, c_{10}$ $c_{11}, c_{12}$	$(y^2 - 3y + 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.41421$ $a = -1.14412$ $b = 0.796180$	2.30291	-4.00000
$u = 1.41421$ $a = 0.437016$ $b = 3.03225$	-5.59278	-4.00000
$u = -1.41421$ $a = 1.14412$ $b = -2.03225$	2.30291	-4.00000
$u = -1.41421$ $a = -0.437016$ $b = 0.203820$	-5.59278	-4.00000

$$\text{IV. } I_1^v = \langle a, b - v + 2, v^2 - 3v + 1 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ v - 2 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -2v + 1 \\ v - 2 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} -2v + 1 \\ 1 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 2v \\ -1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 2v - 1 \\ -1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} v \\ -1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = 14

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$(u - 1)^2$
$c_3, c_4, c_7$ $c_8$	$u^2$
$c_5$	$(u + 1)^2$
$c_6, c_9$	$u^2 - u - 1$
$c_{10}, c_{11}, c_{12}$	$u^2 + u - 1$



(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_5$	$(y - 1)^2$
$c_3, c_4, c_7$ $c_8$	$y^2$
$c_6, c_9, c_{10}$ $c_{11}, c_{12}$	$y^2 - 3y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^v$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$v = 0.381966$ $a = 0$ $b = -1.61803$	$-0.657974$	14.0000
$v = 2.61803$ $a = 0$ $b = 0.618034$	$7.23771$	14.0000

## V. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u-1)^6(u^8 - 3u^7 + 7u^6 - 10u^5 + 11u^4 - 10u^3 + 6u^2 - 4u + 1) \cdot (u^{20} + 24u^{18} + \dots + 3807u + 81)$
$c_2$	$(u-1)^2(u+1)^4(u^8 - u^7 - u^6 + 2u^5 + u^4 - 2u^3 + 2u - 1) \cdot (u^{20} + 4u^{19} + \dots - 93u - 9)$
$c_3, c_4$	$u^2(u^2 - 2)^2(u^8 + u^7 - 3u^6 - 2u^5 + 3u^4 + 2u - 1) \cdot (u^{20} + 2u^{19} + \dots + 12u + 4)$
$c_5$	$(u-1)^4(u+1)^2(u^8 + u^7 - u^6 - 2u^5 + u^4 + 2u^3 - 2u - 1) \cdot (u^{20} + 4u^{19} + \dots - 93u - 9)$
$c_6$	$u^8(u^2 - u - 1)(u^2 + u - 1)^2(u^{20} + 2u^{19} + \dots + 3200u + 256)$
$c_7$	$u^2(u^2 - 2)^2(u^8 + 3u^7 + 7u^6 + 10u^5 + 11u^4 + 10u^3 + 6u^2 + 4u + 1) \cdot (u^{20} - 6u^{19} + \dots - 6212u - 964)$
$c_8$	$u^2(u^2 - 2)^2(u^8 - u^7 - 3u^6 + 2u^5 + 3u^4 - 2u - 1) \cdot (u^{20} + 2u^{19} + \dots + 12u + 4)$
$c_9$	$((u+1)^8)(u^2 - u - 1)^3(u^{20} + 12u^{19} + \dots - 58u + 1)$
$c_{10}$	$u^8(u^2 - u - 1)^2(u^2 + u - 1)(u^{20} + 2u^{19} + \dots + 3200u + 256)$
$c_{11}, c_{12}$	$((u-1)^8)(u^2 + u - 1)^3(u^{20} + 12u^{19} + \dots - 58u + 1)$

## VI. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$(y-1)^6(y^8 + 5y^7 + 11y^6 + 6y^5 - 17y^4 - 34y^3 - 22y^2 - 4y + 1)$ $\cdot (y^{20} + 48y^{19} + \dots - 10684791y + 6561)$
$c_2, c_5$	$(y-1)^6(y^8 - 3y^7 + 7y^6 - 10y^5 + 11y^4 - 10y^3 + 6y^2 - 4y + 1)$ $\cdot (y^{20} + 24y^{18} + \dots - 3807y + 81)$
$c_3, c_4, c_8$	$y^2(y-2)^4(y^8 - 7y^7 + 19y^6 - 22y^5 + 3y^4 + 14y^3 - 6y^2 - 4y + 1)$ $\cdot (y^{20} - 14y^{19} + \dots - 432y + 16)$
$c_6, c_{10}$	$y^8(y^2 - 3y + 1)^3(y^{20} - 60y^{19} + \dots - 5160960y + 65536)$
$c_7$	$y^2(y-2)^4(y^8 + 5y^7 + 11y^6 + 6y^5 - 17y^4 - 34y^3 - 22y^2 - 4y + 1)$ $\cdot (y^{20} + 58y^{19} + \dots - 44905072y + 929296)$
$c_9, c_{11}, c_{12}$	$((y-1)^8)(y^2 - 3y + 1)^3(y^{20} - 44y^{19} + \dots - 2922y + 1)$