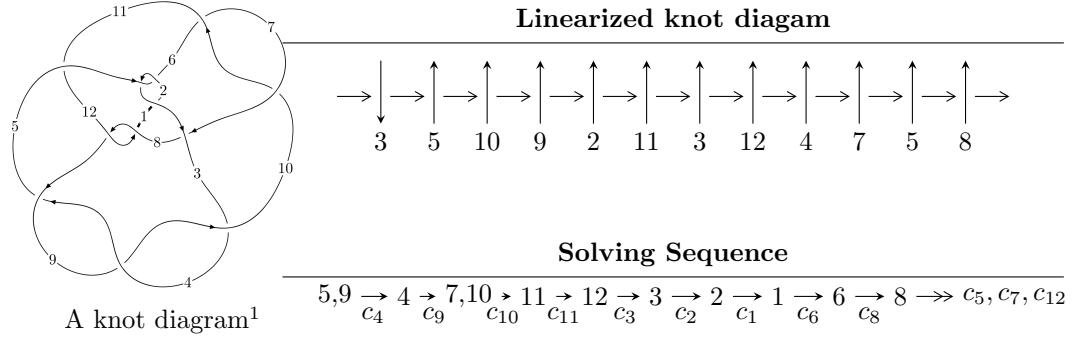


$12n_{0404}$  ( $K12n_{0404}$ )



### Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$\begin{aligned}
 I_1^u &= \langle -u^7 + 2u^6 - 5u^5 + 6u^4 - 6u^3 + 4u^2 + b - u - 1, \\
 &\quad u^{10} - 4u^9 + 13u^8 - 26u^7 + 42u^6 - 50u^5 + 45u^4 - 28u^3 + 9u^2 + 2a + 4u - 3, \\
 &\quad u^{11} - 4u^{10} + 13u^9 - 28u^8 + 48u^7 - 64u^6 + 67u^5 - 52u^4 + 29u^3 - 6u^2 - 3u + 2 \rangle \\
 I_2^u &= \langle u^7 + 3u^5 + 2u^3 + b - u - 1, u^9 + 4u^7 + u^6 + 5u^5 + 3u^4 + u^3 + 2u^2 + a, u^{10} + 5u^8 + 8u^6 + 3u^4 - u^2 + 1 \rangle
 \end{aligned}$$

\* 2 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 21 representations.

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<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$I_1^u = \langle -u^7 + 2u^6 + \dots + b - 1, u^{10} - 4u^9 + \dots + 2a - 3, u^{11} - 4u^{10} + \dots - 3u + 2 \rangle$$

(i) Arc colorings

$$a_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -\frac{1}{2}u^{10} + 2u^9 + \dots - 2u + \frac{3}{2} \\ u^7 - 2u^6 + 5u^5 - 6u^4 + 6u^3 - 4u^2 + u + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u \\ u^3 + u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -\frac{1}{2}u^{10} + 2u^9 + \dots - \frac{5}{2}u^2 + \frac{1}{2} \\ u^{10} - 3u^9 + 9u^8 - 16u^7 + 24u^6 - 27u^5 + 22u^4 - 13u^3 + 3u^2 + 2u - 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} \frac{1}{2}u^{10} - u^9 + \dots + 2u - \frac{1}{2} \\ u^{10} - 3u^9 + 9u^8 - 16u^7 + 24u^6 - 27u^5 + 22u^4 - 13u^3 + 3u^2 + 2u - 1 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u^4 - u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^{10} + 5u^8 + 8u^6 + 3u^4 - u^2 + 1 \\ 9u^{10} - 24u^9 + \dots + 10u - 8 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^8 + 3u^6 + u^4 - 2u^2 + 1 \\ -u^8 - 4u^6 - 4u^4 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} \frac{1}{2}u^{10} - 2u^9 + \dots + u - \frac{3}{2} \\ -u^8 - u^7 - 2u^6 - 5u^5 + 2u^4 - 6u^3 + 4u^2 - 1 \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes

$$= -2u^{10} + 8u^9 - 26u^8 + 54u^7 - 88u^6 + 106u^5 - 94u^4 + 52u^3 - 10u^2 - 18u + 18$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^{11} + 54u^{10} + \cdots + 10929u - 576$
$c_2, c_5$	$u^{11} + 2u^{10} + \cdots + 9u - 24$
$c_3, c_4, c_9$	$u^{11} + 4u^{10} + \cdots - 3u - 2$
$c_6, c_8, c_{10}$ $c_{12}$	$u^{11} - u^{10} + \cdots + u - 1$
$c_7, c_{11}$	$u^{11} - u^{10} + \cdots - 13u - 19$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{11} - 546y^{10} + \cdots + 91631457y - 331776$
$c_2, c_5$	$y^{11} + 54y^{10} + \cdots + 10929y - 576$
$c_3, c_4, c_9$	$y^{11} + 10y^{10} + \cdots + 33y - 4$
$c_6, c_8, c_{10}$ $c_{12}$	$y^{11} + 33y^{10} + \cdots - y - 1$
$c_7, c_{11}$	$y^{11} + 93y^{10} + \cdots + 4083y - 361$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.025290 + 0.573466I$		
$a = 0.522820 + 0.914944I$	$14.2401 + 3.3069I$	$6.51466 - 1.87736I$
$b = -2.15532 - 0.07526I$		
$u = 1.025290 - 0.573466I$		
$a = 0.522820 - 0.914944I$	$14.2401 - 3.3069I$	$6.51466 + 1.87736I$
$b = -2.15532 + 0.07526I$		
$u = -0.041636 + 1.304450I$		
$a = 0.517767 - 0.458534I$	$-3.44187 - 1.25408I$	$7.18214 + 5.23967I$
$b = -0.109577 + 0.529508I$		
$u = -0.041636 - 1.304450I$		
$a = 0.517767 + 0.458534I$	$-3.44187 + 1.25408I$	$7.18214 - 5.23967I$
$b = -0.109577 - 0.529508I$		
$u = 0.564252 + 0.373580I$		
$a = 0.013693 - 0.730377I$	$-1.87019 + 1.75538I$	$8.10394 - 4.89065I$
$b = 0.832660 - 0.220165I$		
$u = 0.564252 - 0.373580I$		
$a = 0.013693 + 0.730377I$	$-1.87019 - 1.75538I$	$8.10394 + 4.89065I$
$b = 0.832660 + 0.220165I$		
$u = 0.21728 + 1.43552I$		
$a = -1.255990 - 0.421534I$	$-7.66219 + 4.64924I$	$5.42003 - 4.56433I$
$b = 1.114610 - 0.376316I$		
$u = 0.21728 - 1.43552I$		
$a = -1.255990 + 0.421534I$	$-7.66219 - 4.64924I$	$5.42003 + 4.56433I$
$b = 1.114610 + 0.376316I$		
$u = 0.40590 + 1.55278I$		
$a = 1.63733 + 1.45401I$	$7.51792 + 8.56204I$	$4.30767 - 3.05307I$
$b = -2.10223 - 0.22168I$		
$u = 0.40590 - 1.55278I$		
$a = 1.63733 - 1.45401I$	$7.51792 - 8.56204I$	$4.30767 + 3.05307I$
$b = -2.10223 + 0.22168I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.342167$		
$a = 0.628767$	0.526767	18.9430
$b = -0.160296$		

$$\text{II. } I_2^u = \langle u^7 + 3u^5 + 2u^3 + b - u - 1, u^9 + 4u^7 + u^6 + 5u^5 + 3u^4 + u^3 + 2u^2 + a, u^{10} + 5u^8 + 8u^6 + 3u^4 - u^2 + 1 \rangle$$

(i) **Arc colorings**

$$\begin{aligned} a_5 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_9 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_4 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_7 &= \begin{pmatrix} -u^9 - 4u^7 - u^6 - 5u^5 - 3u^4 - u^3 - 2u^2 \\ -u^7 - 3u^5 - 2u^3 + u + 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} u \\ u^3 + u \end{pmatrix} \\ a_{11} &= \begin{pmatrix} u^9 - u^8 + 4u^7 - 4u^6 + 5u^5 - 4u^4 + u^3 - 1 \\ -u^9 - 4u^7 - 5u^5 - u^4 - u^3 - 2u^2 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} -u^8 - 4u^6 - 5u^4 - 2u^2 - 1 \\ -u^9 - 4u^7 - 5u^5 - u^4 - u^3 - 2u^2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix} \\ a_2 &= \begin{pmatrix} -u^4 - u^2 + 1 \\ u^4 + 2u^2 \end{pmatrix} \\ a_1 &= \begin{pmatrix} 0 \\ -u^8 - 3u^6 - u^4 + 2u^2 - 1 \end{pmatrix} \\ a_6 &= \begin{pmatrix} u^8 + 3u^6 + u^4 - 2u^2 + 1 \\ -u^8 - 4u^6 - 4u^4 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -u^9 - 5u^7 - 8u^5 - 3u^3 + u \\ u^8 - u^7 + 4u^6 - 3u^5 + 4u^4 - 2u^3 + 2u + 1 \end{pmatrix} \end{aligned}$$

(ii) **Obstruction class** = 1

(iii) **Cusp Shapes** =  $-4u^6 - 12u^4 - 8u^2 + 4$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$(u^5 - 3u^4 + 4u^3 - u^2 - u + 1)^2$
$c_2$	$(u^5 - u^4 + 2u^3 - u^2 + u - 1)^2$
$c_3, c_4, c_9$	$u^{10} + 5u^8 + 8u^6 + 3u^4 - u^2 + 1$
$c_5$	$(u^5 + u^4 + 2u^3 + u^2 + u + 1)^2$
$c_6, c_8, c_{10}$ $c_{12}$	$(u^2 + 1)^5$
$c_7$	$u^{10} - 2u^9 + 5u^8 + 7u^6 + 10u^5 + 24u^4 + 30u^3 + 37u^2 + 40u + 29$
$c_{11}$	$u^{10} + 2u^9 + 5u^8 + 7u^6 - 10u^5 + 24u^4 - 30u^3 + 37u^2 - 40u + 29$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1$	$(y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)^2$
$c_2, c_5$	$(y^5 + 3y^4 + 4y^3 + y^2 - y - 1)^2$
$c_3, c_4, c_9$	$(y^5 + 5y^4 + 8y^3 + 3y^2 - y + 1)^2$
$c_6, c_8, c_{10}$ $c_{12}$	$(y + 1)^{10}$
$c_7, c_{11}$	$y^{10} + 6y^9 + \cdots + 546y + 841$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 1.217740I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -0.37029 - 1.58802I$	-5.69095	2.51890
$b = 1.000000 + 0.766826I$		
$u = -1.217740I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -0.37029 + 1.58802I$	-5.69095	2.51890
$b = 1.000000 - 0.766826I$		
$u = 0.549911 + 0.309916I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = 0.42897 - 1.54636I$	-3.61897 + 1.53058I	3.48489 - 4.43065I
$b = 1.82238 - 0.33911I$		
$u = 0.549911 - 0.309916I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = 0.42897 + 1.54636I$	-3.61897 - 1.53058I	3.48489 + 4.43065I
$b = 1.82238 + 0.33911I$		
$u = -0.549911 + 0.309916I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -0.686530 + 0.668968I$	-3.61897 - 1.53058I	3.48489 + 4.43065I
$b = 0.177625 - 0.339110I$		
$u = -0.549911 - 0.309916I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -0.686530 - 0.668968I$	-3.61897 + 1.53058I	3.48489 - 4.43065I
$b = 0.177625 + 0.339110I$		
$u = -0.21917 + 1.41878I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -0.092267 + 0.641941I$	-9.16243 - 4.40083I	-0.74431 + 3.49859I
$b = -0.200152 - 0.455697I$		
$u = -0.21917 - 1.41878I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -0.092267 - 0.641941I$	-9.16243 + 4.40083I	-0.74431 - 3.49859I
$b = -0.200152 + 0.455697I$		
$u = 0.21917 + 1.41878I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -2.27989 - 1.10735I$	-9.16243 + 4.40083I	-0.74431 - 3.49859I
$b = 2.20015 - 0.45570I$		
$u = 0.21917 - 1.41878I$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	
$a = -2.27989 + 1.10735I$	-9.16243 - 4.40083I	-0.74431 + 3.49859I
$b = 2.20015 + 0.45570I$		

### III. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u^5 - 3u^4 + 4u^3 - u^2 - u + 1)^2)(u^{11} + 54u^{10} + \dots + 10929u - 576)$
$c_2$	$((u^5 - u^4 + 2u^3 - u^2 + u - 1)^2)(u^{11} + 2u^{10} + \dots + 9u - 24)$
$c_3, c_4, c_9$	$(u^{10} + 5u^8 + 8u^6 + 3u^4 - u^2 + 1)(u^{11} + 4u^{10} + \dots - 3u - 2)$
$c_5$	$((u^5 + u^4 + 2u^3 + u^2 + u + 1)^2)(u^{11} + 2u^{10} + \dots + 9u - 24)$
$c_6, c_8, c_{10}$ $c_{12}$	$((u^2 + 1)^5)(u^{11} - u^{10} + \dots + u - 1)$
$c_7$	$(u^{10} - 2u^9 + 5u^8 + 7u^6 + 10u^5 + 24u^4 + 30u^3 + 37u^2 + 40u + 29) \cdot (u^{11} - u^{10} + \dots - 13u - 19)$
$c_{11}$	$(u^{10} + 2u^9 + 5u^8 + 7u^6 - 10u^5 + 24u^4 - 30u^3 + 37u^2 - 40u + 29) \cdot (u^{11} - u^{10} + \dots - 13u - 19)$

#### IV. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$(y^5 - y^4 + 8y^3 - 3y^2 + 3y - 1)^2$ $\cdot (y^{11} - 546y^{10} + \cdots + 91631457y - 331776)$
$c_2, c_5$	$((y^5 + 3y^4 + 4y^3 + y^2 - y - 1)^2)(y^{11} + 54y^{10} + \cdots + 10929y - 576)$
$c_3, c_4, c_9$	$((y^5 + 5y^4 + 8y^3 + 3y^2 - y + 1)^2)(y^{11} + 10y^{10} + \cdots + 33y - 4)$
$c_6, c_8, c_{10}$ $c_{12}$	$((y + 1)^{10})(y^{11} + 33y^{10} + \cdots - y - 1)$
$c_7, c_{11}$	$(y^{10} + 6y^9 + \cdots + 546y + 841)(y^{11} + 93y^{10} + \cdots + 4083y - 361)$