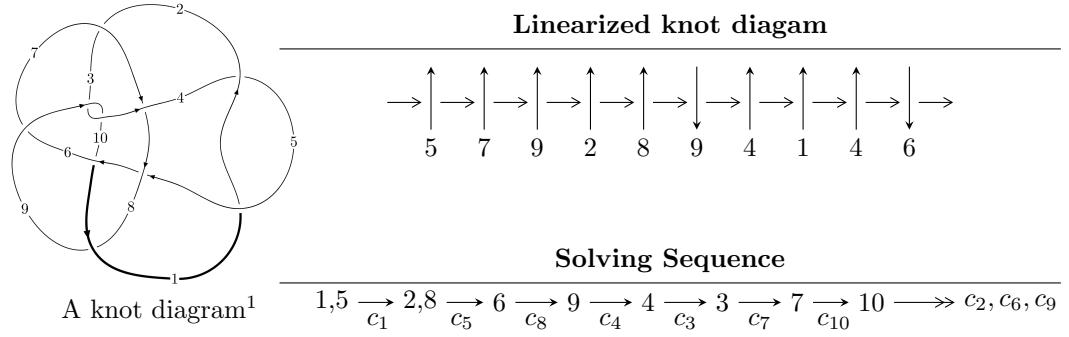


10<sub>165</sub> ( $K10n_{37}$ )



### Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$\begin{aligned}
 I_1^u &= \langle u^{12} + 5u^{11} + 16u^{10} + 34u^9 + 53u^8 + 61u^7 + 48u^6 + 20u^5 - 9u^4 - 24u^3 - 20u^2 + b - 11u - 3, \\
 &\quad - 3u^{12} - 13u^{11} - 41u^{10} - 85u^9 - 136u^8 - 167u^7 - 148u^6 - 90u^5 - 5u^4 + 54u^3 + 63u^2 + 2a + 50u + 14, \\
 &\quad u^{13} + 5u^{12} + 17u^{11} + 39u^{10} + 68u^9 + 91u^8 + 90u^7 + 62u^6 + 15u^5 - 24u^4 - 37u^3 - 30u^2 - 12u - 2 \rangle \\
 I_2^u &= \langle -u^3 - u^2 + b - u - 1, \ u^5 + 2u^4 + 4u^3 + 3u^2 + 2a + 3u + 2, \ u^6 + 2u^5 + 4u^4 + 5u^3 + 5u^2 + 4u + 2 \rangle \\
 I_3^u &= \langle -u^5 + u^4 - 2u^3 + au + u^2 + b - u + 1, \\
 &\quad u^5a - 2u^4a - 2u^5 + 4u^3a + 3u^4 - 4u^2a - 7u^3 + a^2 + 3au + 7u^2 - 2a - 6u + 4, \\
 &\quad u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1 \rangle
 \end{aligned}$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 31 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle u^{12} + 5u^{11} + \dots + b - 3, -3u^{12} - 13u^{11} + \dots + 2a + 14, u^{13} + 5u^{12} + \dots - 12u - 2 \rangle$$

(i) **Arc colorings**

$$\begin{aligned} a_1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_5 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_2 &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_8 &= \begin{pmatrix} \frac{3}{2}u^{12} + \frac{13}{2}u^{11} + \dots - 25u - 7 \\ -u^{12} - 5u^{11} + \dots + 11u + 3 \end{pmatrix} \\ a_6 &= \begin{pmatrix} \frac{3}{2}u^{12} + \frac{13}{2}u^{11} + \dots - 20u - 4 \\ -u^{12} - 5u^{11} + \dots + 15u + 3 \end{pmatrix} \\ a_9 &= \begin{pmatrix} \frac{1}{2}u^{12} + \frac{3}{2}u^{11} + \dots - 14u - 4 \\ -u^{12} - 5u^{11} + \dots + 11u + 3 \end{pmatrix} \\ a_4 &= \begin{pmatrix} -u \\ u^3 + u \end{pmatrix} \\ a_3 &= \begin{pmatrix} \frac{1}{2}u^{12} + \frac{5}{2}u^{11} + \dots - 4u - 1 \\ u^{12} + 4u^{11} + \dots - 5u - 1 \end{pmatrix} \\ a_7 &= \begin{pmatrix} \frac{1}{2}u^{12} + \frac{5}{2}u^{11} + \dots - 26u - 7 \\ -u^{12} - 4u^{11} + \dots + 2u + 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} \frac{1}{2}u^{12} + \frac{3}{2}u^{11} + \dots - 13u - 4 \\ -u^{12} - 5u^{11} + \dots + 10u + 3 \end{pmatrix} \end{aligned}$$

(ii) **Obstruction class** = -1

$$\text{(iii) Cusp Shapes} = 5u^{12} + 24u^{11} + 78u^{10} + 170u^9 + 277u^8 + 342u^7 + 296u^6 + 161u^5 - 15u^4 - 125u^3 - 126u^2 - 82u - 16$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_4$	$u^{13} + 5u^{12} + \cdots - 12u - 2$
$c_2, c_3, c_9$	$u^{13} + 10u^{11} + \cdots + 2u - 1$
$c_5, c_8$	$u^{13} + u^{12} + \cdots + 5u - 1$
$c_6$	$u^{13} - 8u^{12} + \cdots + 18u - 10$
$c_7$	$u^{13} - u^{12} + \cdots + 20u - 7$
$c_{10}$	$u^{13} + 12u^{12} + \cdots - 288u - 64$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$y^{13} + 9y^{12} + \cdots + 24y - 4$
$c_2, c_3, c_9$	$y^{13} + 20y^{12} + \cdots - 10y - 1$
$c_5, c_8$	$y^{13} + 7y^{12} + \cdots + 23y - 1$
$c_6$	$y^{13} - 16y^{12} + \cdots + 1284y - 100$
$c_7$	$y^{13} + 13y^{12} + \cdots + 64y - 49$
$c_{10}$	$y^{13} - 2y^{12} + \cdots + 1024y - 4096$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -1.152860 + 0.170520I$		
$a = 0.717142 - 0.770562I$	$-6.75019 - 5.87953I$	$3.71309 + 4.79533I$
$b = -0.695367 + 1.010640I$		
$u = -1.152860 - 0.170520I$		
$a = 0.717142 + 0.770562I$	$-6.75019 + 5.87953I$	$3.71309 - 4.79533I$
$b = -0.695367 - 1.010640I$		
$u = -0.034812 + 1.171400I$		
$a = 0.739139 + 0.284263I$	$-3.39029 + 0.96735I$	$2.31477 - 3.00161I$
$b = -0.358718 + 0.855935I$		
$u = -0.034812 - 1.171400I$		
$a = 0.739139 - 0.284263I$	$-3.39029 - 0.96735I$	$2.31477 + 3.00161I$
$b = -0.358718 - 0.855935I$		
$u = -0.175701 + 1.175030I$		
$a = -1.067580 - 0.632688I$	$-2.32319 - 3.89550I$	$2.16216 + 1.95849I$
$b = 0.93100 - 1.14328I$		
$u = -0.175701 - 1.175030I$		
$a = -1.067580 + 0.632688I$	$-2.32319 + 3.89550I$	$2.16216 - 1.95849I$
$b = 0.93100 + 1.14328I$		
$u = 0.773330$		
$a = 0.244870$	1.09959	6.33360
$b = 0.189365$		
$u = -0.48596 + 1.43258I$		
$a = 1.058960 + 0.295073I$	$-11.8216 - 11.6031I$	$1.77641 + 5.73851I$
$b = -0.93732 + 1.37365I$		
$u = -0.48596 - 1.43258I$		
$a = 1.058960 - 0.295073I$	$-11.8216 + 11.6031I$	$1.77641 - 5.73851I$
$b = -0.93732 - 1.37365I$		
$u = -0.363253 + 0.187651I$		
$a = -0.56911 - 2.04054I$	0.57483 + 1.68891I	$3.43240 - 5.42565I$
$b = 0.589641 + 0.634441I$		

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.363253 - 0.187651I$		
$a = -0.56911 + 2.04054I$	$0.57483 - 1.68891I$	$3.43240 + 5.42565I$
$b = 0.589641 - 0.634441I$		
$u = -0.67408 + 1.45370I$		
$a = -0.500985 + 0.317553I$	$-10.56050 - 0.87235I$	$-1.56565 + 0.23907I$
$b = -0.123919 - 0.942337I$		
$u = -0.67408 - 1.45370I$		
$a = -0.500985 - 0.317553I$	$-10.56050 + 0.87235I$	$-1.56565 - 0.23907I$
$b = -0.123919 + 0.942337I$		

$$\text{II. } I_2^u = \langle -u^3 - u^2 + b - u - 1, u^5 + 2u^4 + 4u^3 + 3u^2 + 2a + 3u + 2, u^6 + 2u^5 + 4u^4 + 5u^3 + 5u^2 + 4u + 2 \rangle$$

(i) **Arc colorings**

$$\begin{aligned} a_1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_5 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_2 &= \begin{pmatrix} 1 \\ -u^2 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -\frac{1}{2}u^5 - u^4 - 2u^3 - \frac{3}{2}u^2 - \frac{3}{2}u - 1 \\ u^3 + u^2 + u + 1 \end{pmatrix} \\ a_6 &= \begin{pmatrix} \frac{1}{2}u^5 + u^4 + u^3 + \frac{3}{2}u^2 + \frac{1}{2}u \\ -u^4 - u^3 - 2u^2 - u - 1 \end{pmatrix} \\ a_9 &= \begin{pmatrix} -\frac{1}{2}u^5 - u^4 - u^3 - \frac{1}{2}u^2 - \frac{1}{2}u \\ u^3 + u^2 + u + 1 \end{pmatrix} \\ a_4 &= \begin{pmatrix} -u \\ u^3 + u \end{pmatrix} \\ a_3 &= \begin{pmatrix} \frac{1}{2}u^5 + 2u^4 + 3u^3 + \frac{7}{2}u^2 + \frac{3}{2}u + 1 \\ u^5 + 2u^4 + 4u^3 + 3u^2 + 3u + 1 \end{pmatrix} \\ a_7 &= \begin{pmatrix} -\frac{1}{2}u^5 + \frac{3}{2}u^2 + \frac{3}{2}u + 1 \\ u^3 + u^2 + 2u + 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} -\frac{1}{2}u^5 - u^4 - 2u^3 - \frac{3}{2}u^2 - \frac{3}{2}u \\ u^5 + u^4 + 3u^3 + 2u^2 + 2u + 1 \end{pmatrix} \end{aligned}$$

(ii) **Obstruction class = 1**

(iii) **Cusp Shapes** =  $5u^4 + 4u^3 + 9u^2 + 6u + 10$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1$	$u^6 + 2u^5 + 4u^4 + 5u^3 + 5u^2 + 4u + 2$
$c_2, c_3, c_9$	$u^6 + 2u^4 - 2u^2 + 1$
$c_4$	$u^6 - 2u^5 + 4u^4 - 5u^3 + 5u^2 - 4u + 2$
$c_5, c_8, c_{10}$	$u^6 + u^5 - 2u^3 + u + 1$
$c_6$	$u^6 - 5u^5 + 10u^4 - 12u^3 + 11u^2 - 6u + 2$
$c_7$	$u^6 + u^5 + 3u^4 - u^3 + u^2 - 2u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$y^6 + 4y^5 + 6y^4 + 3y^3 + y^2 + 4y + 4$
$c_2, c_3, c_9$	$(y^3 + 2y^2 - 2y + 1)^2$
$c_5, c_8, c_{10}$	$y^6 - y^5 + 4y^4 - 4y^3 + 4y^2 - y + 1$
$c_6$	$y^6 - 5y^5 + 2y^4 + 20y^3 + 17y^2 + 8y + 4$
$c_7$	$y^6 + 5y^5 + 13y^4 + 11y^3 + 3y^2 - 2y + 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.862023 + 0.412869I$		
$a = -0.233003 - 0.750879I$	$1.44750 + 0.78507I$	$8.28869 - 4.60495I$
$b = 0.510869 + 0.551075I$		
$u = -0.862023 - 0.412869I$		
$a = -0.233003 + 0.750879I$	$1.44750 - 0.78507I$	$8.28869 + 4.60495I$
$b = 0.510869 - 0.551075I$		
$u = 0.238984 + 1.138460I$		
$a = 0.176605 + 0.841305I$	$-8.28528 + 1.18132I$	$2.81561 - 0.13577I$
$b = -0.915589 + 0.402116I$		
$u = 0.238984 - 1.138460I$		
$a = 0.176605 - 0.841305I$	$-8.28528 - 1.18132I$	$2.81561 + 0.13577I$
$b = -0.915589 - 0.402116I$		
$u = -0.376961 + 1.214800I$		
$a = -0.943602 - 0.451942I$	$-1.38689 - 5.20040I$	$6.89570 + 6.16090I$
$b = 0.904720 - 0.975923I$		
$u = -0.376961 - 1.214800I$		
$a = -0.943602 + 0.451942I$	$-1.38689 + 5.20040I$	$6.89570 - 6.16090I$
$b = 0.904720 + 0.975923I$		

$$\text{III. } I_3^u = \langle -u^5 + u^4 - 2u^3 + au + u^2 + b - u + 1, u^5a - 2u^5 + \dots - 2a + 4, u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1 \rangle$$

(i) **Arc colorings**

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} a \\ u^5 - u^4 + 2u^3 - au - u^2 + u - 1 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} u^5a - u^4a - u^5 + 2u^3a + u^4 - u^2a - 3u^3 + au + 2u^2 - a - 2u + 2 \\ 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^5 - u^4 + 2u^3 - au - u^2 + a + u - 1 \\ u^5 - u^4 + 2u^3 - au - u^2 + u - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^5 - 3u^4 + 6u^3 - au - 7u^2 + a + 5u - 3 \\ u^3a - 2u^4 - u^2a + 3u^3 + au - 3u^2 + 2u + 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} -u^4a + u^5 + u^3a - u^4 - u^2a + u^3 + a - u \\ -u^4a + 2u^5 + u^3a - 2u^4 - u^2a + 4u^3 - 2u^2 + a + u - 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^5a + u^4a + u^5 - 2u^3a - u^4 + u^2a + 3u^3 - au - 2u^2 + a + 2u - 1 \\ -1 \end{pmatrix}$$

(ii) **Obstruction class** = -1

(iii) **Cusp Shapes** =  $4u^4 - 4u^3 + 8u^2 - 4u + 2$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_4$	$(u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1)^2$
$c_2, c_3, c_9$	$u^{12} + u^{11} + \cdots + 2u + 13$
$c_5, c_8$	$u^{12} + 5u^{11} + \cdots + 6u^2 + 1$
$c_6$	$(u^6 + 5u^5 + 7u^4 - 2u^2 + 3u - 1)^2$
$c_7$	$u^{12} - u^{11} + \cdots - 18u + 23$
$c_{10}$	$(u - 1)^{12}$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$(y^6 + 5y^5 + 9y^4 + 4y^3 - 6y^2 - 5y + 1)^2$
$c_2, c_3, c_9$	$y^{12} + 15y^{11} + \cdots + 360y + 169$
$c_5, c_8$	$y^{12} - y^{11} + \cdots + 12y + 1$
$c_6$	$(y^6 - 11y^5 + 45y^4 - 60y^3 - 10y^2 - 5y + 1)^2$
$c_7$	$y^{12} + 11y^{11} + \cdots - 416y + 529$
$c_{10}$	$(y - 1)^{12}$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.873214$		
$a = 0.211090 + 0.348879I$	1.08035	4.26950
$b = 0.184327 - 0.304646I$		
$u = 0.873214$		
$a = 0.211090 - 0.348879I$	1.08035	4.26950
$b = 0.184327 + 0.304646I$		
$u = -0.138835 + 1.234450I$		
$a = 0.576096 + 0.033593I$	$-9.53998 - 1.97241I$	$-3.42428 + 3.68478I$
$b = -1.96628 - 1.27394I$		
$u = -0.138835 + 1.234450I$		
$a = -0.84220 + 1.68756I$	$-9.53998 - 1.97241I$	$-3.42428 + 3.68478I$
$b = -0.121451 + 0.706495I$		
$u = -0.138835 - 1.234450I$		
$a = 0.576096 - 0.033593I$	$-9.53998 + 1.97241I$	$-3.42428 - 3.68478I$
$b = -1.96628 + 1.27394I$		
$u = -0.138835 - 1.234450I$		
$a = -0.84220 - 1.68756I$	$-9.53998 + 1.97241I$	$-3.42428 - 3.68478I$
$b = -0.121451 - 0.706495I$		
$u = 0.408802 + 1.276380I$		
$a = 1.089440 - 0.275882I$	$-2.88416 + 4.59213I$	$0.58114 - 3.20482I$
$b = -0.511061 - 0.781659I$		
$u = 0.408802 + 1.276380I$		
$a = -0.671738 + 0.185253I$	$-2.88416 + 4.59213I$	$0.58114 - 3.20482I$
$b = 0.79749 + 1.27775I$		
$u = 0.408802 - 1.276380I$		
$a = 1.089440 + 0.275882I$	$-2.88416 - 4.59213I$	$0.58114 + 3.20482I$
$b = -0.511061 + 0.781659I$		
$u = 0.408802 - 1.276380I$		
$a = -0.671738 - 0.185253I$	$-2.88416 - 4.59213I$	$0.58114 + 3.20482I$
$b = 0.79749 - 1.27775I$		

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.413150$		
$a = 2.13731 + 1.92634I$	-5.84089	5.41680
$b = -0.883031 + 0.795869I$		
$u = -0.413150$		
$a = 2.13731 - 1.92634I$	-5.84089	5.41680
$b = -0.883031 - 0.795869I$		

#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$(u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1)^2$ $\cdot (u^6 + 2u^5 + \dots + 4u + 2)(u^{13} + 5u^{12} + \dots - 12u - 2)$
$c_2, c_3, c_9$	$(u^6 + 2u^4 - 2u^2 + 1)(u^{12} + u^{11} + \dots + 2u + 13)$ $\cdot (u^{13} + 10u^{11} + \dots + 2u - 1)$
$c_4$	$(u^6 - 2u^5 + 4u^4 - 5u^3 + 5u^2 - 4u + 2)$ $\cdot ((u^6 - u^5 + 3u^4 - 2u^3 + 2u^2 - u - 1)^2)(u^{13} + 5u^{12} + \dots - 12u - 2)$
$c_5, c_8$	$(u^6 + u^5 - 2u^3 + u + 1)(u^{12} + 5u^{11} + \dots + 6u^2 + 1)$ $\cdot (u^{13} + u^{12} + \dots + 5u - 1)$
$c_6$	$(u^6 - 5u^5 + 10u^4 - 12u^3 + 11u^2 - 6u + 2)$ $\cdot ((u^6 + 5u^5 + 7u^4 - 2u^2 + 3u - 1)^2)(u^{13} - 8u^{12} + \dots + 18u - 10)$
$c_7$	$(u^6 + u^5 + 3u^4 - u^3 + u^2 - 2u + 1)(u^{12} - u^{11} + \dots - 18u + 23)$ $\cdot (u^{13} - u^{12} + \dots + 20u - 7)$
$c_{10}$	$((u - 1)^{12})(u^6 + u^5 - 2u^3 + u + 1)(u^{13} + 12u^{12} + \dots - 288u - 64)$

## V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_4$	$(y^6 + 4y^5 + 6y^4 + 3y^3 + y^2 + 4y + 4) \cdot ((y^6 + 5y^5 + \dots - 5y + 1)^2)(y^{13} + 9y^{12} + \dots + 24y - 4)$
$c_2, c_3, c_9$	$((y^3 + 2y^2 - 2y + 1)^2)(y^{12} + 15y^{11} + \dots + 360y + 169) \cdot (y^{13} + 20y^{12} + \dots - 10y - 1)$
$c_5, c_8$	$(y^6 - y^5 + 4y^4 - 4y^3 + 4y^2 - y + 1)(y^{12} - y^{11} + \dots + 12y + 1) \cdot (y^{13} + 7y^{12} + \dots + 23y - 1)$
$c_6$	$(y^6 - 11y^5 + 45y^4 - 60y^3 - 10y^2 - 5y + 1)^2 \cdot (y^6 - 5y^5 + 2y^4 + 20y^3 + 17y^2 + 8y + 4) \cdot (y^{13} - 16y^{12} + \dots + 1284y - 100)$
$c_7$	$(y^6 + 5y^5 + 13y^4 + 11y^3 + 3y^2 - 2y + 1) \cdot (y^{12} + 11y^{11} + \dots - 416y + 529)(y^{13} + 13y^{12} + \dots + 64y - 49)$
$c_{10}$	$(y - 1)^{12}(y^6 - y^5 + 4y^4 - 4y^3 + 4y^2 - y + 1) \cdot (y^{13} - 2y^{12} + \dots + 1024y - 4096)$