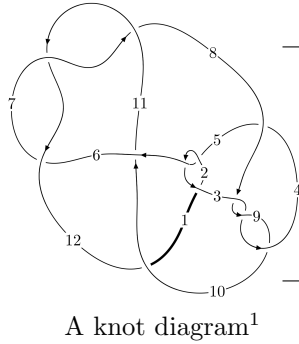
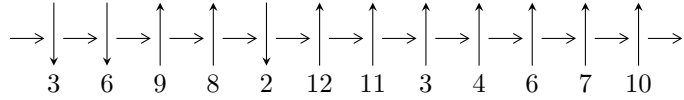


12n<sub>0469</sub> (K12n<sub>0469</sub>)



**Linearized knot diagram**



**Solving Sequence**

$$6, 12 \xrightarrow{c_6} 3, 7 \xrightarrow{c_2} 2 \xrightarrow{c_1} 1 \xrightarrow{c_5} 5 \xrightarrow{c_{11}} 11 \xrightarrow{c_7} 8 \xrightarrow{c_8} 9 \xrightarrow{c_4} 4 \xrightarrow{c_{10}} 10 \Rightarrow c_3, c_9, c_{12}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle 271358647u^{35} - 529043524u^{34} + \dots + 488920819b - 215606766, \\ 334439774u^{35} - 1269481739u^{34} + \dots + 977841638a - 5972341699, u^{36} - 2u^{35} + \dots - 4u - 1 \rangle$$

$$I_2^u = \langle b + 1, -u^2 + a - u - 2, u^3 + u^2 + 2u + 1 \rangle$$

$$I_3^u = \langle b - 1, 2u^2a + a^2 - 2au + 4a + u - 1, u^3 - u^2 + 2u - 1 \rangle$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 45 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\mathbf{I. } I_1^u = \langle 2.71 \times 10^8 u^{35} - 5.29 \times 10^8 u^{34} + \dots + 4.89 \times 10^8 b - 2.16 \times 10^8, 3.34 \times 10^8 u^{35} - 1.27 \times 10^9 u^{34} + \dots + 9.78 \times 10^8 a - 5.97 \times 10^9, u^{36} - 2u^{35} + \dots - 4u - 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -0.342018u^{35} + 1.29825u^{34} + \dots - 14.1375u + 6.10768 \\ -0.555016u^{35} + 1.08206u^{34} + \dots - 2.96956u + 0.440985 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -0.897034u^{35} + 2.38031u^{34} + \dots - 17.1071u + 6.54866 \\ -0.555016u^{35} + 1.08206u^{34} + \dots - 2.96956u + 0.440985 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} u^7 + 4u^5 + 4u^3 \\ -u^7 - 3u^5 - 2u^3 + u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} 1.04147u^{35} - 2.64685u^{34} + \dots + 19.6688u - 5.62579 \\ 0.678551u^{35} - 1.22102u^{34} + \dots + 3.80116u - 0.427325 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ -u^4 - 2u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} 1.07868u^{35} - 2.14851u^{34} + \dots + 24.9712u - 6.81062 \\ -0.00885463u^{35} - 0.0464490u^{34} + \dots + 2.49588u - 1.07868 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.440985u^{35} - 1.43699u^{34} + \dots + 15.6653u - 4.73350 \\ 0.614212u^{35} - 1.25941u^{34} + \dots + 4.73960u - 0.342018 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^3 - 2u \\ u^3 + u \end{pmatrix}$$

(ii) Obstruction class = -1

$$\mathbf{(iii) Cusp Shapes} = \frac{259508424}{488920819}u^{35} - \frac{190324579}{488920819}u^{34} + \dots + \frac{7239334870}{488920819}u + \frac{1820082994}{488920819}$$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{36} + 46u^{35} + \dots + 1827u + 49$
$c_2, c_5$	$u^{36} + 4u^{35} + \dots + 91u - 7$
$c_3, c_8, c_9$	$u^{36} + u^{35} + \dots - 8u - 8$
$c_4$	$u^{36} - 3u^{35} + \dots + 8u - 8$
$c_6, c_7, c_{11}$	$u^{36} - 2u^{35} + \dots - 4u - 1$
$c_{10}$	$u^{36} + 2u^{35} + \dots - 2232u - 481$
$c_{12}$	$u^{36} + 4u^{35} + \dots - 16u + 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{36} - 102y^{35} + \dots - 1510327y + 2401$
$c_2, c_5$	$y^{36} - 46y^{35} + \dots - 1827y + 49$
$c_3, c_8, c_9$	$y^{36} - 29y^{35} + \dots - 832y + 64$
$c_4$	$y^{36} + 55y^{35} + \dots - 1728y + 64$
$c_6, c_7, c_{11}$	$y^{36} + 36y^{35} + \dots - 52y + 1$
$c_{10}$	$y^{36} + 20y^{35} + \dots - 9532084y + 231361$
$c_{12}$	$y^{36} + 44y^{35} + \dots - 532y + 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.746590 + 0.534072I$ $a = -1.14025 - 1.14185I$ $b = 1.71941 + 0.06440I$	$-10.55280 - 2.49065I$	$2.28481 + 2.78156I$
$u = -0.746590 - 0.534072I$ $a = -1.14025 + 1.14185I$ $b = 1.71941 - 0.06440I$	$-10.55280 + 2.49065I$	$2.28481 - 2.78156I$
$u = 0.664083 + 0.631741I$ $a = 1.014470 - 0.540455I$ $b = -1.67188 - 0.14945I$	$-6.71821 - 3.14670I$	$4.48459 + 0.40539I$
$u = 0.664083 - 0.631741I$ $a = 1.014470 + 0.540455I$ $b = -1.67188 + 0.14945I$	$-6.71821 + 3.14670I$	$4.48459 - 0.40539I$
$u = 0.784223 + 0.433590I$ $a = 1.14935 - 1.67783I$ $b = -1.65415 + 0.25580I$	$-6.05301 + 8.02106I$	$5.69652 - 5.48227I$
$u = 0.784223 - 0.433590I$ $a = 1.14935 + 1.67783I$ $b = -1.65415 - 0.25580I$	$-6.05301 - 8.02106I$	$5.69652 + 5.48227I$
$u = -0.754684$ $a = 2.04702$ $b = -1.31973$	0.746951	8.79890
$u = -0.202995 + 1.255340I$ $a = -0.951854 - 0.726217I$ $b = 0.175613 + 0.296050I$	$2.08619 - 3.03413I$	$10.51311 + 3.79199I$
$u = -0.202995 - 1.255340I$ $a = -0.951854 + 0.726217I$ $b = 0.175613 - 0.296050I$	$2.08619 + 3.03413I$	$10.51311 - 3.79199I$
$u = 0.597158 + 0.410952I$ $a = -0.03383 + 1.92390I$ $b = 0.617286 - 0.749282I$	$1.66187 + 4.15562I$	$8.00287 - 6.91393I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.597158 - 0.410952I$ $a = -0.03383 - 1.92390I$ $b = 0.617286 + 0.749282I$	$1.66187 - 4.15562I$	$8.00287 + 6.91393I$
$u = -0.326127 + 1.248070I$ $a = 0.633554 + 0.938984I$ $b = -1.359350 + 0.009058I$	$-3.12452 - 3.89594I$	$3.97835 + 4.14640I$
$u = -0.326127 - 1.248070I$ $a = 0.633554 - 0.938984I$ $b = -1.359350 - 0.009058I$	$-3.12452 + 3.89594I$	$3.97835 - 4.14640I$
$u = 0.504679 + 0.396227I$ $a = -1.116890 - 0.669513I$ $b = 0.659666 + 0.583318I$	$1.51779 - 0.52826I$	$7.65473 - 0.24051I$
$u = 0.504679 - 0.396227I$ $a = -1.116890 + 0.669513I$ $b = 0.659666 - 0.583318I$	$1.51779 + 0.52826I$	$7.65473 + 0.24051I$
$u = -0.631586$ $a = -1.76477$ $b = 0.225322$	$5.91452$	$16.7470$
$u = 0.096853 + 1.373970I$ $a = 0.057014 - 0.382407I$ $b = 0.233991 + 0.646152I$	$-3.76081 + 1.82148I$	$4.18456 - 2.97690I$
$u = 0.096853 - 1.373970I$ $a = 0.057014 + 0.382407I$ $b = 0.233991 - 0.646152I$	$-3.76081 - 1.82148I$	$4.18456 + 2.97690I$
$u = -0.032206 + 1.383860I$ $a = 0.93309 + 1.09446I$ $b = 1.227970 - 0.334502I$	$-1.31636 - 0.59124I$	$2.44003 + 0.I$
$u = -0.032206 - 1.383860I$ $a = 0.93309 - 1.09446I$ $b = 1.227970 + 0.334502I$	$-1.31636 + 0.59124I$	$2.44003 + 0.I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.209916 + 1.383740I$ $a = -0.161570 + 0.134658I$ $b = 0.763567 + 0.425752I$	$-4.01585 + 2.05418I$	$4.74740 + 1.07135I$
$u = 0.209916 - 1.383740I$ $a = -0.161570 - 0.134658I$ $b = 0.763567 - 0.425752I$	$-4.01585 - 2.05418I$	$4.74740 - 1.07135I$
$u = -0.10642 + 1.44688I$ $a = -0.474212 + 0.967148I$ $b = -1.044990 - 0.635725I$	$-7.20243 - 2.62870I$	$0. + 1.56071I$
$u = -0.10642 - 1.44688I$ $a = -0.474212 - 0.967148I$ $b = -1.044990 + 0.635725I$	$-7.20243 + 2.62870I$	$0. - 1.56071I$
$u = 0.21082 + 1.46540I$ $a = 0.586263 + 1.079000I$ $b = 0.693691 - 0.900679I$	$-4.41494 + 7.10948I$	$6.00000 - 5.97206I$
$u = 0.21082 - 1.46540I$ $a = 0.586263 - 1.079000I$ $b = 0.693691 + 0.900679I$	$-4.41494 - 7.10948I$	$6.00000 + 5.97206I$
$u = 0.29360 + 1.49537I$ $a = -0.31496 - 1.59906I$ $b = -1.68265 + 0.33795I$	$-12.2864 + 11.9589I$	$0$
$u = 0.29360 - 1.49537I$ $a = -0.31496 + 1.59906I$ $b = -1.68265 - 0.33795I$	$-12.2864 - 11.9589I$	$0$
$u = -0.334693 + 0.338551I$ $a = 0.58755 + 1.75938I$ $b = -0.790374 - 0.317839I$	$-1.37785 - 1.03574I$	$-0.30331 + 3.73142I$
$u = -0.334693 - 0.338551I$ $a = 0.58755 - 1.75938I$ $b = -0.790374 + 0.317839I$	$-1.37785 + 1.03574I$	$-0.30331 - 3.73142I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.25500 + 1.53176I$ $a = 0.269560 - 1.152100I$ $b = 1.79484 + 0.16936I$	$-17.3048 - 6.1571I$	0
$u = -0.25500 - 1.53176I$ $a = 0.269560 + 1.152100I$ $b = 1.79484 - 0.16936I$	$-17.3048 + 6.1571I$	0
$u = 0.19273 + 1.54392I$ $a = -0.368636 - 0.625846I$ $b = -1.78522 - 0.06510I$	$-13.90280 - 0.08031I$	0
$u = 0.19273 - 1.54392I$ $a = -0.368636 + 0.625846I$ $b = -1.78522 + 0.06510I$	$-13.90280 + 0.08031I$	0
$u = 0.431723$ $a = 0.157248$ $b = 0.236175$	0.680363	15.1480
$u = -0.145505$ $a = 8.22319$ $b = 1.06341$	3.33955	1.75020



$$\text{II. } I_2^u = \langle b + 1, -u^2 + a - u - 2, u^3 + u^2 + 2u + 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^2 + u + 2 \\ -1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} u^2 + u + 1 \\ -1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^2 + u + 2 \\ -1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^2 + u + 2 \\ -1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^2 + 1 \\ -u^2 - u - 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $2u^2 + 4u + 4$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_2$	$(u - 1)^3$
$c_3, c_4, c_8$ $c_9$	$u^3$
$c_5$	$(u + 1)^3$
$c_6, c_7$	$u^3 + u^2 + 2u + 1$
$c_{10}, c_{12}$	$u^3 + u^2 - 1$
$c_{11}$	$u^3 - u^2 + 2u - 1$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_5$	$(y - 1)^3$
$c_3, c_4, c_8$ $c_9$	$y^3$
$c_6, c_7, c_{11}$	$y^3 + 3y^2 + 2y - 1$
$c_{10}, c_{12}$	$y^3 - y^2 + 2y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.215080 + 1.307140I$ $a = 0.122561 + 0.744862I$ $b = -1.00000$	$-4.66906 - 2.82812I$	$-0.18504 + 4.10401I$
$u = -0.215080 - 1.307140I$ $a = 0.122561 - 0.744862I$ $b = -1.00000$	$-4.66906 + 2.82812I$	$-0.18504 - 4.10401I$
$u = -0.569840$ $a = 1.75488$ $b = -1.00000$	$-0.531480$	$2.37010$

$$\text{III. } I_3^u = \langle b - 1, 2u^2a + a^2 - 2au + 4a + u - 1, u^3 - u^2 + 2u - 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} a \\ 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} a + 1 \\ 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -a \\ -1 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u \\ u^2 - u + 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ -u^2 + u - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2a + a + 1 \\ -u^2a + au - 2u^2 - a + u - 2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^2 - u + 2 \\ -au - 2 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u^2 - 1 \\ u^2 - u + 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $4u^2 - 4u + 12$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_5$	$(u - 1)^6$
$c_2$	$(u + 1)^6$
$c_3, c_4, c_8$ $c_9$	$(u^2 - 2)^3$
$c_6, c_7$	$(u^3 - u^2 + 2u - 1)^2$
$c_{10}$	$(u^3 - u^2 + 1)^2$
$c_{11}$	$(u^3 + u^2 + 2u + 1)^2$
$c_{12}$	$(u^3 + u^2 - 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_2, c_5$	$(y - 1)^6$
$c_3, c_4, c_8$ $c_9$	$(y - 2)^6$
$c_6, c_7, c_{11}$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_{10}, c_{12}$	$(y^3 - y^2 + 2y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.215080 + 1.307140I$ $a = 0.814156 - 0.050322I$ $b = 1.00000$	$0.26574 + 2.82812I$	$4.49024 - 2.97945I$
$u = 0.215080 + 1.307140I$ $a = -1.05928 + 1.54005I$ $b = 1.00000$	$0.26574 + 2.82812I$	$4.49024 - 2.97945I$
$u = 0.215080 - 1.307140I$ $a = 0.814156 + 0.050322I$ $b = 1.00000$	$0.26574 - 2.82812I$	$4.49024 + 2.97945I$
$u = 0.215080 - 1.307140I$ $a = -1.05928 - 1.54005I$ $b = 1.00000$	$0.26574 - 2.82812I$	$4.49024 + 2.97945I$
$u = 0.569840$ $a = 0.118556$ $b = 1.00000$	4.40332	11.0200
$u = 0.569840$ $a = -3.62831$ $b = 1.00000$	4.40332	11.0200



#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u - 1)^9)(u^{36} + 46u^{35} + \dots + 1827u + 49)$
$c_2$	$((u - 1)^3)(u + 1)^6(u^{36} + 4u^{35} + \dots + 91u - 7)$
$c_3, c_8, c_9$	$u^3(u^2 - 2)^3(u^{36} + u^{35} + \dots - 8u - 8)$
$c_4$	$u^3(u^2 - 2)^3(u^{36} - 3u^{35} + \dots + 8u - 8)$
$c_5$	$((u - 1)^6)(u + 1)^3(u^{36} + 4u^{35} + \dots + 91u - 7)$
$c_6, c_7$	$((u^3 - u^2 + 2u - 1)^2)(u^3 + u^2 + 2u + 1)(u^{36} - 2u^{35} + \dots - 4u - 1)$
$c_{10}$	$((u^3 - u^2 + 1)^2)(u^3 + u^2 - 1)(u^{36} + 2u^{35} + \dots - 2232u - 481)$
$c_{11}$	$(u^3 - u^2 + 2u - 1)(u^3 + u^2 + 2u + 1)^2(u^{36} - 2u^{35} + \dots - 4u - 1)$
$c_{12}$	$((u^3 + u^2 - 1)^3)(u^{36} + 4u^{35} + \dots - 16u + 1)$

### V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y - 1)^9)(y^{36} - 102y^{35} + \dots - 1510327y + 2401)$
$c_2, c_5$	$((y - 1)^9)(y^{36} - 46y^{35} + \dots - 1827y + 49)$
$c_3, c_8, c_9$	$y^3(y - 2)^6(y^{36} - 29y^{35} + \dots - 832y + 64)$
$c_4$	$y^3(y - 2)^6(y^{36} + 55y^{35} + \dots - 1728y + 64)$
$c_6, c_7, c_{11}$	$((y^3 + 3y^2 + 2y - 1)^3)(y^{36} + 36y^{35} + \dots - 52y + 1)$
$c_{10}$	$((y^3 - y^2 + 2y - 1)^3)(y^{36} + 20y^{35} + \dots - 9532084y + 231361)$
$c_{12}$	$((y^3 - y^2 + 2y - 1)^3)(y^{36} + 44y^{35} + \dots - 532y + 1)$