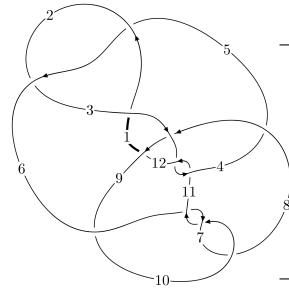
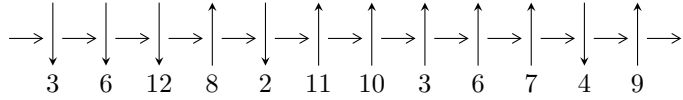


12n<sub>0501</sub> (K12n<sub>0501</sub>)



A knot diagram<sup>1</sup>

**Linearized knot diagram**



**Solving Sequence**

$$6,11 \xrightarrow{c_6} 3,7 \xrightarrow{c_2} 2 \xrightarrow{c_1} 1 \xrightarrow{c_5} 5 \xrightarrow{c_{10}} 10 \xrightarrow{c_7} 8 \xrightarrow{c_4} 4 \xrightarrow{c_9} 9 \xrightarrow{c_{12}} 12 \Rightarrow c_3, c_8, c_{11}$$

**Ideals for irreducible components<sup>2</sup> of  $X_{\text{par}}$**

$$I_1^u = \langle -4u^{32} + 16u^{31} + \dots + 4b + 5, 5u^{32} - 16u^{31} + \dots + 4a + 6, u^{33} - 4u^{32} + \dots - 6u + 1 \rangle$$

$$I_2^u = \langle -au + b, 2u^2a + a^2 + au + 2u^2 + 3a + u + 4, u^3 + u^2 + 2u + 1 \rangle$$

$$I_3^u = \langle u^2 + b + u + 1, -u^2 + a - 1, u^3 + u^2 + 2u + 1 \rangle$$

\* 3 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 42 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/maths/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\langle -4u^{32} + 16u^{31} + \dots + 4b + 5, 5u^{32} - 16u^{31} + \dots + 4a + 6, u^{33} - 4u^{32} + \dots - 6u + 1 \rangle$$

I.  $I_1^u =$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} -\frac{5}{4}u^{32} + 4u^{31} + \dots + \frac{7}{4}u - \frac{3}{2} \\ u^{32} - 4u^{31} + \dots + 9u - \frac{5}{4} \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -\frac{1}{4}u^{32} - \frac{5}{2}u^{30} + \dots + \frac{43}{4}u - \frac{11}{4} \\ u^{32} - 4u^{31} + \dots + 9u - \frac{5}{4} \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -\frac{11}{4}u^{32} + \frac{41}{4}u^{31} + \dots - 23u + \frac{11}{2} \\ \frac{3}{4}u^{32} - 3u^{31} + \dots + \frac{63}{4}u - \frac{15}{4} \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -\frac{1}{4}u^{32} + \frac{3}{4}u^{31} + \dots - 4u + \frac{1}{4} \\ \frac{1}{4}u^{32} - u^{31} + \dots + \frac{13}{4}u - \frac{1}{2} \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ u^3 + u \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ -u^4 - 2u^2 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -\frac{1}{2}u^{32} + \frac{7}{4}u^{31} + \dots - \frac{29}{4}u + \frac{3}{4} \\ \frac{1}{4}u^{32} - u^{31} + \dots + \frac{17}{4}u - \frac{3}{4} \end{pmatrix}$$

$$a_9 = \begin{pmatrix} -u^3 - 2u \\ u^3 + u \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -\frac{9}{4}u^{32} + \frac{33}{4}u^{31} + \dots - \frac{37}{2}u + \frac{19}{4} \\ \frac{3}{4}u^{32} - \frac{11}{4}u^{31} + \dots + 13u - \frac{13}{4} \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $-\frac{9}{2}u^{32} + \frac{35}{2}u^{31} + \dots - \frac{111}{2}u + \frac{23}{4}$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1$	$u^{33} + 46u^{32} + \dots + 42u + 1$
$c_2, c_5$	$u^{33} + 4u^{32} + \dots + 4u - 1$
$c_3, c_{11}$	$u^{33} - 4u^{32} + \dots + 10u - 1$
$c_4$	$u^{33} - 3u^{32} + \dots - 31624u - 99623$
$c_6, c_7, c_{10}$	$u^{33} + 4u^{32} + \dots - 6u - 1$
$c_8$	$u^{33} + u^{32} + \dots + 128u^2 - 512$
$c_9$	$u^{33} - 4u^{32} + \dots - 744u - 137$
$c_{12}$	$u^{33} + 26u^{31} + \dots + 1410u - 2071$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1$	$y^{33} - 114y^{32} + \dots + 1162y - 1$
$c_2, c_5$	$y^{33} - 46y^{32} + \dots + 42y - 1$
$c_3, c_{11}$	$y^{33} + 26y^{32} + \dots + 42y - 1$
$c_4$	$y^{33} + 39y^{32} + \dots - 19254673246y - 9924742129$
$c_6, c_7, c_{10}$	$y^{33} + 34y^{32} + \dots - 6y - 1$
$c_8$	$y^{33} + 49y^{32} + \dots + 131072y - 262144$
$c_9$	$y^{33} + 26y^{32} + \dots - 461086y - 18769$
$c_{12}$	$y^{33} + 52y^{32} + \dots - 8631988y - 4289041$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.783955 + 0.550569I$ $a = 1.50759 - 1.19961I$ $b = -1.84235 + 0.11041I$	$-10.85950 + 2.60786I$	$-1.64483 - 2.62552I$
$u = 0.783955 - 0.550569I$ $a = 1.50759 + 1.19961I$ $b = -1.84235 - 0.11041I$	$-10.85950 - 2.60786I$	$-1.64483 + 2.62552I$
$u = 0.745301 + 0.601492I$ $a = -1.40874 + 1.23496I$ $b = 1.79275 - 0.07307I$	$-6.97586 - 2.52993I$	$1.181503 + 0.379753I$
$u = 0.745301 - 0.601492I$ $a = -1.40874 - 1.23496I$ $b = 1.79275 + 0.07307I$	$-6.97586 + 2.52993I$	$1.181503 - 0.379753I$
$u = 0.799754 + 0.494935I$ $a = -1.61442 + 1.19019I$ $b = 1.88021 - 0.15283I$	$-6.63229 + 7.68298I$	$1.86058 - 5.38110I$
$u = 0.799754 - 0.494935I$ $a = -1.61442 - 1.19019I$ $b = 1.88021 + 0.15283I$	$-6.63229 - 7.68298I$	$1.86058 + 5.38110I$
$u = -0.071136 + 1.190150I$ $a = 0.658204 - 0.211530I$ $b = -0.204931 - 0.798410I$	$1.03666 - 2.07532I$	$3.43845 + 3.26136I$
$u = -0.071136 - 1.190150I$ $a = 0.658204 + 0.211530I$ $b = -0.204931 + 0.798410I$	$1.03666 + 2.07532I$	$3.43845 - 3.26136I$
$u = -0.688247 + 0.161266I$ $a = 0.110580 + 0.396771I$ $b = 0.140092 + 0.255244I$	$3.61024 - 0.82218I$	$5.86040 + 0.81952I$
$u = -0.688247 - 0.161266I$ $a = 0.110580 - 0.396771I$ $b = 0.140092 - 0.255244I$	$3.61024 + 0.82218I$	$5.86040 - 0.81952I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.182052 + 1.337320I$ $a = -0.217943 - 0.164939I$ $b = -0.260253 + 0.261432I$	$-3.43336 - 2.41737I$	0
$u = -0.182052 - 1.337320I$ $a = -0.217943 + 0.164939I$ $b = -0.260253 - 0.261432I$	$-3.43336 + 2.41737I$	0
$u = -0.294839 + 1.343030I$ $a = -0.011030 + 0.323726I$ $b = 0.431523 + 0.110260I$	$-1.11961 - 4.40985I$	0
$u = -0.294839 - 1.343030I$ $a = -0.011030 - 0.323726I$ $b = 0.431523 - 0.110260I$	$-1.11961 + 4.40985I$	0
$u = -0.212327 + 0.555507I$ $a = -0.012738 - 0.990392I$ $b = -0.552874 - 0.203211I$	$1.64764 - 2.26500I$	$1.52805 + 4.62369I$
$u = -0.212327 - 0.555507I$ $a = -0.012738 + 0.990392I$ $b = -0.552874 + 0.203211I$	$1.64764 + 2.26500I$	$1.52805 - 4.62369I$
$u = 0.09222 + 1.42628I$ $a = -0.711100 - 0.954177I$ $b = -1.29535 + 1.10223I$	$-1.79243 + 4.81413I$	0
$u = 0.09222 - 1.42628I$ $a = -0.711100 + 0.954177I$ $b = -1.29535 - 1.10223I$	$-1.79243 - 4.81413I$	0
$u = 0.02669 + 1.47035I$ $a = 0.484236 + 0.756490I$ $b = 1.099380 - 0.732184I$	$-7.20920 + 1.09488I$	0
$u = 0.02669 - 1.47035I$ $a = 0.484236 - 0.756490I$ $b = 1.099380 + 0.732184I$	$-7.20920 - 1.09488I$	0

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.06717 + 1.48459I$ $a = -0.340624 - 0.578506I$ $b = -0.881727 + 0.466828I$	$-4.89325 - 3.34061I$	0
$u = -0.06717 - 1.48459I$ $a = -0.340624 + 0.578506I$ $b = -0.881727 - 0.466828I$	$-4.89325 + 3.34061I$	0
$u = -0.497661$ $a = -0.331095$ $b = -0.164773$	0.854180	12.3760
$u = 0.29075 + 1.52597I$ $a = -0.056410 + 1.324390I$ $b = 2.03738 - 0.29899I$	$-13.1973 + 11.6833I$	0
$u = 0.29075 - 1.52597I$ $a = -0.056410 - 1.324390I$ $b = 2.03738 + 0.29899I$	$-13.1973 - 11.6833I$	0
$u = 0.27105 + 1.54914I$ $a = 0.016042 - 1.254950I$ $b = -1.94844 + 0.31531I$	$-17.7298 + 6.4903I$	0
$u = 0.27105 - 1.54914I$ $a = 0.016042 + 1.254950I$ $b = -1.94844 - 0.31531I$	$-17.7298 - 6.4903I$	0
$u = 0.23795 + 1.56175I$ $a = 0.049432 + 1.196740I$ $b = 1.85725 - 0.36197I$	$-14.1120 + 1.0765I$	0
$u = 0.23795 - 1.56175I$ $a = 0.049432 - 1.196740I$ $b = 1.85725 + 0.36197I$	$-14.1120 - 1.0765I$	0
$u = 0.362886 + 0.190995I$ $a = 0.83437 - 3.02007I$ $b = -0.879597 + 0.936581I$	$3.51710 + 3.29019I$	$-0.48267 - 6.06797I$

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.362886 - 0.190995I$		
$a = 0.83437 + 3.02007I$	$3.51710 - 3.29019I$	$-0.48267 + 6.06797I$
$b = -0.879597 - 0.936581I$		
$u = 0.154044 + 0.322324I$		
$a = -0.12190 + 2.14236I$	$-1.241050 + 0.560760I$	$-5.36805 - 2.52603I$
$b = 0.709313 - 0.290729I$		
$u = 0.154044 - 0.322324I$		
$a = -0.12190 - 2.14236I$	$-1.241050 - 0.560760I$	$-5.36805 + 2.52603I$
$b = 0.709313 + 0.290729I$		



$$\text{II. } I_2^u = \langle -au + b, 2u^2a + a^2 + au + 2u^2 + 3a + u + 4, u^3 + u^2 + 2u + 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} a \\ au \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} au + a \\ au \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2a - 2u^2 - a - u - 4 \\ au + u^2 + a + 2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} -u^2a - au - a \\ u^2 + a + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} -u^2a - au - u^2 - 2a - 2 \\ -au + 2u^2 + a + 2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^2a - au - 2u^2 - a - u - 4 \\ -u^2a + au + u^2 + a + 2 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $5au + 5u^2 + 2a + 5u + 8$

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_{10}, c_{11}$	$(u^3 - u^2 + 2u - 1)^2$
$c_2, c_9$	$(u^3 + u^2 - 1)^2$
$c_3, c_6, c_7$	$(u^3 + u^2 + 2u + 1)^2$
$c_4$	$u^6 + u^5 + 4u^4 + u^3 + 2u^2 - 2u + 1$
$c_5$	$(u^3 - u^2 + 1)^2$
$c_8$	$u^6$
$c_{12}$	$(u^3 - u - 1)^2$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_3, c_6$ $c_7, c_{10}, c_{11}$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_2, c_5, c_9$	$(y^3 - y^2 + 2y - 1)^2$
$c_4$	$y^6 + 7y^5 + 18y^4 + 21y^3 + 16y^2 + 1$
$c_8$	$y^6$
$c_{12}$	$(y^3 - 2y^2 + y - 1)^2$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.215080 + 1.307140I$ $a = 0.447279 - 0.744862I$ $b = 0.877439 + 0.744862I$	$-5.65624I$	$3.89456 + 5.95889I$
$u = -0.215080 + 1.307140I$ $a = 0.092519 + 0.562280I$ $b = -0.754878$	$-4.13758 - 2.82812I$	$-4.97655 + 4.84887I$
$u = -0.215080 - 1.307140I$ $a = 0.447279 + 0.744862I$ $b = 0.877439 - 0.744862I$	$5.65624I$	$3.89456 - 5.95889I$
$u = -0.215080 - 1.307140I$ $a = 0.092519 - 0.562280I$ $b = -0.754878$	$-4.13758 + 2.82812I$	$-4.97655 - 4.84887I$
$u = -0.569840$ $a = -1.53980 + 1.30714I$ $b = 0.877439 - 0.744862I$	$4.13758 + 2.82812I$	$8.08199 - 1.11003I$
$u = -0.569840$ $a = -1.53980 - 1.30714I$ $b = 0.877439 + 0.744862I$	$4.13758 - 2.82812I$	$8.08199 + 1.11003I$

$$\text{III. } I_3^u = \langle u^2 + b + u + 1, -u^2 + a - 1, u^3 + u^2 + 2u + 1 \rangle$$

(i) Arc colorings

$$a_6 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u^2 + 1 \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} -u \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 - 2u - 1 \\ -u^2 \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u + 2 \\ u \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} -u \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} u^2 + 1 \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 1 \\ -u^2 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} u^2 + 1 \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u \\ -u^2 - u - 1 \end{pmatrix}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes = 0

(iv) u-Polynomials at the component

Crossings	u-Polynomials at each crossing
$c_1, c_{10}, c_{11}$ $c_{12}$	$u^3 - u^2 + 2u - 1$
$c_2, c_9$	$u^3 + u^2 - 1$
$c_3, c_6, c_7$	$u^3 + u^2 + 2u + 1$
$c_4$	$u^3 - 3u^2 + 2u + 1$
$c_5$	$u^3 - u^2 + 1$
$c_8$	$u^3$

(v) Riley Polynomials at the component

Crossings	Riley Polynomials at each crossing
$c_1, c_3, c_6$ $c_7, c_{10}, c_{11}$ $c_{12}$	$y^3 + 3y^2 + 2y - 1$
$c_2, c_5, c_9$	$y^3 - y^2 + 2y - 1$
$c_4$	$y^3 - 5y^2 + 10y - 1$
$c_8$	$y^3$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.215080 + 1.307140I$ $a = -0.662359 - 0.562280I$ $b = 0.877439 - 0.744862I$	0	0
$u = -0.215080 - 1.307140I$ $a = -0.662359 + 0.562280I$ $b = 0.877439 + 0.744862I$	0	0
$u = -0.569840$ $a = 1.32472$ $b = -0.754878$	0	0



#### IV. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1$	$((u^3 - u^2 + 2u - 1)^3)(u^{33} + 46u^{32} + \dots + 42u + 1)$
$c_2$	$((u^3 + u^2 - 1)^3)(u^{33} + 4u^{32} + \dots + 4u - 1)$
$c_3$	$((u^3 + u^2 + 2u + 1)^3)(u^{33} - 4u^{32} + \dots + 10u - 1)$
$c_4$	$(u^3 - 3u^2 + 2u + 1)(u^6 + u^5 + 4u^4 + u^3 + 2u^2 - 2u + 1)$ $\cdot (u^{33} - 3u^{32} + \dots - 31624u - 99623)$
$c_5$	$((u^3 - u^2 + 1)^3)(u^{33} + 4u^{32} + \dots + 4u - 1)$
$c_6, c_7$	$((u^3 + u^2 + 2u + 1)^3)(u^{33} + 4u^{32} + \dots - 6u - 1)$
$c_8$	$u^9(u^{33} + u^{32} + \dots + 128u^2 - 512)$
$c_9$	$((u^3 + u^2 - 1)^3)(u^{33} - 4u^{32} + \dots - 744u - 137)$
$c_{10}$	$((u^3 - u^2 + 2u - 1)^3)(u^{33} + 4u^{32} + \dots - 6u - 1)$
$c_{11}$	$((u^3 - u^2 + 2u - 1)^3)(u^{33} - 4u^{32} + \dots + 10u - 1)$
$c_{12}$	$((u^3 - u - 1)^2)(u^3 - u^2 + 2u - 1)(u^{33} + 26u^{31} + \dots + 1410u - 2071)$

### V. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1$	$((y^3 + 3y^2 + 2y - 1)^3)(y^{33} - 114y^{32} + \dots + 1162y - 1)$
$c_2, c_5$	$((y^3 - y^2 + 2y - 1)^3)(y^{33} - 46y^{32} + \dots + 42y - 1)$
$c_3, c_{11}$	$((y^3 + 3y^2 + 2y - 1)^3)(y^{33} + 26y^{32} + \dots + 42y - 1)$
$c_4$	$(y^3 - 5y^2 + 10y - 1)(y^6 + 7y^5 + 18y^4 + 21y^3 + 16y^2 + 1)$ $\cdot (y^{33} + 39y^{32} + \dots - 19254673246y - 9924742129)$
$c_6, c_7, c_{10}$	$((y^3 + 3y^2 + 2y - 1)^3)(y^{33} + 34y^{32} + \dots - 6y - 1)$
$c_8$	$y^9(y^{33} + 49y^{32} + \dots + 131072y - 262144)$
$c_9$	$((y^3 - y^2 + 2y - 1)^3)(y^{33} + 26y^{32} + \dots - 461086y - 18769)$
$c_{12}$	$(y^3 - 2y^2 + y - 1)^2(y^3 + 3y^2 + 2y - 1)$ $\cdot (y^{33} + 52y^{32} + \dots - 8631988y - 4289041)$