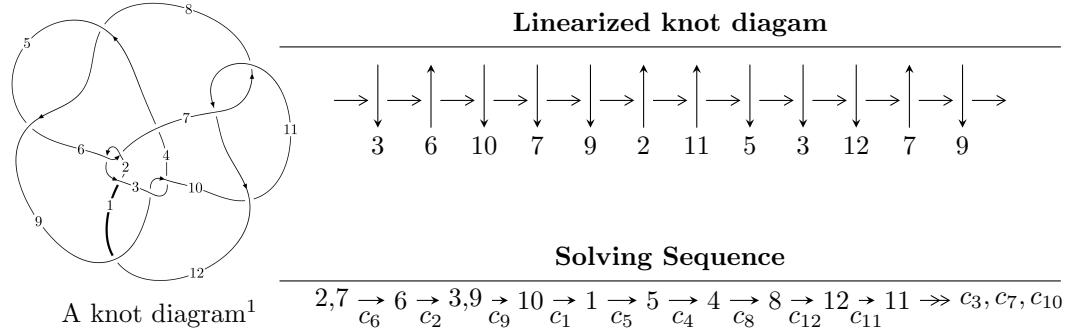


## $12n_{0509}$ ( $K12n_{0509}$ )



### Ideals for irreducible components<sup>2</sup> of $X_{\text{par}}$

$$\begin{aligned}
 I_1^u &= \langle -u^6 + 3u^5 - 6u^4 + 7u^3 - 6u^2 + b + 3u - 1, u^6 - 3u^5 + 6u^4 - 7u^3 + 5u^2 + a - 3u, \\
 &\quad u^7 - 3u^6 + 7u^5 - 10u^4 + 10u^3 - 8u^2 + 3u - 1 \rangle \\
 I_2^u &= \langle -3u^{13} + 14u^{12} + \dots + 4b + 4, u^{13} - 8u^{12} + \dots + 8a - 4, u^{14} - 6u^{13} + \dots - 28u + 8 \rangle \\
 I_3^u &= \langle -u^2 + b + a - 1, a^2 + au + 2u^2 - a + u, u^3 + u^2 + 2u + 1 \rangle \\
 I_4^u &= \langle -u^6 - u^5 - 2u^4 - u^3 - 2u^2 + b - u - 1, u^6 + u^5 + 2u^4 + u^3 + 3u^2 + a + u + 2, \\
 &\quad u^7 + u^6 + 3u^5 + 2u^4 + 4u^3 + 2u^2 + 3u + 1 \rangle \\
 I_5^u &= \langle -2u^2a - au - 2u^2 + b - 2a - u - 4, 2u^2a + a^2 + 2u^2 + 3a + 2u + 4, u^3 + u^2 + 2u + 1 \rangle \\
 I_6^u &= \langle -u^2a + 2u^2 + 2b + u + 3, a^2 + 3u^2 + 3u + 2, u^3 + u^2 + 2u + 1 \rangle \\
 I_7^u &= \langle u^6 + u^5 + 2u^4 + u^3 + u^2 + b + u + 1, -2u^7 - 2u^6 - 5u^5 - 3u^4 - 3u^3 - 3u^2 + a - 4u - 1, \\
 &\quad u^8 + u^7 + 3u^6 + 2u^5 + 3u^4 + 2u^3 + 3u^2 + u + 1 \rangle
 \end{aligned}$$

\* 7 irreducible components of  $\dim_{\mathbb{C}} = 0$ , with total 54 representations.

<sup>1</sup>The image of knot diagram is generated by the software “**Draw programme**” developed by Andrew Bartholomew(<http://www.layer8.co.uk/math/draw/index.htm#Running-draw>), where we modified some parts for our purpose(<https://github.com/CATsTAILs/LinksPainter>).

<sup>2</sup>All coefficients of polynomials are rational numbers. But the coefficients are sometimes approximated in decimal forms when there is not enough margin.

$$\text{I. } I_1^u = \langle -u^6 + 3u^5 - 6u^4 + 7u^3 - 6u^2 + b + 3u - 1, u^6 - 3u^5 + 6u^4 - 7u^3 + 5u^2 + a - 3u, u^7 - 3u^6 + 7u^5 - 10u^4 + 10u^3 - 8u^2 + 3u - 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_7 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u \\ u^3 + u \end{pmatrix} \\ a_9 &= \begin{pmatrix} -u^6 + 3u^5 - 6u^4 + 7u^3 - 5u^2 + 3u \\ u^6 - 3u^5 + 6u^4 - 7u^3 + 6u^2 - 3u + 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} -u^6 + 2u^5 - 4u^4 + 4u^3 - 3u^2 + 2u \\ -u^3 - u \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^3 \\ u^5 + u^3 + u \end{pmatrix} \\ a_5 &= \begin{pmatrix} -u^5 + 2u^4 - 3u^3 + 3u^2 - u + 1 \\ -u^6 + 3u^5 - 6u^4 + 7u^3 - 6u^2 + 3u - 1 \end{pmatrix} \\ a_4 &= \begin{pmatrix} -u^6 + 2u^5 - 4u^4 + 4u^3 - 3u^2 + 2u \\ -u^6 + 3u^5 - 6u^4 + 7u^3 - 6u^2 + 3u - 1 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -u^6 + 2u^5 - 4u^4 + 4u^3 - u^2 + u + 1 \\ -u^2 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} u^5 - 2u^4 + 4u^3 - 4u^2 + 2u - 1 \\ u \end{pmatrix} \\ a_{11} &= \begin{pmatrix} u^5 - 2u^4 + 4u^3 - 4u^2 + u - 1 \\ u \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $2u^6 - 8u^5 + 20u^4 - 32u^3 + 36u^2 - 30u + 5$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_{10}$	$u^7 + 5u^6 + 9u^5 - 2u^4 - 24u^3 - 24u^2 - 7u - 1$
$c_2, c_6, c_7$ $c_{11}$	$u^7 - 3u^6 + 7u^5 - 10u^4 + 10u^3 - 8u^2 + 3u - 1$
$c_3, c_5, c_8$ $c_9$	$u^7 + 5u^6 + 8u^5 + 4u^4 + 2u^3 + 5u^2 + 3u + 1$
$c_4, c_{12}$	$u^7 - u^6 - 8u^5 + 5u^4 + 21u^3 + 14u^2 + 4u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_{10}$	$y^7 - 7y^6 + 53y^5 - 210y^4 + 364y^3 - 244y^2 + y - 1$
$c_2, c_6, c_7$ $c_{11}$	$y^7 + 5y^6 + 9y^5 - 2y^4 - 24y^3 - 24y^2 - 7y - 1$
$c_3, c_5, c_8$ $c_9$	$y^7 - 9y^6 + 28y^5 - 28y^4 + 2y^3 - 21y^2 - y - 1$
$c_4, c_{12}$	$y^7 - 17y^6 + 116y^5 - 325y^4 + 239y^3 - 38y^2 - 12y - 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_1^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.136302 + 1.137730I$	$-4.44698 + 2.33074I$	$-9.82174 - 3.80507I$
$a = -1.052660 + 0.165088I$		
$b = 0.776815 + 0.145062I$		
$u = 0.136302 - 1.137730I$	$-4.44698 - 2.33074I$	$-9.82174 + 3.80507I$
$a = -1.052660 - 0.165088I$		
$b = 0.776815 - 0.145062I$		
$u = 1.24390$		
$a = 0.333091$	$-11.4456$	$-6.73760$
$b = 2.21419$		
$u = 0.194340 + 0.463986I$		
$a = 0.726250 + 0.493271I$	$-0.189704 + 0.962753I$	$-3.67202 - 7.06800I$
$b = 0.096235 - 0.312929I$		
$u = 0.194340 - 0.463986I$		
$a = 0.726250 - 0.493271I$	$-0.189704 - 0.962753I$	$-3.67202 + 7.06800I$
$b = 0.096235 + 0.312929I$		
$u = 0.54741 + 1.45600I$		
$a = 1.65987 + 0.82195I$	$18.5842 + 12.7630I$	$-10.63743 - 5.46514I$
$b = -2.48014 + 0.77211I$		
$u = 0.54741 - 1.45600I$		
$a = 1.65987 - 0.82195I$	$18.5842 - 12.7630I$	$-10.63743 + 5.46514I$
$b = -2.48014 - 0.77211I$		

$$\text{II. } I_2^u = \langle -3u^{13} + 14u^{12} + \dots + 4b + 4, u^{13} - 8u^{12} + \dots + 8a - 4, u^{14} - 6u^{13} + \dots - 28u + 8 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_7 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u \\ u^3 + u \end{pmatrix} \\ a_9 &= \begin{pmatrix} -\frac{1}{8}u^{13} + u^{12} + \dots - \frac{3}{4}u + \frac{1}{2} \\ \frac{3}{4}u^{13} - \frac{7}{2}u^{12} + \dots + 5u - 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} -\frac{1}{8}u^{13} + \frac{3}{4}u^{11} + \dots + \frac{21}{4}u - \frac{3}{2} \\ -\frac{5}{4}u^{13} + \frac{13}{2}u^{12} + \dots - 17u + 5 \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^3 \\ u^5 + u^3 + u \end{pmatrix} \\ a_5 &= \begin{pmatrix} \frac{1}{8}u^{13} - u^{12} + \dots + \frac{21}{4}u - 1 \\ \frac{1}{4}u^{13} - u^{12} + \dots + \frac{13}{2}u - 3 \end{pmatrix} \\ a_4 &= \begin{pmatrix} \frac{3}{8}u^{13} - 2u^{12} + \dots + \frac{47}{4}u - 4 \\ \frac{1}{4}u^{13} - u^{12} + \dots + \frac{13}{2}u - 3 \end{pmatrix} \\ a_8 &= \begin{pmatrix} \frac{5}{8}u^{13} - \frac{13}{4}u^{12} + \dots + 7u - 1 \\ -\frac{1}{2}u^{13} + \frac{5}{2}u^{12} + \dots - \frac{13}{2}u + 3 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} -\frac{1}{8}u^{13} + u^{12} + \dots - \frac{17}{4}u + 1 \\ \frac{3}{4}u^{13} - 4u^{12} + \dots + \frac{31}{2}u - 5 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} -\frac{7}{8}u^{13} + 5u^{12} + \dots - \frac{79}{4}u + 6 \\ \frac{3}{4}u^{13} - 4u^{12} + \dots + \frac{31}{2}u - 5 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

$$(iii) \text{ Cusp Shapes} = -2u^{13} + 12u^{12} - 43u^{11} + 105u^{10} - 196u^9 + 295u^8 - 375u^7 + 414u^6 - 395u^5 + 319u^4 - 217u^3 + 130u^2 - 70u + 18$$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_{10}$	$u^{14} + 8u^{13} + \cdots + 240u + 64$
$c_2, c_6, c_7$ $c_{11}$	$u^{14} - 6u^{13} + \cdots - 28u + 8$
$c_3, c_5, c_8$ $c_9$	$(u^7 - 2u^6 - 3u^5 + 7u^4 - 3u^2 + 1)^2$
$c_4, c_{12}$	$u^{14} - 2u^{13} + \cdots + 15u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_{10}$	$y^{14} + 72y^{12} + \cdots + 13056y + 4096$
$c_2, c_6, c_7$ $c_{11}$	$y^{14} + 8y^{13} + \cdots + 240y + 64$
$c_3, c_5, c_8$ $c_9$	$(y^7 - 10y^6 + 37y^5 - 61y^4 + 46y^3 - 23y^2 + 6y - 1)^2$
$c_4, c_{12}$	$y^{14} - 24y^{13} + \cdots - 53y + 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.625244 + 0.634655I$	$0.091886 + 0.891330I$	$-6.03895 - 5.74662I$
$a = 0.339092 + 0.351980I$		
$b = 0.233020 - 0.022062I$		
$u = 0.625244 - 0.634655I$	$0.091886 - 0.891330I$	$-6.03895 + 5.74662I$
$a = 0.339092 - 0.351980I$		
$b = 0.233020 + 0.022062I$		
$u = -0.378126 + 1.062500I$	$-1.03722 - 4.17104I$	$-10.56067 + 2.18298I$
$a = -1.121480 - 0.077660I$		
$b = 0.404958 + 1.328870I$		
$u = -0.378126 - 1.062500I$	$-1.03722 + 4.17104I$	$-10.56067 - 2.18298I$
$a = -1.121480 + 0.077660I$		
$b = 0.404958 - 1.328870I$		
$u = 0.643460 + 1.009790I$	$-1.03722 + 4.17104I$	$-10.56067 - 2.18298I$
$a = -0.104502 - 0.643158I$		
$b = 0.260752 - 0.097466I$		
$u = 0.643460 - 1.009790I$	$-1.03722 - 4.17104I$	$-10.56067 + 2.18298I$
$a = -0.104502 + 0.643158I$		
$b = 0.260752 + 0.097466I$		
$u = 1.204680 + 0.069237I$	$-16.0632 + 6.5463I$	$-8.56192 - 3.00206I$
$a = -0.311651 - 0.047691I$		
$b = -2.19150 + 0.08443I$		
$u = 1.204680 - 0.069237I$	$-16.0632 - 6.5463I$	$-8.56192 + 3.00206I$
$a = -0.311651 + 0.047691I$		
$b = -2.19150 - 0.08443I$		
$u = -0.321436 + 0.722211I$	$0.091886 + 0.891330I$	$-6.03895 - 5.74662I$
$a = 1.113680 + 0.162158I$		
$b = 0.125852 - 0.871897I$		
$u = -0.321436 - 0.722211I$	$0.091886 - 0.891330I$	$-6.03895 + 5.74662I$
$a = 1.113680 - 0.162158I$		
$b = 0.125852 + 0.871897I$		

Solutions to $I_2^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.63252 + 1.42544I$		
$a = 1.28836 + 1.02059I$	19.2127	$-10.67691 + 0.I$
$b = -2.20148 + 0.65782I$		
$u = 0.63252 - 1.42544I$		
$a = 1.28836 - 1.02059I$	19.2127	$-10.67691 + 0.I$
$b = -2.20148 - 0.65782I$		
$u = 0.59366 + 1.46472I$		
$a = -1.45351 - 0.86882I$	$-16.0632 + 6.5463I$	$-8.56192 - 3.00206I$
$b = 2.36839 - 0.76461I$		
$u = 0.59366 - 1.46472I$		
$a = -1.45351 + 0.86882I$	$-16.0632 - 6.5463I$	$-8.56192 + 3.00206I$
$b = 2.36839 + 0.76461I$		

$$\text{III. } I_3^u = \langle -u^2 + b + a - 1, \ a^2 + au + 2u^2 - a + u, \ u^3 + u^2 + 2u + 1 \rangle$$

(i) Arc colorings

$$a_2 = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

$$a_7 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$a_6 = \begin{pmatrix} 1 \\ u^2 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} u \\ -u^2 - u - 1 \end{pmatrix}$$

$$a_9 = \begin{pmatrix} a \\ u^2 - a + 1 \end{pmatrix}$$

$$a_{10} = \begin{pmatrix} u^2 a + au + 2a - u - 1 \\ u^2 + u + 1 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} -u^2 - 2u - 1 \\ u^2 + 2u \end{pmatrix}$$

$$a_5 = \begin{pmatrix} u^2 a + au + u^2 + a - u \\ -u^2 + a - 1 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} u^2 a + au + 2a - u - 1 \\ -u^2 + a - 1 \end{pmatrix}$$

$$a_8 = \begin{pmatrix} -u^2 a - au + 3u^2 - a + 3u + 2 \\ -u^2 \end{pmatrix}$$

$$a_{12} = \begin{pmatrix} -u^2 a + u^2 - a - u - 1 \\ u \end{pmatrix}$$

$$a_{11} = \begin{pmatrix} -u^2 a + u^2 - a - 2u - 1 \\ u \end{pmatrix}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $8u^2 + 8u + 2$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_{10}$	$(u^3 + 3u^2 + 2u - 1)^2$
$c_2, c_6, c_7$ $c_{11}$	$(u^3 + u^2 + 2u + 1)^2$
$c_3, c_5, c_8$ $c_9$	$u^6 + u^5 - 2u^4 + 5u^3 + 14u^2 - 8$
$c_4, c_{12}$	$u^6 - 4u^5 - 3u^4 + 14u^3 + 14u^2 + 2u - 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_{10}$	$(y^3 - 5y^2 + 10y - 1)^2$
$c_2, c_6, c_7$ $c_{11}$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_3, c_5, c_8$ $c_9$	$y^6 - 5y^5 + 22y^4 - 97y^3 + 228y^2 - 224y + 64$
$c_4, c_{12}$	$y^6 - 22y^5 + 149y^4 - 266y^3 + 146y^2 - 32y + 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_3^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.215080 + 1.307140I$		
$a = -1.276330 - 0.394337I$	$-10.98310 - 5.65624I$	$-13.0195 + 5.9589I$
$b = 0.613967 - 0.167943I$		
$u = -0.215080 + 1.307140I$		
$a = 2.49141 - 0.91280I$	$-10.98310 - 5.65624I$	$-13.0195 + 5.9589I$
$b = -3.15376 + 0.35052I$		
$u = -0.215080 - 1.307140I$		
$a = -1.276330 + 0.394337I$	$-10.98310 + 5.65624I$	$-13.0195 - 5.9589I$
$b = 0.613967 + 0.167943I$		
$u = -0.215080 - 1.307140I$		
$a = 2.49141 + 0.91280I$	$-10.98310 + 5.65624I$	$-13.0195 - 5.9589I$
$b = -3.15376 - 0.35052I$		
$u = -0.569840$		
$a = 1.51738$	$-2.70789$	0.0390210
$b = -0.192667$		
$u = -0.569840$		
$a = 0.0524558$	$-2.70789$	0.0390210
$b = 1.27226$		

$$\text{IV. } I_4^u = \langle -u^6 - u^5 - 2u^4 - u^3 - 2u^2 + b - u - 1, u^6 + u^5 + 2u^4 + u^3 + 3u^2 + a + u + 2, u^7 + u^6 + 3u^5 + 2u^4 + 4u^3 + 2u^2 + 3u + 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_7 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u \\ u^3 + u \end{pmatrix} \\ a_9 &= \begin{pmatrix} -u^6 - u^5 - 2u^4 - u^3 - 3u^2 - u - 2 \\ u^6 + u^5 + 2u^4 + u^3 + 2u^2 + u + 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} -u^6 - 2u^4 - 3u^2 - 2 \\ -u^3 - u \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^3 \\ u^5 + u^3 + u \end{pmatrix} \\ a_5 &= \begin{pmatrix} 2u^6 + u^5 + 4u^4 + u^3 + 5u^2 + u + 3 \\ -u^6 - u^5 - 2u^4 - u^3 - 2u^2 - u - 1 \end{pmatrix} \\ a_4 &= \begin{pmatrix} u^6 + 2u^4 + 3u^2 + 2 \\ -u^6 - u^5 - 2u^4 - u^3 - 2u^2 - u - 1 \end{pmatrix} \\ a_8 &= \begin{pmatrix} u^6 + 2u^4 + 3u^2 - u + 1 \\ -u^2 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} u^5 + 2u^3 + 2u - 1 \\ -u \end{pmatrix} \\ a_{11} &= \begin{pmatrix} u^5 + 2u^3 + 3u - 1 \\ -u \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = 1

(iii) Cusp Shapes =  $2u^6 - 4u^3 - 4u^2 - 2u - 11$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_{10}$	$u^7 - 5u^6 + 13u^5 - 22u^4 + 24u^3 - 16u^2 + 5u + 1$
$c_2, c_7$	$u^7 - u^6 + 3u^5 - 2u^4 + 4u^3 - 2u^2 + 3u - 1$
$c_3, c_8$	$u^7 + u^6 - 4u^5 - 4u^4 + 6u^3 + 3u^2 - 3u + 1$
$c_4$	$u^7 + u^6 - u^4 + 3u^3 - 4u^2 + 2u - 1$
$c_5, c_9$	$u^7 - u^6 - 4u^5 + 4u^4 + 6u^3 - 3u^2 - 3u - 1$
$c_6, c_{11}$	$u^7 + u^6 + 3u^5 + 2u^4 + 4u^3 + 2u^2 + 3u + 1$
$c_{12}$	$u^7 - u^6 + u^4 + 3u^3 + 4u^2 + 2u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_{10}$	$y^7 + y^6 - 3y^5 - 10y^4 + 12y^3 + 28y^2 + 57y - 1$
$c_2, c_6, c_7$ $c_{11}$	$y^7 + 5y^6 + 13y^5 + 22y^4 + 24y^3 + 16y^2 + 5y - 1$
$c_3, c_5, c_8$ $c_9$	$y^7 - 9y^6 + 36y^5 - 76y^4 + 82y^3 - 37y^2 + 3y - 1$
$c_4, c_{12}$	$y^7 - y^6 + 8y^5 + 11y^4 + 3y^3 - 6y^2 - 4y - 1$

(vi) Complex Volumes and Cusp Shapes

Solutions to $I_4^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.537424 + 0.962927I$		
$a = -0.662429 - 0.173919I$	$-0.10098 + 4.46523I$	$-0.57946 - 5.51833I$
$b = 0.300834 - 0.861081I$		
$u = 0.537424 - 0.962927I$		
$a = -0.662429 + 0.173919I$	$-0.10098 - 4.46523I$	$-0.57946 + 5.51833I$
$b = 0.300834 + 0.861081I$		
$u = -0.132251 + 1.213860I$		
$a = 1.92214 - 0.71131I$	$-10.69670 - 3.59676I$	$-12.51315 + 1.73858I$
$b = -1.46616 + 1.03238I$		
$u = -0.132251 - 1.213860I$		
$a = 1.92214 + 0.71131I$	$-10.69670 + 3.59676I$	$-12.51315 - 1.73858I$
$b = -1.46616 - 1.03238I$		
$u = -0.723592 + 0.997572I$		
$a = -0.752025 + 0.832832I$	$-3.82765 - 5.64420I$	$-9.10487 + 5.57424I$
$b = 0.223589 + 0.610838I$		
$u = -0.723592 - 0.997572I$		
$a = -0.752025 - 0.832832I$	$-3.82765 + 5.64420I$	$-9.10487 - 5.57424I$
$b = 0.223589 - 0.610838I$		
$u = -0.363162$		
$a = -2.01537$	$-3.64806$	$-10.6050$
$b = 0.883481$		

$$\text{V. } I_5^u = \langle -2u^2a - au - 2u^2 + b - 2a - u - 4, 2u^2a + a^2 + 2u^2 + 3a + 2u + 4, u^3 + u^2 + 2u + 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_7 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u \\ -u^2 - u - 1 \end{pmatrix} \\ a_9 &= \begin{pmatrix} a \\ 2u^2a + au + 2u^2 + 2a + u + 4 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} u^2a + au - u^2 + a - 1 \\ u^2a + au + u^2 + 2a + 2 \end{pmatrix} \\ a_1 &= \begin{pmatrix} -u^2 - 2u - 1 \\ u^2 + 2u \end{pmatrix} \\ a_5 &= \begin{pmatrix} u^2a + 4u^2 + 2a + 2u + 7 \\ -u^2a - 2u^2 - 2a - 2u - 4 \end{pmatrix} \\ a_4 &= \begin{pmatrix} 2u^2 + 3 \\ -u^2a - 2u^2 - 2a - 2u - 4 \end{pmatrix} \\ a_8 &= \begin{pmatrix} 4u^2a + 2au + 8u^2 + 8a + 3u + 14 \\ u^2a + 2u^2 + a + u + 4 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} u^2a + u^2 + 2a + 1 \\ 2u^2a + au + 3u^2 + 3a + u + 5 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} -u^2a - au - 2u^2 - a - u - 4 \\ 2u^2a + au + 3u^2 + 3a + u + 5 \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $4u^2a + 4au + 8u^2 + 8a + 4u + 6$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_{10}$	$(u^3 + 3u^2 + 2u - 1)^2$
$c_2, c_6, c_7$ $c_{11}$	$(u^3 + u^2 + 2u + 1)^2$
$c_3, c_9, c_{12}$	$(u^3 - u^2 - 4u + 5)^2$
$c_4$	$u^6 + u^5 + 6u^4 - 3u^3 + 10u^2 + 8$
$c_5, c_8$	$(u^3 + u^2 - 1)^2$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_{10}$	$(y^3 - 5y^2 + 10y - 1)^2$
$c_2, c_6, c_7$ $c_{11}$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_3, c_9, c_{12}$	$(y^3 - 9y^2 + 26y - 25)^2$
$c_4$	$y^6 + 11y^5 + 62y^4 + 127y^3 + 196y^2 + 160y + 64$
$c_5, c_8$	$(y^3 - y^2 + 2y - 1)^2$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_5^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.215080 + 1.307140I$		
$a = 0.824718 - 0.424452I$	$-6.84548 - 2.82812I$	$-6.49024 + 2.97945I$
$b = -0.732199 + 0.986732I$		
$u = -0.215080 + 1.307140I$		
$a = -0.50000 + 1.54901I$	$-10.9831$	$-13.01951 + 0.I$
$b = 0.94728 - 2.29387I$		
$u = -0.215080 - 1.307140I$		
$a = 0.824718 + 0.424452I$	$-6.84548 + 2.82812I$	$-6.49024 - 2.97945I$
$b = -0.732199 - 0.986732I$		
$u = -0.215080 - 1.307140I$		
$a = -0.50000 - 1.54901I$	$-10.9831$	$-13.01951 + 0.I$
$b = 0.94728 + 2.29387I$		
$u = -0.569840$		
$a = -1.82472 + 0.42445I$	$-6.84548 - 2.82812I$	$-6.49024 + 2.97945I$
$b = 0.284920 + 0.882689I$		
$u = -0.569840$		
$a = -1.82472 - 0.42445I$	$-6.84548 + 2.82812I$	$-6.49024 - 2.97945I$
$b = 0.284920 - 0.882689I$		

$$\text{VI. } I_6^u = \langle -u^2a + 2u^2 + 2b + u + 3, a^2 + 3u^2 + 3u + 2, u^3 + u^2 + 2u + 1 \rangle$$

(i) Arc colorings

$$\begin{aligned} a_2 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_7 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u \\ -u^2 - u - 1 \end{pmatrix} \\ a_9 &= \begin{pmatrix} a \\ \frac{1}{2}u^2a - u^2 - \frac{1}{2}u - \frac{3}{2} \end{pmatrix} \\ a_{10} &= \begin{pmatrix} \frac{1}{2}u^2a + \frac{1}{2}au + \frac{3}{2}a + \frac{1}{2} \\ \frac{1}{2}u^2a - \frac{1}{2}u^2 + \frac{1}{2}a - \frac{1}{2}u - 1 \end{pmatrix} \\ a_1 &= \begin{pmatrix} -u^2 - 2u - 1 \\ u^2 + 2u \end{pmatrix} \\ a_5 &= \begin{pmatrix} u^2a + \frac{1}{2}au + 2u^2 + \frac{3}{2}a + \frac{3}{2}u + 1 \\ -2u^2 - u - 2 \end{pmatrix} \\ a_4 &= \begin{pmatrix} u^2a + \frac{1}{2}au + \frac{3}{2}a + \frac{1}{2}u - 1 \\ -2u^2 - u - 2 \end{pmatrix} \\ a_8 &= \begin{pmatrix} \frac{1}{2}u^2 - \frac{3}{2}a + u - \frac{1}{2} \\ \frac{1}{2}u^2 + \frac{1}{2}a + \frac{3}{2} \end{pmatrix} \\ a_{12} &= \begin{pmatrix} \frac{1}{2}au + \frac{1}{2}u^2 + \frac{3}{2}u + \frac{3}{2} \\ \frac{1}{2}u^2a + a - \frac{1}{2}u - \frac{1}{2} \end{pmatrix} \\ a_{11} &= \begin{pmatrix} -\frac{1}{2}u^2a + \frac{1}{2}au + \frac{1}{2}u^2 - a + 2u + 2 \\ \frac{1}{2}u^2a + a - \frac{1}{2}u - \frac{1}{2} \end{pmatrix} \end{aligned}$$

(ii) Obstruction class = -1

(iii) Cusp Shapes =  $2u^2a + 2u^2 + 2a + 2u - 6$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_{10}$	$(u^3 + 3u^2 + 2u - 1)^2$
$c_2, c_6, c_7$ $c_{11}$	$(u^3 + u^2 + 2u + 1)^2$
$c_3, c_9$	$(u^3 + u^2 - 1)^2$
$c_4, c_5, c_8$	$(u^3 - u^2 - 4u + 5)^2$
$c_{12}$	$u^6 + u^5 + 6u^4 - 3u^3 + 10u^2 + 8$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_{10}$	$(y^3 - 5y^2 + 10y - 1)^2$
$c_2, c_6, c_7$ $c_{11}$	$(y^3 + 3y^2 + 2y - 1)^2$
$c_3, c_9$	$(y^3 - y^2 + 2y - 1)^2$
$c_4, c_5, c_8$	$(y^3 - 9y^2 + 26y - 25)^2$
$c_{12}$	$y^6 + 11y^5 + 62y^4 + 127y^3 + 196y^2 + 160y + 64$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_6^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = -0.215080 + 1.307140I$		
$a = 1.98708 - 0.56228I$	-10.9831	$-13.01951 + 0.I$
$b = -1.53980 - 0.18258I$		
$u = -0.215080 + 1.307140I$		
$a = -1.98708 + 0.56228I$	-6.84548 - 2.82812I	$-6.49024 + 2.97945I$
$b = 2.07960$		
$u = -0.215080 - 1.307140I$		
$a = 1.98708 + 0.56228I$	-10.9831	$-13.01951 + 0.I$
$b = -1.53980 + 0.18258I$		
$u = -0.215080 - 1.307140I$		
$a = -1.98708 - 0.56228I$	-6.84548 + 2.82812I	$-6.49024 - 2.97945I$
$b = 2.07960$		
$u = -0.569840$		
$a = 1.124560I$	-6.84548 - 2.82812I	$-6.49024 + 2.97945I$
$b = -1.53980 + 0.18258I$		
$u = -0.569840$		
$a = -1.124560I$	-6.84548 + 2.82812I	$-6.49024 - 2.97945I$
$b = -1.53980 - 0.18258I$		

$$\text{VII. } I_7^u = \langle u^6 + u^5 + 2u^4 + u^3 + u^2 + b + u + 1, -2u^7 - 2u^6 + \dots + a - 1, u^8 + u^7 + 3u^6 + 2u^5 + 3u^4 + 2u^3 + 3u^2 + u + 1 \rangle$$

(i) **Arc colorings**

$$\begin{aligned} a_2 &= \begin{pmatrix} 0 \\ u \end{pmatrix} \\ a_7 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ a_6 &= \begin{pmatrix} 1 \\ u^2 \end{pmatrix} \\ a_3 &= \begin{pmatrix} u \\ u^3 + u \end{pmatrix} \\ a_9 &= \begin{pmatrix} 2u^7 + 2u^6 + 5u^5 + 3u^4 + 3u^3 + 3u^2 + 4u + 1 \\ -u^6 - u^5 - 2u^4 - u^3 - u^2 - u - 1 \end{pmatrix} \\ a_{10} &= \begin{pmatrix} u^7 + u^6 + 2u^5 + u^4 + u^3 + u^2 + 2u \\ -u^7 - 2u^6 - 3u^5 - 4u^4 - 2u^3 - 3u^2 - 2u - 2 \end{pmatrix} \\ a_1 &= \begin{pmatrix} u^3 \\ u^5 + u^3 + u \end{pmatrix} \\ a_5 &= \begin{pmatrix} 2u^7 + 3u^6 + 6u^5 + 5u^4 + 5u^3 + 4u^2 + 6u + 3 \\ -u^6 - u^5 - 2u^4 - u^3 - u^2 - u - 2 \end{pmatrix} \\ a_4 &= \begin{pmatrix} 2u^7 + 2u^6 + 5u^5 + 3u^4 + 4u^3 + 3u^2 + 5u + 1 \\ -u^6 - u^5 - 2u^4 - u^3 - u^2 - u - 2 \end{pmatrix} \\ a_8 &= \begin{pmatrix} -3u^6 - 2u^5 - 7u^4 - 3u^3 - 5u^2 - 3u - 5 \\ -u^7 - 2u^5 + u^4 - u^3 + u^2 - u + 2 \end{pmatrix} \\ a_{12} &= \begin{pmatrix} -2u^7 - u^6 - 4u^5 - u^4 - 2u^3 - u^2 - 3u + 1 \\ u^7 + u^6 + 3u^5 + 2u^4 + 3u^3 + 2u^2 + 3u + 1 \end{pmatrix} \\ a_{11} &= \begin{pmatrix} -3u^7 - 2u^6 - 7u^5 - 3u^4 - 5u^3 - 3u^2 - 6u \\ u^7 + u^6 + 3u^5 + 2u^4 + 3u^3 + 2u^2 + 3u + 1 \end{pmatrix} \end{aligned}$$

(ii) **Obstruction class = 1**

(iii) **Cusp Shapes** =  $-u^7 - 4u^6 - 5u^5 - 10u^4 - 7u^3 - 7u^2 - 5u - 14$

**(iv) u-Polynomials at the component**

Crossings	u-Polynomials at each crossing
$c_1, c_{10}$	$u^8 - 5u^7 + 11u^6 - 16u^5 + 19u^4 - 16u^3 + 11u^2 - 5u + 1$
$c_2, c_7$	$u^8 - u^7 + 3u^6 - 2u^5 + 3u^4 - 2u^3 + 3u^2 - u + 1$
$c_3, c_8$	$(u^4 - u^3 - 2u^2 + 2u + 1)^2$
$c_4$	$u^8 - 4u^7 + 5u^6 - u^5 - u^4 - u^3 + 4u^2 - 3u + 1$
$c_5, c_9$	$(u^4 + u^3 - 2u^2 - 2u + 1)^2$
$c_6, c_{11}$	$u^8 + u^7 + 3u^6 + 2u^5 + 3u^4 + 2u^3 + 3u^2 + u + 1$
$c_{12}$	$u^8 + 4u^7 + 5u^6 + u^5 - u^4 + u^3 + 4u^2 + 3u + 1$

**(v) Riley Polynomials at the component**

Crossings	Riley Polynomials at each crossing
$c_1, c_{10}$	$y^8 - 3y^7 - y^6 + 24y^5 + 43y^4 + 24y^3 - y^2 - 3y + 1$
$c_2, c_6, c_7$ $c_{11}$	$y^8 + 5y^7 + 11y^6 + 16y^5 + 19y^4 + 16y^3 + 11y^2 + 5y + 1$
$c_3, c_5, c_8$ $c_9$	$(y^4 - 5y^3 + 10y^2 - 8y + 1)^2$
$c_4, c_{12}$	$y^8 - 6y^7 + 15y^6 - 11y^5 + 17y^4 - 5y^3 + 8y^2 - y + 1$

**(vi) Complex Volumes and Cusp Shapes**

Solutions to $I_7^u$	$\sqrt{-1}(\text{vol} + \sqrt{-1}CS)$	Cusp shape
$u = 0.589362 + 0.807869I$		
$a = 0.618883 - 0.314577I$	0.398187	$-4.14403 + 0.I$
$b = 0.500000 + 0.685376I$		
$u = 0.589362 - 0.807869I$		
$a = 0.618883 + 0.314577I$	0.398187	$-4.14403 + 0.I$
$b = 0.500000 - 0.685376I$		
$u = -0.756438 + 0.654065I$		
$a = 0.313901 - 0.842956I$	-2.83064	$-8.01125 + 0.I$
$b = 0.500000 - 0.432332I$		
$u = -0.756438 - 0.654065I$		
$a = 0.313901 + 0.842956I$	-2.83064	$-8.01125 + 0.I$
$b = 0.500000 + 0.432332I$		
$u = -0.112930 + 0.707515I$		
$a = -0.42892 + 2.19402I$	$-8.65338 + 2.52742I$	$-12.42236 - 1.86858I$
$b = -0.779254 - 0.555127I$		
$u = -0.112930 - 0.707515I$		
$a = -0.42892 - 2.19402I$	$-8.65338 - 2.52742I$	$-12.42236 + 1.86858I$
$b = -0.779254 + 0.555127I$		
$u = -0.219994 + 1.378280I$		
$a = -1.50387 + 0.55124I$	$-8.65338 - 2.52742I$	$-12.42236 + 1.86858I$
$b = 1.77925 - 0.55513I$		
$u = -0.219994 - 1.378280I$		
$a = -1.50387 - 0.55124I$	$-8.65338 + 2.52742I$	$-12.42236 - 1.86858I$
$b = 1.77925 + 0.55513I$		

### VIII. u-Polynomials

Crossings	u-Polynomials at each crossing
$c_1, c_{10}$	$((u^3 + 3u^2 + 2u - 1)^6)(u^7 - 5u^6 + \dots + 5u + 1)$ $\cdot (u^7 + 5u^6 + 9u^5 - 2u^4 - 24u^3 - 24u^2 - 7u - 1)$ $\cdot (u^8 - 5u^7 + 11u^6 - 16u^5 + 19u^4 - 16u^3 + 11u^2 - 5u + 1)$ $\cdot (u^{14} + 8u^{13} + \dots + 240u + 64)$
$c_2, c_7$	$(u^3 + u^2 + 2u + 1)^6(u^7 - 3u^6 + 7u^5 - 10u^4 + 10u^3 - 8u^2 + 3u - 1)$ $\cdot (u^7 - u^6 + 3u^5 - 2u^4 + 4u^3 - 2u^2 + 3u - 1)$ $\cdot (u^8 - u^7 + 3u^6 - 2u^5 + 3u^4 - 2u^3 + 3u^2 - u + 1)$ $\cdot (u^{14} - 6u^{13} + \dots - 28u + 8)$
$c_3, c_8$	$(u^3 - u^2 - 4u + 5)^2(u^3 + u^2 - 1)^2(u^4 - u^3 - 2u^2 + 2u + 1)^2$ $\cdot (u^6 + u^5 - 2u^4 + 5u^3 + 14u^2 - 8)(u^7 - 2u^6 - 3u^5 + 7u^4 - 3u^2 + 1)^2$ $\cdot (u^7 + u^6 - 4u^5 - 4u^4 + 6u^3 + 3u^2 - 3u + 1)$ $\cdot (u^7 + 5u^6 + 8u^5 + 4u^4 + 2u^3 + 5u^2 + 3u + 1)$
$c_4$	$(u^3 - u^2 - 4u + 5)^2(u^6 - 4u^5 - 3u^4 + 14u^3 + 14u^2 + 2u - 1)$ $\cdot (u^6 + u^5 + 6u^4 - 3u^3 + 10u^2 + 8)$ $\cdot (u^7 - u^6 - 8u^5 + 5u^4 + 21u^3 + 14u^2 + 4u + 1)$ $\cdot (u^7 + u^6 - u^4 + 3u^3 - 4u^2 + 2u - 1)$ $\cdot (u^8 - 4u^7 + 5u^6 - u^5 - u^4 - u^3 + 4u^2 - 3u + 1)$ $\cdot (u^{14} - 2u^{13} + \dots + 15u + 1)$
$c_5, c_9$	$(u^3 - u^2 - 4u + 5)^2(u^3 + u^2 - 1)^2(u^4 + u^3 - 2u^2 - 2u + 1)^2$ $\cdot (u^6 + u^5 - 2u^4 + 5u^3 + 14u^2 - 8)(u^7 - 2u^6 - 3u^5 + 7u^4 - 3u^2 + 1)^2$ $\cdot (u^7 - u^6 - 4u^5 + 4u^4 + 6u^3 - 3u^2 - 3u - 1)$ $\cdot (u^7 + 5u^6 + 8u^5 + 4u^4 + 2u^3 + 5u^2 + 3u + 1)$
$c_6, c_{11}$	$(u^3 + u^2 + 2u + 1)^6(u^7 - 3u^6 + 7u^5 - 10u^4 + 10u^3 - 8u^2 + 3u - 1)$ $\cdot (u^7 + u^6 + 3u^5 + 2u^4 + 4u^3 + 2u^2 + 3u + 1)$ $\cdot (u^8 + u^7 + 3u^6 + 2u^5 + 3u^4 + 2u^3 + 3u^2 + u + 1)$ $\cdot (u^{14} - 6u^{13} + \dots - 28u + 8)$
$c_{12}$	$(u^3 - u^2 - 4u + 5)^2(u^6 - 4u^5 - 3u^4 + 14u^3 + 14u^2 + 2u - 1)$ $\cdot (u^6 + u^5 + 6u^4 - 3u^3 + 10u^2 + 8)(u^7 - u^6 + u^4 + 3u^3 + 4u^2 + 2u + 1)$ $\cdot (u^7 - u^6 - 8u^5 + 5u^4 + 21u^3 + 14u^2 + 4u + 1)$ $\cdot (u^8 + 4u^7 + 5u^6 + u^5 - u^4 + u^3 + 4u^2 + 3u + 1)$ $\cdot (u^{14} - 2u^{13} + \dots + 15u + 1)$

## IX. Riley Polynomials

Crossings	Riley Polynomials at each crossing
$c_1, c_{10}$	$(y^3 - 5y^2 + 10y - 1)^6$ $\cdot (y^7 - 7y^6 + 53y^5 - 210y^4 + 364y^3 - 244y^2 + y - 1)$ $\cdot (y^7 + y^6 - 3y^5 - 10y^4 + 12y^3 + 28y^2 + 57y - 1)$ $\cdot (y^8 - 3y^7 - y^6 + 24y^5 + 43y^4 + 24y^3 - y^2 - 3y + 1)$ $\cdot (y^{14} + 72y^{12} + \dots + 13056y + 4096)$
$c_2, c_6, c_7$ $c_{11}$	$(y^3 + 3y^2 + 2y - 1)^6(y^7 + 5y^6 + 9y^5 - 2y^4 - 24y^3 - 24y^2 - 7y - 1)$ $\cdot (y^7 + 5y^6 + 13y^5 + 22y^4 + 24y^3 + 16y^2 + 5y - 1)$ $\cdot (y^8 + 5y^7 + 11y^6 + 16y^5 + 19y^4 + 16y^3 + 11y^2 + 5y + 1)$ $\cdot (y^{14} + 8y^{13} + \dots + 240y + 64)$
$c_3, c_5, c_8$ $c_9$	$(y^3 - 9y^2 + 26y - 25)^2(y^3 - y^2 + 2y - 1)^2$ $\cdot (y^4 - 5y^3 + 10y^2 - 8y + 1)^2$ $\cdot (y^6 - 5y^5 + 22y^4 - 97y^3 + 228y^2 - 224y + 64)$ $\cdot (y^7 - 10y^6 + 37y^5 - 61y^4 + 46y^3 - 23y^2 + 6y - 1)^2$ $\cdot (y^7 - 9y^6 + 28y^5 - 28y^4 + 2y^3 - 21y^2 - y - 1)$ $\cdot (y^7 - 9y^6 + 36y^5 - 76y^4 + 82y^3 - 37y^2 + 3y - 1)$
$c_4, c_{12}$	$(y^3 - 9y^2 + 26y - 25)^2$ $\cdot (y^6 - 22y^5 + 149y^4 - 266y^3 + 146y^2 - 32y + 1)$ $\cdot (y^6 + 11y^5 + 62y^4 + 127y^3 + 196y^2 + 160y + 64)$ $\cdot (y^7 - 17y^6 + 116y^5 - 325y^4 + 239y^3 - 38y^2 - 12y - 1)$ $\cdot (y^7 - y^6 + 8y^5 + 11y^4 + 3y^3 - 6y^2 - 4y - 1)$ $\cdot (y^8 - 6y^7 + 15y^6 - 11y^5 + 17y^4 - 5y^3 + 8y^2 - y + 1)$ $\cdot (y^{14} - 24y^{13} + \dots - 53y + 1)$